

STUDY OF BEAM LIFETIME IN PLS STORAGE RING

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Abstract

A simulation that uses a technique to obtain tail distributions is applied to the PLS storage ring. This program makes it possible to investigate tail distribution in simple and fast simulation technique. This simulation now includes rare random processes such as intra-beam scattering, beam-residual gas scattering and beam-residual gas bremsstrahlung. An estimate of the beam lifetimes due to these processes in the PLS storage ring is presented. It is shown that the estimated beam lifetime shows a good agreement with operational beam lifetime in PLS.

1 INTRODUCTION

The distribution of particles in a beam can be divided into two regions: the core region at small amplitudes and the tail region at large amplitudes. The core distribution determines the brilliance and the tail distribution affects the beam lifetime. Since it is not always possible to obtain the beam distributions by analytical treatments, brute-force tracking can be used to simulate the beam tails. However, it requires a tremendous amount of CPU time. In particular, when rare random processes contribute to beam tails, it is actually impossible to obtain results with sufficient statistics. In that case, it is desirable to investigate the beam tails by available simulation methods.

A simple and fast simulation method was proposed to obtain the beam tail due to rare random processes [1, 2]. The simulation method investigates the beam tails caused by rare and large-amplitude processes from the core distribution. In this study, a simulation method is applied to the PLS storage ring in order to understand the beam tails from both transverse and longitudinal random processes. To see the effects of tails, we consider the processes of intra-beam scattering, beam-residual gas scattering and beam-residual gas bremsstrahlung. In the presence of apertures, these processes can also lead to a steady loss of particles. The beam lifetime in the PLS storage ring is also estimated in a simulation by counting the number of the particles extending beyond the apertures.

2 DESCRIPTION OF THE SIMULATION METHOD

Suppose that an electron undergoes elastic collision with residual gas present in a vacuum chamber. We describe a method which can be used to simulate these processes and to find the equilibrium distribution.

The simulation starts with n macroparticles that are given randomly with specified variances in six-dimensional phase space. Each macroparticle (i) has a particle number

(N_i). Let p be the probability that an electron undergoes a random process during one turn. Once an electron in a macroparticle undergoes this process (the probability P is $N_i p$), we create a new macroparticle ($n + 1$)th. This new macroparticle has one particle ($N_{i+1} = 1$) and the macroparticle which has undergone a random process now has a number of particles ($N_i - 1$).

We assume that the variation in the random variable due to a random process is limited to a range between a minimum and a maximum value. To obtain the variation, first, calculate the probability (P), and generate one uniform random number ($0 \leq x \leq 1$). If $x < P$, a random process occurs for the macroparticle. Second, generate a uniform random number (θ_1) in the interval between the minimum value (θ_c) and the maximum value (θ_m) and one uniform random number in the interval $0 < y < (d\sigma(\theta)/d\theta)_{max}$, and compare y and $(d\sigma(\theta)/d\theta)_{\theta=\theta_1}$ is the cross section corresponding to θ' . If $y < (d\sigma(\theta)/d\theta)_{\theta=\theta_1}$, a random variation corresponding θ_1 is given to an electron. If $y > (d\sigma(\theta)/d\theta)_{\theta=\theta_1}$, discard these θ_1 and y , and generate new θ_1 and y until the relation $y < (d\sigma(\theta)/d\theta)_{\theta=\theta_1}$ holds.

One should find a reasonable minimum cutoff value for each of the random processes by testing several values. The choice of too small a cutoff value reduces the efficiency of this simulation, without giving any contribution to the beam tails; and a large cutoff value can result in larger statistical fluctuations. The equilibrium beam distributions should not be affected by variations of this parameter.

Each macroparticle in the simulation is tracked as follows:

1. *Input* We use the following normalized variables in tracking:

$$X = \frac{x}{\sigma_x^o}, P = \frac{\beta_x P_x}{\sigma_x^o}, Y = \frac{y}{\sigma_y^o} \quad (1)$$

$$Q = \frac{\beta_y P_y}{\sigma_y^o}, Z = \frac{z}{\sigma_z^o}, E = \frac{\epsilon'}{E_o \sigma_{\epsilon^o}} \quad (2)$$

Here, the σ_x^o, σ_y^o and β are nominal values of the transverse beam sizes and betatron function, respectively. $E_o, \sigma_z^o, \sigma_{\epsilon^o}$ and $\epsilon' (= E - E_o)$ are the nominal energy, nominal bunch length, relative energy spread and energy deviation due to a random process, respectively.

2. *Random process* When a longitudinal random process, such as beam-residual gas bremsstrahlung, occurs, the energy of a particle is varied by

$$E = E - \frac{\epsilon'}{E_o \sigma_{\epsilon^o}}, \quad (3)$$

where ϵ' is given by values between the minimum cutoff energy and energy aperture of the beam.

