EMITTANCE CONTROL FOR VERY SHORT BUNCHES

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Abstract

Many recent accelerator projects call for the production of high energy bunches of electrons or positrons that are simultaneously short, intense, and have small emittances. Examples of such projects are the Self-Amplified Spontaneous Emission (SASE) FEL’s, such as the Linac Coherent Light Source (LCLS). A major challenge is keeping in check forces that increase beam emittances in accelerator components, such as: wakefields of accelerator structures and surface roughness, and coherent synchrotron radiation. We describe such forces and discuss emittance control.

INTRODUCTION

In recent accelerator projects one often finds high energy bunches of electrons or positrons that are simultaneously short, intense, and have small emittances. In the Next Linear Collider (NLC) trains of 1 nC, 100 μm long bunches are accelerated through 10 km of linac, on their way the final focus and the collision point [1]. In the Linac Coherent Light Source (LCLS) a 1 nC bunch is compressed to a length of 20 μm, accelerated in 500 m of linac, before entering the undulator for lasing [2]. In both cases a major challenge is keeping in check wakefields that are induced in various parts of the accelerator and that tend to increase emittances, thereby degrading luminosity (in the former case) or lasing (in the latter).

As bunches become shorter new sources of wakefields become important—e.g. the roughness wake—and familiar wakefields display unfamiliar behavior—e.g. the resistive wall wake—, and measurement and emittance control methods need to be modified. How do we quantify short? One simple way to define “short” is if σ/z/a ≪ 1, where σ/z is bunch length and a is beam pipe radius. Under this criterion both the NLC bunch in the main linac (σ/z/a = 0.02) and the LCLS bunch in the SLAC linac (σ/z/a = 0.002) can be considered short. This definition can be extended to include bunches with significant high frequency content (e.g. the somewhat rectangular final LCLS bunch shape), and to bunches undergoing micro-oscillation (micro-bunching).

In this report, we discuss wakefields that are important for short bunches and discuss emittance control. As a concrete example, we will consider parts of the LCLS project. Fig. 1 gives a schematic of the LCLS, displaying important machine parameters and the beam properties of energy, rms bunch length, rms relative energy spread (δσ), at various locations. The LCLS comprises an rf gun, four S-band accelerator regions, an X-band structure, two chicane bunch compressors (named BC1 and BC2), and an undulator. We will focus on the effect of coherent synchrotron radiation (CSR) in the BC2 chicane, the accelerator structure wake in Linac-3, and the resistive wall and roughness wakes in the beam pipe of the undulator.

Figure 1: Schematic of the LCLS.

WAKES AND IMPEDANCES

Consider a point particle of unit charge moving at the speed of light c through a structure, that is followed, at distance s, by a test particle, that is also moving at c. The longitudinal wake W(s) is the voltage loss experienced by the test particle, typically given in units [V/C] for a single structure, in [V/C/m] for a periodic one. The wake is zero if the test particle is in front (s < 0). For a bunch of longitudinal charge distribution Az, the bunch wake W(s)—the voltage gain for a test particle at position s—is given by

\[ W(s) = -\int_0^\infty W(s')\lambda_z(s-s') ds' \quad (1) \]

Note that integrating by parts gives

\[ W(s) = -\int_0^\infty S(s')\lambda_z'(s-s') ds' \quad (2) \]

with S(s) = \int_0^s W(s') ds', an equation that is often used to obtain the wake of smooth, not-too-short bunches given an asymptotic form of the wake (e.g. the resistive wall wake, the CSR wake). The average of minus the bunch wake, \( -\langle W \rangle \), gives the loss factor; the rms Wrms gives the energy spread increase: δE_{rms} = eNLW_{rms}, with eN the charge and L the length of structure (in the periodic case).

