ANTIPROTON MOMENTUM DISTRIBUTIONS AS A MEASURE OF ELECTRON COOLING FORCE AT THE FERMILAB RECYCLER*

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Abstract

The Fermilab Recycler is a fixed 8 GeV kinetic energy storage ring located in the Fermilab Main Injector tunnel near the ceiling. Electron cooling of high energy antiprotons has recently been demonstrated [1] at the Recycler. Antiproton beam Schottky signals were used to measure the antiproton momentum distribution at equilibrium between a calibrated broadband diffusion source and electron cooling. The large Recycler momentum aperture, the dependence of the electron cooling force as a function of the antiproton momentum deviation and the calibrated diffusion source combine to give a unique spectral measurement of the antiproton beam momentum distribution.

INTRODUCTION

The beam distribution function, ψ , is governed by the Fokker-Planck equation,

$$\frac{\partial \psi}{\partial t}=\frac{\partial}{\partial p}\left(-F\psi+\frac{D}{2}\frac{\partial \psi}{\partial p}\right) \qquad (1)$$

where the distribution function is normalized and the $\lim_{p\to\infty}(p^2\psi)=0$ so that the second moment exists and is finite. F and D are the cooling and diffusion force coefficients respectively.

With no cooling force, F=0, and a constant diffusion, the time evolution of Eq. (1) gives, after integration over p and t,

$$w^2=w_0^2+Dt \qquad (2)$$

where w^2 is the second moment of the distribution function. With constant non-zero cooling and diffusion sources in equilibrium, the Fokker-Planck equation reduces to

$$F=\frac{D}{2}\frac{d\ln(\psi)}{dp}. \qquad (3)$$

The right hand side of Eq. (3) is simply the derivative of an appropriately scaled equilibrium Schottky beam momentum spectrum. The essence of this measurement is a programmable constant diffusion source, measured with Eq. (2) and in equilibrium with the electron cooling force, Eq. (3).

Table 1: Electron Cooler Parameters

Parameter	Symbol	Value	Unit
Electron energy	E_b	4.34	MeV
Beam current (for cooling)	I_b	0.1	A
Terminal voltage ripple, rms	δU	250	V
Cooling section (CS) length	L	20	m
Solenoid field in CS	B_{cs}	105	G
Beam radius in CS	r_b	3.5	mm

EXPERIMENTAL CONDITIONS

Electron cooling of 8.9 GeV/c antiprotons requires a dc electron beam with kinetic energy of 4.3 MeV and a beam current of 0.1-0.5 A. The main parameters of the cooler [2] are listed in Table 1. For this experiment, there were ~9.5e10 antiprotons stored un-bunched in the Recycler. The antiproton 95% transverse emittance was controlled by the betatron stochastic cooling systems and remained less than 1.0 π mm-mrad throughout the experiment.

DIFFUSION SOURCE

Electron cooling requires that the electron and antiproton beams have the same velocity, are colinear and have similar transverse sizes. The Recycler momentum aperture is ~1.5% of the beam momentum and betatron stochastic cooling for small intensities, ~1e11, can attain 95% transverse emittances less than 3π mm mrad (< 11 mm antiproton beam diameter in the electron cooling solenoids). To improve the chances of detecting electron cooling, a method for filling the Recycler momentum aperture with a low intensity beam and small transverse emittance was implemented. The same noise source will be used for this cooling force measurement.

Programmable Noise Source

The Agilent Vector Signal Analyzer (VSA) has a digital noise source which may be configured to generate noise which has a nearly uniform power density over a given bandwidth, Fig. 1. The total width of the distributions in Fig. 1 is 2.4 times the respective resolution bandwidths. It

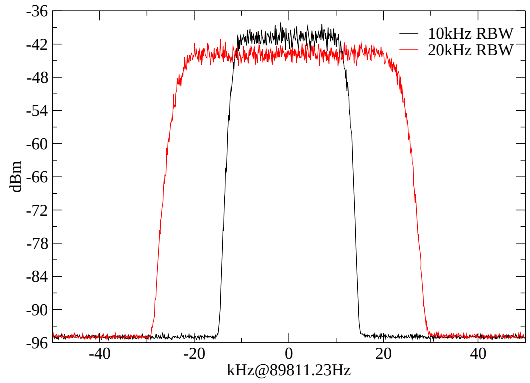

Figure 1: Spectrum of VSA noise source for two different resolution bandwidths as detected by a swept spectrum analyzer. The VSA source power is kept constant between the two traces. The VSA noise source and the spectrum analyzer are centered on the Recycler revolution frequency.