When a transverse random process, such as beam-residual gas scattering, occurs, the momenta of a particle are varied by

$$P = P - \frac{\theta}{\sigma'_x}, Q = Q - \frac{\theta}{\sigma'_y}. \quad (4)$$

Here, the scattering angle (θ) is given by values between the minimum cutoff angle and the transverse aperture of the beam. $\sigma'_x = \sigma_x/\beta_x$ and $\sigma'_y = \sigma_y/\beta_y$, where, $\sigma_x, \sigma_y, \beta_x$ and β_y are the horizontal beam size, the vertical beam size, the horizontal betatron function and vertical betatron function at the position where the scattering process occurs, respectively.

3. *Betatron oscillation*
4. *Synchrotron oscillation*
5. *Synchrotron radiation*

3 BEAM TAIL DISTRIBUTIONS DUE TO THE RARE RANDOM PROCESSES

We performed a weak-strong simulation with 40000 macroparticles in the phase spaces. The results of a simulation for the machine parameters of PLS will be obtained.

3.1 Beam-Residual Gas Bremsstrahlung

An electron with energy E_o , which passes a molecule of the residual gas, is deflected in the electric field of the nucleus. The electron loses its energy due to the radiation emitted when an electron is deflected. There is a certain probability that a photon with energy u is emitted, producing an electron with energy E' , where $E' + u = E_o$. The differential cross section for an energy loss due to bremsstrahlung between E and $E + dE$ is given by [3]

$$d\sigma = 4\alpha r_e^2 Z(Z+1) \frac{du}{u} \frac{E'}{E_o} \left[\left(\frac{E_o^2 + E'^2}{E_o E'} - \frac{2}{3} \right) \log \frac{183}{Z^{1/3}} + \frac{1}{9} \right], \quad (5)$$

where Z, α and r_e denote the atomic number, the fine-structure constant and the classical electron radius, respectively. If we expand Eq.(5) by $\frac{u}{E_o}$ and take first-order term, we obtain

$$d\sigma_{BR} = 4\alpha r_e^2 Z(Z+1) \left(\frac{4}{3} \log \frac{183}{Z^{1/3}} + \frac{1}{9} \right) \frac{du}{u}. \quad (6)$$

We assume that CO molecule uniformly exists in the ring, so that

$$N = Q\sigma_{BR}c, \quad (7)$$

where σ_{BR} is the cross section of the beam-residual gas bremsstrahlung occurring between the minimum cutoff energy and maximum energy, c is the velocity of the light and Q is the number of gas molecules in a unit volume, which is given by $Q = 2.65 \times 10^{20} n P_a$. Here, n is the number of atoms in each gas molecule and P_a is the partial pressure of the gas in pascals.

3.2 Beam-residual gas scattering

The cross section of the elastic scattering with an atom is given by[3]

$$\frac{d\sigma}{d\Omega} = \left(\frac{2Zr_e}{\gamma} \right)^2 \frac{1}{(\theta^2 + \theta_{min}^2)^2}, \quad (8)$$

where Ω is the solid angle, θ the scattering angle, Z the atomic number, r_e the classical electron radius, γ the Lorentz factor and the screening of the atomic electrons is accounted by the angle θ_{min} , which is determined by the uncertainty principle as $\theta_{min} = Z^{1/3}\alpha/\gamma$, where α is fine-structure constant. If we consider the scattering occurring between angle θ_a and angle θ_b in the betatron phase space, we obtain

$$\sigma_{BS} = 2\pi \left(\frac{2Zr_e}{\gamma} \right)^2 \left[\frac{1}{2(\theta_a^2 + \theta_{min}^2)} - \frac{1}{2(\theta_b^2 + \theta_{min}^2)} \right]. \quad (9)$$

We also assume that there is only CO molecule so that $N = Q\sigma_{BS}c$. The scattering angle when a particle undergoes scattering can be obtained as follows. First, calculate the probability(P) that is scattered at higher angles θ than minimum scattering angle θ_a , and generate one uniform random number ($0 \leq x \leq 1$) each turn to decide whether the scattering occurs or not. If $x < P$, the scattering angle is defined by

$$\theta = \theta_a / \sqrt{R}, \quad (10)$$

where R ($0 < R < 1$) is the other uniform random number.