The impedance is the Fourier transform of the wake:

\[ Z(k) = \int_0^\infty W(s)e^{ikx} ds \quad (3) \]

with k the wave number. The transverse (dipole) wake Wx is similarly defined for the transverse force experienced by the test particle per unit offset, given in [V/C/m²] in the periodic case. The transverse impedance Zx is, by convention, taken to be times the Fourier transform of Wx.
Considerations especially relevant for short bunches are:

**Catch-up distance:** In calculating the effect on a beam, the wake is typically taken to act instantaneously. For very short bunches there can be a significant lag between the generation of radiation by the head of a bunch and its effect on tail particles. When a head particle passes a vacuum chamber object, such as the beginning of a cavity, that information cannot arrive at a tail particle until the distance \( z = a^2/2s \), where \( a \) is the beam pipe radius and \( s \) is the distance between the two particles. For example, if \( a = 1 \) cm and \( s = 20 \mu m \), then the catch-up distance is 2.5 m.

**Transients:** Similarly, for periodic structures, the interaction with a short bunch will entail an initial transient region before the steady-state wake is reached (see the example shown in Fig. 2). For a Gaussian bunch with width \( \sigma_z \), the transient regime will last \( z \sim a^2/2\sigma_z \). As a bunch becomes shorter, the transients become ever more important.

Limiting value of wake: For periodic, cylindrically symmetric structures whose closest approach to the axis is \( a \), the steady state wakes have the property

\[
W(0^+) = \frac{Z_0 c}{\pi a^2} \quad \text{and} \quad W'_+(0^+) = \frac{2Z_0 c}{\pi a^3},
\]

with \( W(0^+) = 0 \), where \( Z_0 = 377 \Omega \). This is true for a resistive pipe [3], a disk-loaded accelerator structure [4], a pipe with small periodic corrugations [5, 6], and a dielectric tube within a pipe [7]; it seems safe to assume that it is generally true. For a non-rounnd structure the result will be a different constant [e.g. for 2 parallel, resistive plates separated by \( 2a \), \( W(0^+) = \pi Z_0 c/16a^2 [8] \)], but again a constant dependent on transverse dimensions only and independent of material properties. We see that for short bunches the longitudinal wake approaches a maximum strength, and the transverse wake approaches zero.

**Finite energy:** The impedance will drop sharply to zero for frequencies \( k > \gamma/a \), with \( \gamma \) the Lorentz energy factor. In the time domain when \( \sigma_z < a/\gamma \), \( \sigma_z \) should be replaced by \( a/\gamma \) in wakefield formulas. For example, if \( a = 1 \) cm and energy \( E = 14 \) GeV this occurs when \( \sigma_z = 0.4 \mu m \).

### TYPES OF SHORT BUNCH WAKES

We discuss four short bunch wakes that are important for the LCLS. Note that all are periodic wakes that give the steady-state beam-environment interaction.

**Resistive Wall Wake**

The longitudinal wakefield excited by an ultra-relativistic particle in a metallic beam pipe has long been known to be given by [3]

\[
W(s) = -\frac{c}{4\pi^{3/2}a} \sqrt{\frac{Z_0}{\sigma}} \frac{1}{s^{3/2}},
\]

(5)

with \( \sigma \) is the conductivity of the metal, provided that \( s \gg s_0 \), where

\[
s_0 = \left( \frac{2a^2}{Z_0 \sigma} \right)^{1/3}
\]

(6)

\((s > 0)\) is implied in Eq. 5 and in following wakefield equations. The general solution (including small \( s \)), however, is [9]

\[
W = \frac{4Z_0 c}{\pi a^2} \left( \frac{e^{-s/s_0}}{3} \cos \frac{\sqrt{3} s}{s_0} - \frac{\sqrt{3}}{\pi} \int_0^\infty dx x^2 e^{-x^2 s/s_0} \right)
\]

(7)

(see Fig. 3). Note that a similar equation exists for the transverse wake [9].

Figure 3: Plot of Eq. 7, the resistive wall wake. The long-range asymptote (Eq. 5) is shown in dashes.

For the LCLS undulator, for example, the beam pipe is copper coated with \( a = 3 \) mm, and \( s_0 = 9 \mu m \). For bunch lengths \( \sigma_z < 10 \mu m \) other effects, such as frequency dependence of conductivity [9], can be included in the calculation. Unfortunately, other physics that is not well understood (e.g. the room temperature anomalous skin effect) may also manifest and modify details of the wake. However, in light of Eqs. 4 the wake, over the length of a very short bunch, has not much room to deviate.

