CRYSTALLINE BEAMS AT HIGH ENERGIES*

Jie Wei†, Brookhaven National Laboratory, USA
Hiromi Okamoto, Shuta Ochi, Hiroshima University, Japan
Yosuke Yuri, Japan Atomic Energy Agency, Japan
Andrew Sessler, Lawrence Berkeley National Laboratory, USA
Shinji Machida, Rutherford Appleton Laboratory, United Kingdom

Abstract

Previously it was shown that by crystallizing each of the two counter-circulating beams, a much larger beam-beam tune shift can be tolerated during the beam-beam collisions; thus a higher luminosity can be reached for colliding beams [1]. On the other hand, crystalline beams can only be formed at energies below the transition energy (γ_T) of the accelerators [2]. In this paper, we investigate the formation of crystals in a high-γ_T lattice that also satisfies the maintenance condition for a crystalline beam [3].

INTRODUCTION

For high-energy colliders, the luminosity is usually limited by the Coulomb interactions between particles of either the opposite beams (the beam-beam effect) or the same beam (intra-beam scattering) causing growth in the beam emittance. Beam cooling is often considered to reduce the emittance and enhance the luminosity performance. Crystalline-beam state is the ultimate state when the beam emittance approaches zero.

Previously it was shown that by crystallizing each of the two counter-circulating beams, a much larger beam-beam tune shift can be tolerated during the beam-beam collisions; thus a higher luminosity can be reached for colliding beams [1]. On the other hand, present studies on beam crystallization is limited to very low energies, due partly to lack of effective cooling techniques at high energies, and partly to conditions that prohibits crystal formation above the transition energy of the accelerator ring [2]. In this paper, we explore crystal formation in ring lattices where the transition energy is made either much higher than the transverse tunes of the ring or imaginary.

CONDITIONS FOR CRYSTALLIZATION

The Hamiltonian of particles in an AG-focusing machine can be expressed in terms of the action-angle variables J_x, J_y and momentum $\bar{P}_z$ as [4]

$$\bar{H} = \nu_x J_x + \nu_y J_y + \frac{1-\gamma^2 F_z}{2}\bar{P}_z^2 + \bar{V}_C, \qquad (1)$$

where the transverse tunes ν_x and ν_y are positive for a stable machine lattice, $\bar{V}_C$ is the Coulomb potential, and F_z is related to the normalized horizontal dispersion D as [4]

$$F_z = \begin{cases} D + DD'' + (D')^2 & \text{(bending section)} \\ DD'' + (D')^2 & \text{(straight section).} \end{cases} \qquad (2)$$

The average value of F_z can be obtained as

$$\langle F_z \rangle = \frac{\rho}{2\pi R}\oint F_z dt = \frac{\rho}{2\pi R}\oint_{\text{bend}} D dt \equiv \frac{1}{\gamma_T^2}, \qquad (3)$$

where γ_T is the transition energy of the machine. For a stable crystalline beam, the Coulomb force must on the average provide focusing in the azimuthal direction,

$$\bar{V}_C \approx \frac{k_z}{2}\bar{z}^2, \quad \text{for } \bar{z} \ll \Delta_z, \qquad (4)$$

where $k_z \geq 0$ is the effective Coulomb focusing strength. It can thus be seen from Eqs. 1 and 3 that the azimuthal motion will not be bounded if $\gamma \geq \gamma_T$. Hence, the crystalline beam can not form when the beam energy γ is above the transition energy γ_T. This is the so-called crystal formation condition or the first condition [2]. Eq. 1 also indicates that crystals may be formed if the transition energy is imaginary (negative momentum-compaction).

The second condition is on the crystal maintenance. The condition derived from the linear-resonance criteria states that the frequency of ring lattice variation seen by the circulating crystalline beam must not equal the sum of any two phonon frequencies of the crystal [5]. This condition is satisfied for beams of any density when the transverse phase advance across the lattice superperiod is less than $\pi/\sqrt{2}$ (or 127°) [3, 5].

Considering more practical aspects of cooling a high-temperature beam and the need to suppress linear resonances over the whole density and temperature region, a more stringent version of the second condition is that the transverse phase advance across the lattice superperiod is less than $\pi/4$ (or 90°) [6, 7].

