IONIZATION COOLING OF MUON BEAMS

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Abstract

Ionization cooling is the preferred method for transverse cooling of a muon beam in a neutrino factory. We design an alternating solenoid cooling channel and show that the transverse beam emittance through the cooling channel is cooled down by a factor of 6.5. The cooling channel design has been carried out by invoking the ICOOL program. Transmission efficiency, transverse beam emittance, growth of longitudinal bunch length, and momentum spread are investigated. It is shown that the cooling channel provides adequate cooling performance for a neutrino factory. It is also shown that the numerical results agree well with those of analytical formulae.

1 INTRODUCTION

A challenge in the design of a neutrino factory is the cooling of a muon beam. When muons are produced from a target, they occupy a large phase-space area. A successful design of a neutrino factory therefore requires that the transverse emittance of the muon beam after the capture, phase rotation, and buncher channels be sufficiently reduced so that the muon beam can be accelerated efficiently in downstream sections. Furthermore, fast cooling is required due to short lifetime of the muon beam. The lifetime of a muon is given by $\tau_\mu = 2.197 \times 10^{-6} E_\mu/m_\mu$ seconds where E_μ is the energy and m_μ is the rest mass of a muon. A cooling method that can meet such a requirement is ionization cooling [1, 2, 3]. In ionization cooling, particles pass through a material medium and lose energy through ionization interactions. The losses are parallel to particle motion and therefore both transverse and longitudinal momenta are lost. Only longitudinal momentum is then restored by acceleration through rf cavities. This results in a reduction of the angular spread of a beam, and therefore leads to a decrease in the transverse beam emittance. However, the random process of multiple Coulomb scattering in material medium leads to an adverse effect, namely the beam emittance increases. When an absorber material is placed in a strong focusing field, the heating can be minimized.

The equation for the normalized transverse emittance ϵ_n is given by [3]

$$\frac{d\epsilon_n}{dz} = -\frac{1}{\beta^2}\frac{dE_\mu}{dz}\frac{\epsilon_n}{E_\mu} + \frac{1}{\beta^3}\frac{\beta_\perp E_s^2}{2E_\mu m_\mu c^2 L_R}, \qquad (1)$$

where z is the path length, E_μ is the muon beam energy, $\beta = v/c$, L_R is the radiation length of the absorber material, $\beta_\perp$ is the betatron function, and E_s is the characteristic scattering energy which is approximately 13.6 MeV. The normalized emittance ϵ_n is related to the conventional unnormalized geometric emittance ϵ by ϵ_n=$\epsilon\beta_\perp\gamma$, where γ is the conventional relativistic factor. In eq.1, the first term describes the cooling and the second term represents the heating due to multiple scattering. As the emittance approaches an equilibrium value, the cooling term is balanced by the heating due to multiple scattering. Since the heating is proportional to $\beta_\perp$ and $1/L_R$, small $\beta_\perp$ (i.e., strong focusing) and a material with high L_R (i.e., a low-Z absorber) should be considered to maximize the cooling.

In this paper, we consider an alternating solenoid cooling channel. In the code ICOOL [4], energy loss is modeled by the Vavilov distribution function and multiple scattering is modeled by the Moliere distribution. We compare results of the ICOOL simulation with the analytical method by Kim and Wang [5]. Engineering constraint in the design of the cooling channels is also investigated by considering realistic physical parameters.

2 DESCRIPTION OF COOLING CHANNEL

In general, a cooling channel consists of a series of identical cells. The length of each cell is chosen to avoid betatron and synchro-betatron resonances. These resonances can lead to large particle loss when the betatron wavelength equals the period of the magnetic field or the synchrotron-oscillation wavelength.

When cooling channels are designed, many different conditions have to be simultaneously satisfied to be an acceptable cooling channel. First, we use liquid hydrogen (LH) as an absorber material since it provides the lowest possible transverse emittance. The thickness of the absorber window is a critical parameter. It must be thick enough to sustain the pressure from the LH and as thin as possible to reduce multiple scattering. The thickness and the radius of the LH absorber and the LH window affect particle losses in the transverse direction. A thicker LH absorber yields more particle loss and therefore it requires a larger rf gradient or a longer channel length per cell. However, thick LH absorber makes it possible to use a thin LH window which can reduce multiple scattering. The LH is assumed to be contained in a vessel with very thin windows on both ends. The length of the LH is chosen so that the increase in $\beta_\perp$ near the ends of the vessel is not significant. The total number of cells in the cooling channel is determined by the total ionization loss and the rf phase at which the muon beam is accelerated. An appropriate gap between the LH window and the rf cavity is considered for rf assembly and maintenance.