**Accelerator Structure Wake**

For a short bunch \( (\sigma_z/a \ll 1) \) passing through a single cavity connected to long beam pipes the diffraction model
applies [3]:

\[ W(s) = \frac{Z_0 c}{\sqrt{2\pi^2 a^2}} \sqrt{\frac{g}{s^3}} \equiv \frac{1}{\sqrt{2\pi s^3}} \frac{Z_0 c}{\sqrt{a^2}} \equiv \frac{Z_0 c}{\sqrt{2\pi a^2}} \sqrt{\frac{g}{s^3}}, \] (8)

where \( g \) is the gap; the impedance varies as \( Z \sim k^{-1/2} \).

For a periodic collection of cavities of period \( p \), the asymptotic, high frequency impedance is given by [4, 10],

\[ Z(k) \approx \frac{iZ_0}{\pi ka^2} \left[ 1 + (1 + i) \frac{\alpha(g/p)}{a} \left( \frac{\pi}{kg} \right)^{1/2} \right]^{-1}, \] (9)

with \( \alpha(x) \approx 1 - 0.465x - 0.070x^2 \); the real part of the impedance \( \Re(Z(k)) \sim k^{-3/2} \). Inverse Fourier transforming, one obtains an analytical expression for the very short-range wake.

Obtaining the short-range wake numerically and fitting to a simple function, we obtain a result that is valid over a larger \( s \) range and over a useful range of structure parameters [11]:

\[ W(s) = \frac{Z_0 c}{\pi a^2} \exp \left( -\sqrt{s/s_1} \right), \] (10)

with

\[ s_1 = 0.41 \frac{a^{1.8} g^{1.6}}{p^{2.4}}. \] (11)

The result is valid for \( s/p \leq 0.15 \), \( 0.34 \leq a/p \leq 0.69 \), and \( 0.54 \leq g/p \leq 0.89 \). For the SLAC linac \((a = 11.6 \text{ mm}, g = 29.2 \text{ mm}, p = 35.0 \text{ mm}) \), \( s_1 \approx 1.5 \text{ mm} \). It has been numerically verified that, for a Gaussian bunch in a periodic accelerator structure, the steady state result becomes valid after a distance of \( z \approx a^2/2\sigma_z \) [12].

The high frequency transverse impedance, similar to the low frequency case, is related to the longitudinal impedance by \( Z_x = 2Z/ka^2 \) [13]. An equation equivalent to Eq. 10 has also been found for the transverse, short-range wake [14]:

\[ W_x(s) = \frac{4Z_0 c^2}{\pi a^4} \left( 1 - \left( 1 + \sqrt{\frac{s}{s_2}} \right) \exp \left( -\sqrt{\frac{s}{s_2}} \right) \right), \] (12)

with \( s_2 = 0.17a^{1.79} g^{0.38}/p^{1.17} \).

**Roughness Impedance**

A metallic beam pipe with a rough surface has an impedance that is enhanced at high frequencies. Two approaches to modelling the impedance of a rough surface are with: (i) a random collection of bumps on a surface and (ii) a wall with small periodic corrugations.

An early model of roughness impedance assumes a random, non-interacting collection of bumps of various shapes, with the total impedance given by the sum of the individual impedances. Consider a beam pipe of radius \( a \) on which there is a small hemispherical bump of radius \( h \). At low frequencies \( k \ll 1/h \) the impedance is inductive and given by [16]

\[ Z(k) = i\alpha L_1 = i\alpha Z_0 h^3 \frac{1}{4\pi a^2}, \] (13)

with \( L_1 \) the inductance of the bump. If the hemispherical bump is replaced by one of a different shape but with about the same size, the above equation is multiplied by a form factor of order 1. For many randomly distributed bumps the inductance per unit length \( \mathcal{L}/L \) can be written as

\[ \mathcal{L}/L = \frac{2\alpha f a L_1}{h^2} = \frac{\alpha f Z_0 h}{2\pi a c}, \] (14)

where \( \alpha \) is the filling factor of bumps and \( f \) is the effective form factor. The rms wake of a Gaussian bunch is given by \( W_{\text{rms}} \approx 0.06c^2 \mathcal{L}/L \sigma_z^2 \).