HIGH-γ_T LATTICES

3-cell Missing-dipole Lattice

A conventional method to achieve small or negative momentum-compaction (high or imaginary γ_T) is to use modules of three FODO cells with missing dipole in the middle cell (Fig 1 [8, 9]). The horizontal phase advance of about 90° per cell excites the dispersion oscillation so

* Work performed under the auspices of the U.S. Department of Energy.
† jwei@bnl.gov

that high dispersion occurs only at locations of missing dipole (Eq. 3). However, this lattice is not suitable for beam crystallization since the phase advance (∼270°) across the module is much higher than 127°.

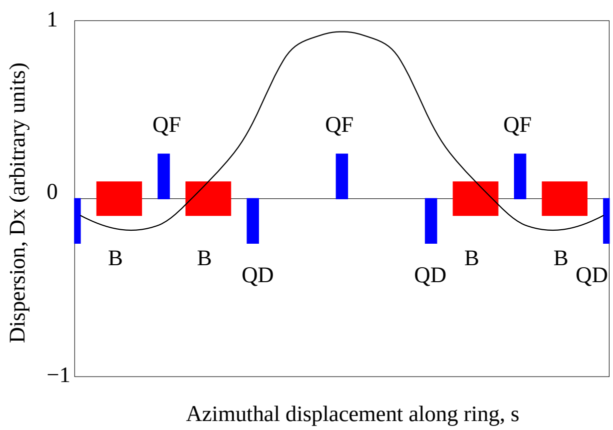

Figure 1: 3-cells missing-dipole low-momentum-compaction lattice module. B, QF, and QD denote dipole, focusing, and defocusing quadrupole magnets, respectively. The horizontal phase advance across the module is approximately 270°.

Negative-bend Lattice

A low-momentum-compaction lattice that satisfies the maintenance condition is shown in Fig. 2. The short, negative-bend dipoles at the high-dispersion region compensate for the long, regular dipoles at the low-dispersion region. Such lattice was proposed in 1955 to avoid transition crossing [10]. A variation of this structure is proposed for the Fixed Field Alternating Gradient (FFAG) accelerator combining B and QD as one magnet while keeping the cell phase advance below 90° [11], as shown in Fig. 3.

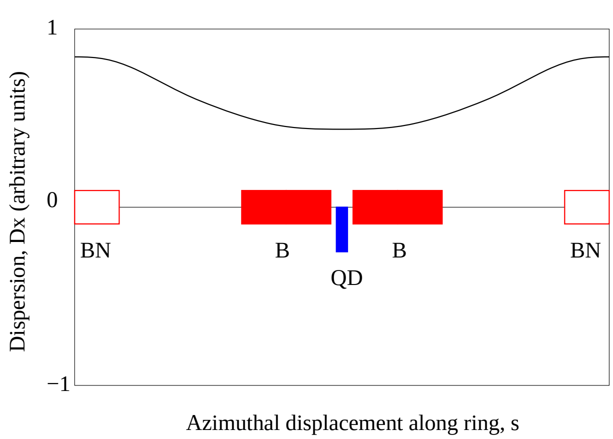

Figure 2: Negative-bend low-momentum-compaction lattice. B, BN, and QD denote dipole, negative-bend dipole, and defocusing quadrupole magnets, respectively. The horizontal phase advance across the module is usually below 127°.

MOLECULAR-DYNAMICS RESULTS

We evaluate the formation and stability of crystalline beams with various types of lattices using the molecular-dynamics (MD) computer simulation. With the 3-cell missing-dipole lattice, only 1D crystalline string may be formed. In the following, we discuss three cases of the negative-bend lattice: imaginary-γ_T with combined-function magnets and phase advance per cell below 90°; high-γ_T with combined-function magnets and phase below 90°; and high-γ_T with separate-function magnets and phase above 90°.

$^{24}Mg^{+}$ ions of various densities are stored in rings each consisting of 12 superperiods. Positive and negative bends are of equal bending radius. In addition to the transverse cooling, tapered cooling is applied for every lattice period in the longitudinal direction [12]. If a crystalline state is reached, the cooling force is removed to test the stability of the formed crystal.