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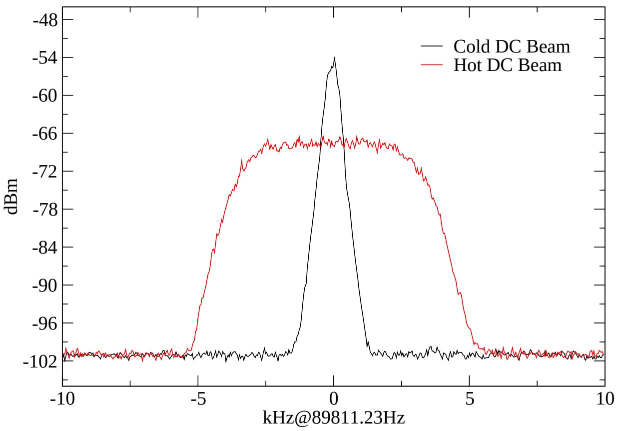

Figure 2:

is immediately apparent that the integrated power in both traces of Fig. 1 is the same (3dB is equivalent to linear factor of two). The power density, mW/Hz, is obviously tunable and limited only by the VSA source rms power. This digital noise source is used to drive the Recycler wide-band RF system [3]. By limiting the resolution bandwidth of the noise source, noise power is applied only to the longitudinal beam momentum.

Noise Source Example

Fig. 2 is an example of the digital noise source applied to stochastically cooled dc antiprotons. Flying wire measurements indicated that the transverse emittance remained constant throughout this test. Using this method to fill the Recycler momentum aperture allowed for the first detection of medium energy electron cooling, measurements of chromaticity and dynamic momentum aperture. The stochastically cooled beam shown in Fig. 2 has a fractional frequency width $\Delta f / f_0 \sim 10^{-3}$ which is 100 times larger than that achievable with electron cooling.

EXPERIMENTAL PROCEDURE

The antiproton beam is un-bunched (no RF structure) and the betatron motion is stochastically cooled. Ion trapping prohibits large antiproton DC currents so this experiment was performed with ~9.5e10 antiprotons. The noise source applied is intended to be much larger than other diffusion sources. This is another reason for a low antiproton beam current which decreases the diffusion from intrabeam scattering.

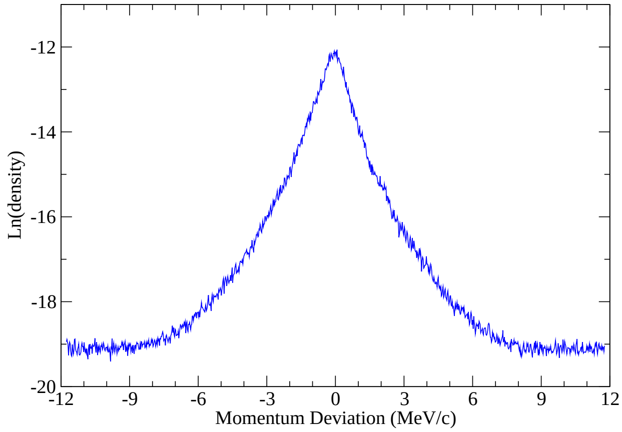

Figure 3:

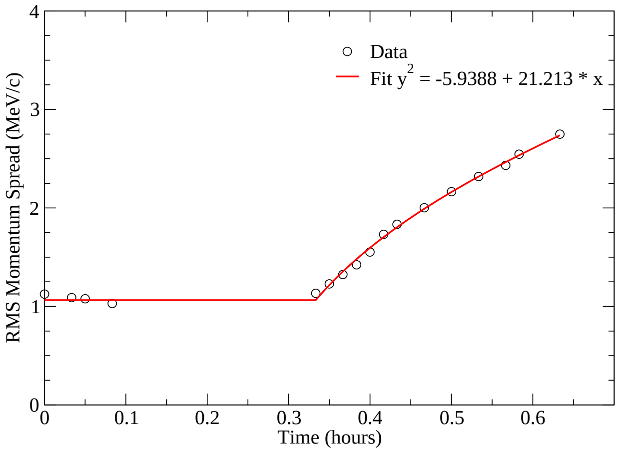

Figure 4:

Equilibrium

After the small emittance antiproton beam is cooled with the electron beam, the noise source is applied and adjusted. Fig. 3 shows the measured equilibrium momentum distribution measured with a Schottky detector and swept spectrum analyzer rescaled to the natural logarithm. This distribution is an average of ~6 minutes worth of swept spectrum analyzer measurements with resolution bandwidth 300 Hz. Eq. 3 indicates that the spectrum will be “triangular” on a semi-log plot for constant cooling and diffusion sources which is indeed the case in Fig. 3.

Diffusion Calibration

Once equilibrium has been achieved, the diffusion is measured by recording the time evolution of the beam distribution second moment, Fig. 4. Eq. 2 is used to fit for the diffusion constant, *D*, in the Fokker-Planck equation. The forward difference of the spectrum in Fig. 3 may now be rescaled in a completely consistent manner to give the measured longitudinal electron cooling force as a function of antiproton momentum offset.

ELECTRON COOLING MODEL

The cooling force seen by the antiprotons in the beam frame of reference can be written for the non-magnetized case as

$$\vec{F} = -\frac{4\pi n e^4}{m_e} L_C \vec{J} \equiv -F_0 \vec{J}$$

with the collision integral

$$\vec{J} = \int d^3\vec{v}\,\frac{\vec{u}}{u^3}\,g(\vec{v})$$

where n is the electron density, $g(\vec{v})$ is the electron velocity distribution, L_C is the Coulomb logarithm and $\vec{u} = \vec{v}_p - \vec{v}$ is the difference between the proton and electron velocities.