The simple idea of this model has been systematized so that one can, from measurements of the contour of a surface, obtain the inductance per length of the surface [17]

\[ \mathcal{L}/L = \frac{Z_0}{2\pi a} \int_{-\infty}^{\infty} \frac{k_z^2}{\sqrt{k_\theta^2 + k_z^2}} S(k_z, k_\theta) dk_z dk_\theta, \] (15)

where \( S(k_z, k_\theta) \) is the spectrum of the surface profile—

-the square of the absolute value of the Fourier transform of the surface variation—and \( k_z, k_\theta \), are the wave numbers in the longitudinal and azimuthal directions. Finally, note that since, for an inductive model, the wake of a bunch is given by the derivative of the bunch distribution, such a model cannot be applied to a rectangular or other non-smooth bunch shape.

The second modelling approach is to consider a beam pipe with small periodic corrugations. The motivation for such a model came from numerical simulations of many randomly placed, small cavities on a beam pipe; it was found that, in steady state, the short range wake is very similar to the truly periodic case [5]. Consider a beam pipe with small, random, periodic corrugations, with \( h \) the half-depth, \( g \) the gap \((= p/2)\), and \( p \) the period. In the case \( h/p > 1 \) the wake is dominated by one mode of relatively low frequency \((k \ll 1/h) \) [5, 6]:

\[ W(s) \approx \frac{Z_0 c}{\pi a^2} \cos k_0 s \quad \text{with} \quad k_0 = \frac{2}{\sqrt{ah}}, \] (16)

For smooth bunches, with \( k_0 \sigma_z \) large, the wake for this model becomes inductive with \( \mathcal{L}/L = Z_0 h/4ac \), very similar to the first model. However, even for non-smooth distributions, such as the roughly rectangular bunch shape found in the undulator region of the LCLS, this model can still be applied.

This single resonator model is valid when the depth-to-period of the surface roughness is not small compared to 1. However, measurements of copper surfaces with good finish show that the opposite tends to be true: that the depth-to-period \( \lesssim 0.01 \) (see Fig. 4) [18]. As the depth-to-period ratio becomes small the dominant, low frequency mode is replaced by many weak, closely spaced, modes beginning just above \( k = \pi/p \) [19]. For a sinusoidally oscillating wall with amplitude \( h \) and period \( p \), where \( h \ll p \), the wake becomes [20]:

\[ W(s) = \frac{Z_0 c h^2 k_z^3}{4\pi a} f(k_1 s), \] (17)
\[ f(\zeta) = -\frac{1}{2\sqrt{\pi}} \frac{\partial}{\partial \zeta} \cos(\zeta/2) + \sin(\zeta/2) \frac{1}{\sqrt{\zeta}} \] (18)

with \( k_1 = 2\pi/p \). To satisfy the requirement on \( W_z(0^+) \), we can artificially set \( W(s) = Z_0 c/\pi a^2 \) for \( 0^+ < s < 0.11 k_1 h^{3/3} a^{2/3} \).

For \( h_1 s \lesssim 1 \) (but not too small), \( W \sim s^{-3/2} \) and for a bunch \( W \sim \sigma_z^{-3/2} \). Note that for given \( h \) the bunch wake here is weaker, by \( \sim h/p \), than for the single mode model described earlier. Note also that the wake has the form of the long-range resistive wall wake; it can be described as the wake of a metal with conductivity \( \sigma = 16/Z_0 h^4 k_1^4 \). If we believe Fig. 4 is representative of the undulator beam pipe (\( h \approx 0.5 \mu m, p \approx 100 \mu m \)), and \( \sigma_z = 20 \mu m \), our parameters are in the regime of this model; in this case the roughness wake is small, only \( \sim 0.15 \) the strength of the resistive wall wake (for copper).