We use high field solenoids for focusing since they provide a large angular acceptance as well as simultaneous strong focusing of a beam in both transverse directions. Engineering feasibility in magnets is investigated by considering such parameters as magnetic field, current density and stress on the conductor. Our design follows the engineering constraint that is shown in a feasibility study report of the neutrino factory [6]. A conservative rule of thumb (based on keeping the hoop stress within manageable limits) for solenoids built from an Nb_3Sn superconductor is given by $BJR < 350$ MPa, where B is the field at the coil, J the area current density, and R the radius of solenoid from the z-axis. Sheets are used to generate magnetic fields. An appropriate interval between sheets is considered for a power supply from klystron. Radius, length, height and current density of sheets are set to generate appropriate magnetic fields.

The space between absorber materials is filled with rf cavities to compensate for beam energy loss in absorber materials. The cavity length is chosen to provide sufficient acceleration to match the energy loss in the LH. We take the pill-box type of the TM010 mode, with $\pi/2$ phase-advance per cell, equipped with thin beryllium windows of 125 μm thickness and five cells per each rf section in the simulation. Rf frequency and rf gradient are 201.25 MHz and 15 MV/m, respectively. Total power deposited on an rf window is proportional to the fourth power of the radius of the rf window. The radius of rf window is critical to the particle losses when the rf window with a small radius is located at the position with large beta function.

3 ALTERNATING SOLENOID COOLING CHANNEL

In this channel, a lattice of solenoids with alternating direction is used to achieve the required focusing. Fig. 1 shows the configuration of the cooling channel with a cell length of 2.2 m. The cooling channel has a maximum magnetic field of 3.4 T on the longitudinal axis and 6 T on the solenoid with a current density of 58 A/mm^2. Minimum beta function is 20 cm and maximum beta function is 110 cm. The magnetic field peaks with a magnitude of 3.4 T near the center of the absorbers. The field falls to zero and alternates in direction at the center of the rf cavities. Fig. 2 shows the magnetic field (B_z) in one cell of alternating solenoid along the longitudinal axis. The maximum and minimum values of magnetic field B_z occur near the midpoint of absorber and rf cavities, respectively. The maximum value of the beta function occurs at the midpoint of the rf cavities. The minimum value occurs near the midpoint of absorbers. The reason is that particle loss is mainly generated at the position of the rf window where the beam size is large.

3.1 Input beam

The initial beam distribution is generated by a random Gaussian with 11000 mm-mrad transverse and 16 mm longitudinal emittances. A beam with a longitudinal momentum of P_z=214 MeV/c with an rms momentum spread of 9% is used. The properties of the initial beam used in the simulation are shown in Table II.

3.2 Parameters of the alternating solenoid cooling channel

The designed cooling channels have 73 cells with 3.4 T magnetic field B_z. Cell length of all cooling channels is 2.2 m. The parameters of the alternating cooling channel are shown in Table I. For the 3.4 T alternating solenoid channel, the relevant values are B= 6 T at the coil, J=58 A/mm^2, and R=0.805, giving $BJR = 280$ MPa.

3.3 Cooling performance

The transverse emittance in both the x and y planes is reduced to 15% of its initial value, while the longitudinal emittance grows by a factor of two. The overall normalized six-dimensional emittance is reduced to 13% of its initial value. Table II shows the final beam parameters of rms beam size σ_x, rms bunch length σ_z and rms longitudinal momentum spread.

3.4 Comparison with analytical result

The simulation result by ICOOL is compared with the result of the analytical method [5]. The analytical formula for the reduction of the transverse emittance is given by

$$\epsilon(z) = \frac{e^{-\zeta_-(z)}}{2}[\epsilon^0 + L^0 + \mathcal{D}^-(z)] + \frac{e^{-\zeta_+(z)}}{2}[\epsilon^0 - L^0 + \mathcal{D}^+(z)], \qquad (2)$$

where

$$\zeta_\pm(z) = \int_0^s d\bar{z}\eta(\bar{z})[1 \pm \kappa(\bar{z})\beta(\bar{z})] \qquad (3)$$

$$\mathcal{D}^\pm(z) = \int_0^z d\bar{z} e^{\zeta_\pm(\bar{z})}\beta(\bar{z})\xi(\bar{z}). \qquad (4)$$