Lab Frame Longitudinal Cooling Force

Transformation to the lab reference frame gives,

$$F_0 = 4(3600)\left(\frac{r_e}{r_b}\right)^2 \frac{m_e}{\gamma} \frac{I L_C \eta}{e}$$

where *I* is the electron beam current, r_e the classical electron radius, r_b the electron beam radius, m_e the electron mass, *e* the electron charge and *η* the fractional cooling section length. The units of F_0 written this way are (MeV/c/hour)·c².

To perform the collision integral, a Maxwellian distribution of electron velocities,

$$g(\vec{v}) = \frac{1}{(2\pi)^{3/2} \Delta_{\parallel} \Delta_{\perp}^2} e^{-\left(\frac{v_\perp^2}{2\Delta_\perp^2} + \frac{v_\parallel^2}{2\Delta_\parallel^2}\right)}$$

where $v_\perp^2 = v_x^2 + v_y^2$, is assumed [4]. The perpendicular and longitudinal velocity spreads of the electron distribution are $\Delta_\perp$ and $\Delta_\parallel$ respectively. By experimental design, the longitudinal cooling force may be approximated in the electron and antiproton velocity regimes $v_p \gg v$ and $\vec{v}_p \approx (0,0,v_{p\parallel})$ as

$$F_\parallel = -F_0 \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{2\pi v_\perp dv_\perp dv_\parallel g(\vec{v})(v_\parallel - v_p)}{(v_\perp^2 + (v_\parallel - v_p)^2)^{3/2}}. \qquad (4)$$

In units compatible with F_0 above, the velocities are given as fractions of the speed of light.

Data Analysis

The actual function used in the least squares fit [5] to the data involves the numerical integration of Eq 4. The fit parameters are the electron velocity spreads, a velocity offset and an overall scale factor. The numerical integration over $dv_\parallel$ is approximated with sufficient accuracy using the interval $\left[-6\Delta_\parallel, 6\Delta_\parallel\right]$. F_0 can be calculated from the measured electron beam parameters in Table 1 and the Recycler circumference of 3.3 km.

The data used to fit for the electron velocity spreads are restricted to $v_p \leq 0.0004$c (*Δp<3.5* MeV/c). Noise in the tails of the antiproton momentum distribution has no significance. Table 2 contains fit results for different combinations of free parameters.

The overall normalization fit parameter is presented as a change in the current density. However, assuming the current is well measured, this is equivalent to making a change to the electron beam radius while leaving the estimates for the Coulomb logarithm and cooling section length fixed. From several measurements, the angular divergence of the electron beam is estimated to be 0.2 mrad. The data and fit results are also shown in Fig. 5.

CONCLUSIONS

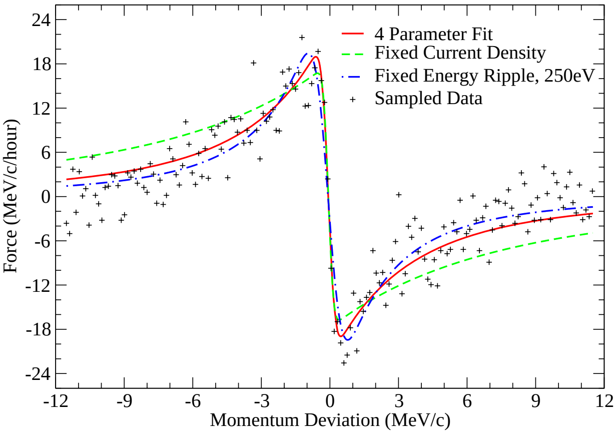

Figure 5:

Other measurements have been made using the voltage jump method [2] which is a time consuming method to sample many antiproton velocities. The procedure presented here achieves results with the benefit of sampling a spectrum of antiproton velocities with one measurement.

The least squares analysis gives unbiased estimates which are a factor of 2-3 smaller from measured values of the electron beam radius and energy spread and a factor of 2-4 in electron beam divergence. However, the asphericity in the electron velocity distributions, $\Delta_\perp / \Delta_\parallel$, for the free fit is 60% of the estimated value of 30 from the measured electron beam parameters. One explanation is this measurement samples the center of the electron beam which would have smaller velocity spreads than the measured values using the entire electron beam.

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Table 2: Analysis Results

Parameter	Free Fit	Fixed	Fixed
$I/\pi r_b^2$(A/m²)	711.08	2598.45 fixed	369.00
δU (eV)	129.27	96.16	250 fixed
θ_e (μrad)	50.43	108.32	31.30
Intercept (MeV/c)	-0.07	-0.07	-0.09
Peak Value (MeV/c/hour)	18.97	16.73	19.44
Inflection (MeV/c)	±0.55	±0.53	±0.86

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