**CSR Wake**

The effect on a bunch of coherent synchrotron radiation can be described in terms of a wakefield, although there are some differences from normal wakes. For example, for an ultra-relativistic particle moving in a circle of radius \( R \) in free space, the wake (for a test particle on the same path) is non-zero ahead of the radiating particle (\( s < 0 \)), because the radiation moves on a shorter, straight path. The wake experienced by the test particle is sketched in Fig. 5. For \( -(s) \gg R/\gamma^3 \) it is given by [21, 22]

\[ W(s) = -\frac{Z_0 c}{2 \cdot 3^{\frac{3}{2}} \pi R^{2/3} (-s)^{4/3}} \quad s < 0, \] (19)

while \( W(0^-) = Z_0 c \gamma^4 / 3 \pi R^2 \). For a bunch the wake scales as \( \sim R^{-2/3} \sigma_z^{-4/3} \); for a Gaussian the coefficient (in units of \( Z_0 c \)) for average wake is \( \sim 0.028 \), for rms 0.020. Fourier transforming the wake gives the impedance [21, 22]

\[ Z(k) = \frac{Z_0}{2 \cdot 3^{\frac{3}{2}} \pi} \Gamma \left( \frac{2}{3} \right) e^{i \pi / 6} k^{1/3} R^{2/3}, \] (20)

with \( \Gamma(2/3) = 1.35 \), valid to very high frequencies (\( k \sim \gamma^3 / R \)).

For particles moving on a circle through a beam pipe the wake will be modified, an effect that can be calculated using image charges in the time domain [21], or be dealt with in the frequency domain [23]. The pipe suppresses the wake of a bunch, provided that \( \sigma_z/a > (a/R)^{1/2} \) [23]. If we consider the beam in the last bend of BC2 of the LCLS, and take \( \sigma_z = 20 \mu m, a = 1 \) cm (here the half the vertical beam pipe aperture), and \( R = 15 \) m, we see that the bunch length is 13 times too short to feel the effect of shielding.

Chicanes are composed of 3 or 4 bends with drifts in between. Instead of one continuous circle, the beam moves in circular arcs (in the bends) and straight lines (outside the bends). There will be transients when the beam enters a bend and also after it leaves. The catch-up distance is the distance a test particle, ahead of the radiating particle, needs to travel to catch up to the radiated wave. The distance of the incoming transient region is \( z \approx (24 R^2 \sigma_z)^{1/3} \) (assuming \( z/R \) is small) [22].

One can consider the potential energy change (the “compression work”) that the beam undergoes in being compressed. The average kinetic energy change to balance this is approximately (assuming the compression factor is large, for a Gaussian bunch) [24, 25]

\[ \langle \delta E \rangle = -\frac{e N Z_0 c}{4 \pi^{3/2} \sigma_z} \ln \left( \frac{\gamma \sigma_z}{\sigma_x + \sigma_y} \right), \] (21)

where beam sizes are final quantities, and the rms spread \( \delta E_{rms} \approx -0.4 \langle \delta E \rangle \). Interestingly, for example chicanes, Eq. 21 was found to roughly agree with the total energy change obtained by detailed CSR simulations [25].

To simulate the CSR force in a chicane compressor, computer programs that slice the bunch into macro-particles and solve the Liénard-Wiechert equations have been written [26]-[28]. The bunch can have transverse as well as longitudinal dimensions, shielding can be added, and the orbit/forces can be computed in a self-consistent manner. These programs, however, can be time consuming to run. However, analytical solutions for the 1D wake of an ultra-relativistic particle entering, traversing, and leaving a bend without shielding have been derived [29]. These formulas, when used in a tracking program such as ELEGANT [30], give results that are quick to obtain and agree reasonably well with more detailed simulations for typical beam/chicane parameters [28]. (Ref. [28] gives a good comparison of simulation programs.)
EMITTANCE CONTROL

In a collider, such as the NLC, preserving the projected emittances is what is most important. In an FEL, although projected emittance has its importance, it is primarily the slice emittance, i.e., the emittance over a slippage length, that we need to preserve (for the LCLS 0.5 \(\mu m\) v.s. a bunch length of 20 \(\mu m\)). Wake forces are head-tail forces and only weakly affect the slice emittance directly. A compressor, however, can couple head-tail effects into slice emittance. Forces that can affect the slice emittance directly are e.g. space charge, (incoherent) synchrotron radiation, or intra-beam scattering.