Imaginary-γ_T Lattice

Lattice functions of a ring superperiod is shown in Fig. 3. The ring circumference is 84 m. The phase advances per cell are $\mu_x = 87°$ and $\mu_y = 87°$. The tunes (ν_x, ν_y) are $(2.9, 2.9)$. The transition energy is imaginary, $\gamma_T = i23$.

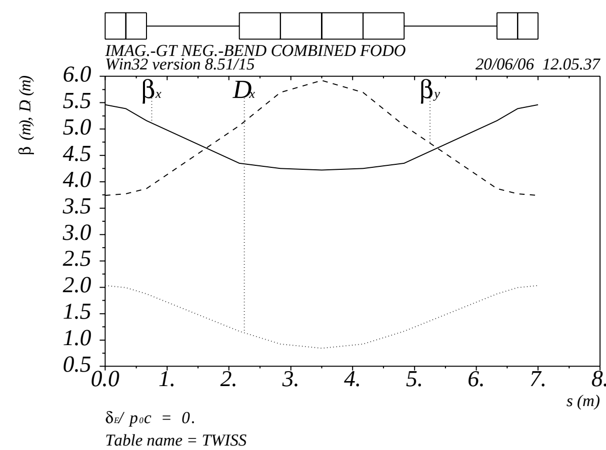

Figure 3: Imaginary-γ_T negative-bend lattice with 87° horizontal phase advance. The middle (positive) bend is of combined-function (dipole and defocusing quadrupole).

Fig. 4 shows a stable, multi-shell (3D) crystal formed at energy $\gamma = 1.45$ with a line density of 2.1×10^7/m. Formation of stable 3D crystals becomes increasingly difficult for higher beam energies. When γ is higher than the horizontal tune ν_x of the ring, only 2D crystals are formed. Due to reduction of the effective horizontal focusing (effective focusing strength is $\nu_x^2 - \gamma^2$ in the horizontal and ν_y^2 in the vertical directions under the smooth approximation [5]), the zig-zag structure extends in the horizontal plane. Fig. 5 shows a stable, 2D crystal formed at $\gamma = 5.5$ with a line density of 1.6×10^6/m.

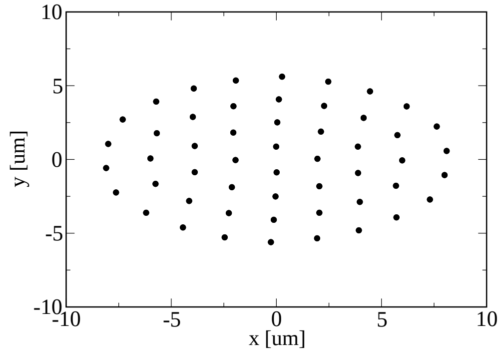

Figure 4: A multi-shell crystalline beam formed with the lattice in Fig. 3 with $\gamma = 1.45$ and density 2.1×10^7/m.

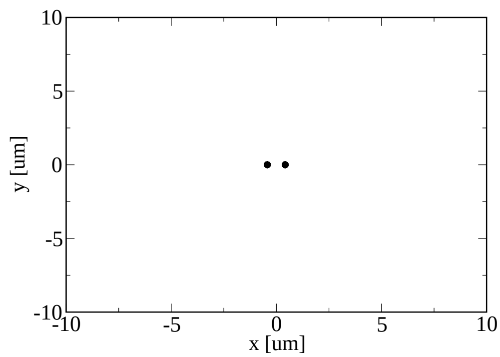

Figure 5: A 2D zig-zag crystalline beam formed with the lattice in Fig. 3 with $\gamma = 5.5$ and density 1.6×10^6/m.

High-γ_T, Low-tune Lattice

With a structure similar to Fig. 3, a high-γ_T lattice is used for the MD study. The phase advances per cell are $\mu_x = 88°$ and $\mu_y = 85°$. The tunes (ν_x, ν_y) are $(2.9, 2.8)$. The transition energy is real, $\gamma_T = 102$. The crystal properties are similar to those formed with the imaginary-γ_T lattice.