η is the scaled parameter specifying the energy loss per unit length in the absorber material and $\xi(z)$ is the angular excitation due to the stochastic kick arising from multiple scattering. The parameters ϵ^0, β and L^0 are initial transverse emittance, betatron function and angular momentum, respectively. They are explicitly given by

$$\eta = \frac{1}{p_s v}\frac{dE}{dz} = \frac{1}{p_z}\frac{dp}{dz} \qquad (5)$$

$$\xi(z) = \left(\frac{13.6MeV}{pv}\right)^2 \frac{1}{L_{rad}}, \qquad (6)$$

where L_{rad} is the radiation length of the material. The simulation result by ICOOL is compared with that of the analytical method. The results show good agreements, as shown in Fig. 3.

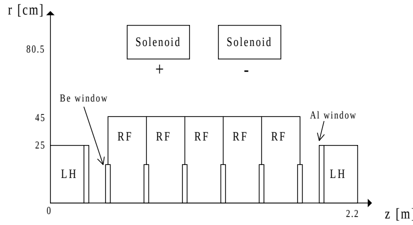

Figure 1: One cell of the alternating cooling channel. The magnetic field of the adjacent cell has a mirror symmetric configuration.

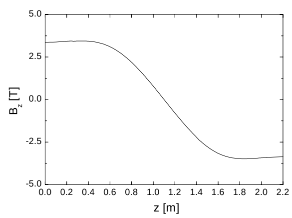

Figure 2: Magnetic field (B_z) in one cell of the cooling channel.

4 CONCLUSION

An alternating solenoid cooling channel is designed and its cooling performance is calculated with the help of the ICOOL program. It is shown that adequate cooling parameters for the neutrino factory can be achieved by this cooling channel. The purpose of our study is to obtain efficient transverse cooling channels that satisfy engineering constraints. Detailed simulation studies have been performed to obtain the optimal parameters for each cooling channel. This study will be valuable in selecting the best design based on cooling performance, engineering constraint and cost. It is also shown that the simulation results agree well with those of analytical method.

5 REFERENCES

[1] A. N. Skrinsky and V. V. Parkhomchuk: Sov. J. Nucl. Phys. 12 (1981) 3.

[2] E. A. Perevedentsev, A. N. Skrinsky: Proc. 12th Int. Conf. High Energy Accelerators, 1983, p.485.

[3] D. Neuffer: Nucl. Instrum. & Methods A 362 (1995) 213.

[4] R. Fernow: Proc. 1999 Particle Accelerator Conf., p.3020.

[5] K. J. Kim and C. Wang: Phys. Rev. Lett. 85 (2000) 760.

[6] N. Holtkamp: submitted to Phys. Rev. Special Top.: Accel. Beams, May, 2000.

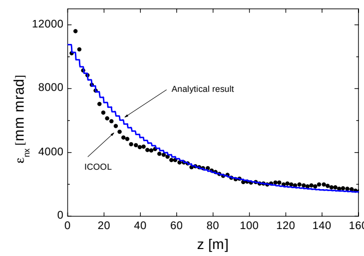

Figure 3: Transverse emittances ϵ_x (mm mrad) as a function of channel length by ICOOL simulation and analytical formulae. Analytical result is shown by a step function.

Table 1: Parameters of designed cooling channels.

Rf frequency	201.25 MHz
Rf gradient	15 MV/m
Cell length	2.2 m
Magnetic field on the z-axis	3.4 T
Total channel length	160 m
Minimum beta function	20 cm
Maximum beta function	110 cm
LH length/cell	30 cm
LH window thickness	400 μm
LH window radius	25 cm
Be window thickness	125 μm
Be window radius	20 cm
Solenoid length	55 cm
Solenoid height	15 cm
Solenoid current density (J)	58 A/mm^2
Solenoid radius (R)	80.5 cm
Magnetic field on solenoid (B)	6 T
BJR	280 MPa

Table 2: Initial and final beam parameters in 160 m alternating solenoid channel.

	Initial	Final
Particles tracked	1000	515
Normalized transverse emit.(mm mrad)	10227	1585
Normalized longitudinal emit.(mm)	16	30
Normalized 6D emit.($\times 10^{-8}$) (m rad)^3	197	27
Rms bunch length (σ_z) (cm)	10.2	14.4
Rms beam size (σ_x) (cm)	3.9	1.1
Rms momentum spread (%)	9	25