Let us consider three regions at the end of the LCLS to study wake effects: BC2, Linac-3 and the undulator. The charge \(eN = 1\) nC, we take the bunch shape to be uniform with rms length \(\sigma_z = 20\) \(\mu m\), the normalized emittance \(\epsilon_n = 10^{-6}\) m; before Linac-3, \(E = 4.5\) GeV, after \(E = 14\) GeV. The length of Linac-3 is \(L = 550\) m, of the undulator \(L = 130\) m.

For short bunches, the transverse wake becomes weak, and in the short-bunch regions of the LCLS the transverse wake is not important. In Linac-3, for example, we can approximate the wake \(W(x) = 2Z_0c^2/\pi a^4\). The emittance growth due to a betatron oscillation with amplitude of the beam size is (if it is small) \(\delta \epsilon/\epsilon \approx \nu^2/2\), with strength parameter \(\nu = e^2NL\langle \chi_x \rangle /\beta E\) (\(\beta\) is beta function) [3]. Taking \(\beta = 45\) m, \(E = 9\) GeV (the average), we find that \(\nu = 0.06\) and \(\delta \epsilon/\epsilon\) is insignificant.

The longitudinal wake in Linac-3 is part of the compression process, used to take out the residual chirp left in the beam at the end of BC-2. For short bunches, the longitudinal wake approaches its maximum. In Linac-3 we approximate it by \(W(x) = Z_0c^2/\pi a^2\), and the bunch wake \(\delta \epsilon/\epsilon\) is nearly linear (for a uniform bunch). The rms of the chirp induced by the wake \(\delta \epsilon/\epsilon = e^2N\langle W_{xx} \rangle /\beta\) (\(\beta\) is beta function) [3]. In the undulator region the resistive wall wake dominates over the roughness wake, assuming a copper pipe with the type of surface finish shown in Fig. 4. The rms induced energy spread \(\delta \epsilon/\epsilon = e^2N\langle W_{xx} \rangle /\beta\) is 0.3%. In the undulator region the resistive wall wake dominates over the roughness wake, assuming a copper pipe with the type of surface finish shown in Fig. 4. The rms induced energy spread \(\delta \epsilon/\epsilon = e^2N\langle W_{xx} \rangle /\beta\) is 0.3%.

For compressor BC-2, Eq. 21 yields (now for a Gaussian bunch, taking \(\sigma_z = \sigma_y = 25\) \(\mu m\)) \(\delta \epsilon/\epsilon = 0.018\%\), whereas 1D simulation yields 0.016% (leading to 38% emittance growth) [31]. An estimate of energy spread that is often used, though it is an underestimate, is an approximation to the spread due to the last bend of the chicane: \(\delta \epsilon/\epsilon = e^2N\langle W_{xx} \rangle L_b/\beta\) (\(\beta\) is for the CSR wake, \(L_b\) is bend length) which here yields 0.008%.

Finally we should mention that emittance control can also mean increasing the emittance. For example, the slice energy spread of the beam leaving the gun may be too low: a cold beam is susceptible to instabilities, driven, e.g. by the longitudinal space charge impedance, that can cause initial noise on the beam to amplify through the compressors and linacs. Using a laser to heat the beam can inhibit such instabilities [32]. Also, an idea to obtain a shorter pulse of light from the LCLS has been proposed: to place a thin beryllium foil with a narrow slot in the middle of an LCLS chicane. Most particles pass through the foil, their emittance increases through scattering, and they do not lase; those passing through the slot become a short sub-pulse that lases normally [33].

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REFERENCES