High-γ_T, High-tune Lattice

Fig. 6 shows a lattice with separate-function positive and negative bending and defocusing magnets. The phase advances per cell are $\mu_x = 95.4°$ and $\mu_y = 90.3°$ exceeding the $90°$ value. The tunes (ν_x, ν_y) are $(3.18, 3.01)$. The transition energy is real, $\gamma_T = 19$.

The crystal properties are similar to those formed with lattices of cell phase advance below 90°. At a high energy, stable 2D structures in the horizontal plane are formed at $\gamma = 9$, much higher than the transverse tunes.

DISCUSSIONS AND SUMMARY

We develop low-momentum-compaction lattices that would allow the formation of crystalline beams at high energies (γ higher than the machine tunes). Lattices containing negative bending are adopted to keep the transverse phase advance per lattice superperiod below $\pi/\sqrt{2}$.

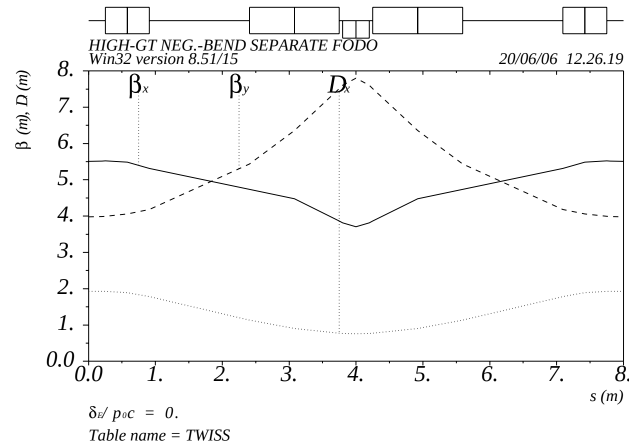

Figure 6: High-γ_T negative-bend lattice with $95.4°$ horizontal phase advance.

With these lattices, we formed crystals beyond 1D at energies (γ) much higher than the machine tunes ($\nu_{x,y}$). However, when $\gamma > \nu_{x,y}$, stable crystals turn to extend in the horizontal plane due to reduction of the effective horizontal focusing.

Under the smooth approximation, the transition γ_T is about equal to the horizontal tune of the ring. This approximation no longer holds for low-momentum-compaction lattices whose γ_T deviates significantly from the tune. More systematic analysis is in progress to evaluate the change of effective focusing with energy in such lattices.

We thank X.-P. Li and D. Trbojevic for many helpful discussions.

REFERENCES

[1] J. Wei, A.M. Sessler, EPAC 1998, Stockholm (1998) 862

[2] J. Wei, X-P. Li, A. M. Sessler, Phys. Rev. Lett., **73**, (1994) 3089

[3] J. Wei, H. Okamoto, and A.M. Sessler, Phys. Rev. Lett., **80**, (1998) 2606

[4] J. Wei, X.-P. Li, A.M. Sessler, Proc. 6th Adv. Accel. Conf. (1994) 224

[5] X.-P. Li, H. Enokizono, H. Okamoto, Y. Yuri, A.M. Sessler, J. Wei, Phys. Rev. ST-AB, **9** (2006) 034201

[6] K. Okabe, H. Okamoto, Jpn. J. Appl. Phys. **42** (2003) 4584

[7] I. Hofmann, L. Laslett, L. Smith, I. Haber, Part. Accel. **13** (1983) 145

[8] R.C. Gupta, J.I. Botman, M.K. Craddock, IEEE Trans. Nucl. Sci. **NS-32** (1985) 2308

[9] D. Trbojevic, D. Finley, R. Gerig, S. Holmes, EPAC90 (1990) 1536

[10] V.V. Vladimirski, E.K. Tarasov, Theoretical problems of the ring accelerators, USSR Academy of Science (1955)

[11] C. Johhnstone, E. Keil, A.M. Sessler, and D. Trbojevic, private communications

[12] H. Okamoto, J. Wei, Phys. Rev. E, **58** (1998) 3817