On the other hand, beam-residual gas scattering causes the changes of momenta of a particle in the horizontal and the vertical directions. Then, third random number is used to define azimuthal angle ϕ which is the angle between the horizontal and scattering planes. To obtain the changes of momenta due to the scattering in the normalized momenta, we have to multiply $\theta_x = \theta \cos \phi$ and $\theta_y = \theta \sin \phi$ with the value $\beta_x/\sigma_x, \beta_y/\sigma_y$, respectively, taken at the position where the elastic scattering takes place. It follows that vertical distribution of a beam is more affected than horizontal distribution by the elastic beam-residual gas scattering due to the relation $\beta_y/\sigma_y > \beta_x/\sigma_x$.

3.3 Intra-beam scattering

Particle scattering in a bunch is called intra-beam scattering. In the moving frame of the bunch the motion becomes purely transverse, neglecting the slow synchrotron motion. Coulomb scattering will occur for particles having different transverse velocities and will transfer their transverse momenta into longitudinal momenta [4]. The intra-beam scattering differential cross section for electrons is given by the Moller formula

$$\frac{d\sigma}{d\Omega} = \frac{4r_o^2}{(v/c)^4} \left[\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta} \right] \quad (11)$$

where v is the relative velocity in the c.m system. The momentum transfer into the longitudinal direction is $\Delta P =$

$p_x |\cos \xi|$, $2p_x$ being the relative transverse momentum. The particle is lost if $\gamma \Delta P \geq \epsilon_{RF}$ where ϵ_{RF} is the momentum acceptance of the RF. These particle losses contribute to the Touschek lifetime.

4 LIFETIME AS A FUNCTION OF APERTURES

Once a particle's amplitude exceeds an aperture, this particle would be lost. The lost particles are counted at one position of the ring per turn by comparing amplitudes of the particle with the apertures. Beam lifetime is defined by

$$\tau = \frac{N}{-\frac{dN}{dt}}, \quad (12)$$

where N is initial number of particles in a beam and $-\frac{dN}{dt}$ is number of the particles that exceed each aperture. We obtain the lifetime by using average β in stead of $\beta(s)$ in the ring. In our simulation we use vacuum pressure of 0.6 nTorr , $5.5\text{m } \beta_x$ and $4\text{m } \beta_y$ as average betatron function value. Table-2 shows beam lifetimes obtained from our simulation method. The lifetimes are obtained when the horizontal, the vertical and the energy apertures are assumed to $100\sigma_x$, $80\sigma_y$ and 1.5% ($E=15$), respectively. The estimated beam lifetime shows a good agreement with operational beam lifetime in PLS storage ring that is observed to be 17 hours in 180 mA of 400 bunches.

5 CONCLUSION

We have established the simulation method to obtain beam distribution due to the incoherent random processes. This simulation method provides a simple and fast means to obtain the tail distributions and to estimate the beam lifetimes due to various random processes in the storage rings. Beam tail distributions in the PLS storage ring are estimated by the simulation method. It is also shown that simulated beam lifetime agrees well with that of normal operation in PLS storage ring.

6 REFERENCES

- [1] Eun-San Kim, Part. Accel. 56, 249 (1997).
- [2] Eun-San Kim, Part. Accel. 63, 1 (1999).
- [3] Heiliter, W., The Quantum Field Theory, Oxford Univ. Press (1954).
- [4] J. Le Duff, CERN 95-06, p.573 (1995).

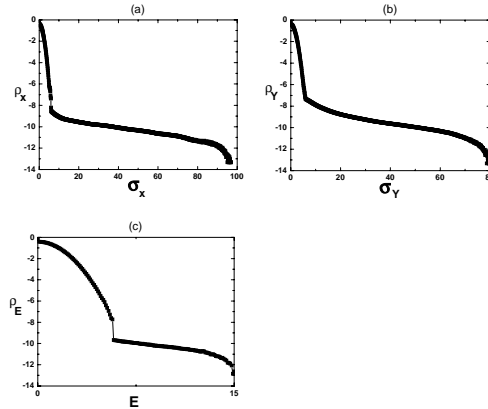


Figure 1: (a) Horizontal, (b)vertical and (c) energy density distributions in a logarithmic scaled obtained from the simulation method in the PLS storage ring. Beam current is 0.45 mA . Vacuum pressure in the ring is 0.6 nTorr .

Table 1: Parameters of the PLS storage ring

Energy	GeV	2.5
Circumference	m	280.56
Natural Bunch Length	$\sigma_z (mm)$	8
Energy Spread	σ_E	8.5×10^{-4}
Particles/bunch	N	2.8×10^9
Beam Current	mA	180
Betatron tune	ν_x/ν_y	14.28/8.18
Synchrotron Tune	ν_s	0.01
Emittance	nm	18.7
Damping time	$ms(x/z)$	8.5/4.2
Beam size	$\mu s(x/y)$	320/85
Number of bunches		400

Table 2: Simulated lifetimes under the Table-1.

Effect	Lifetime
Beam-residual gas scattering	292 hours
Beam-residual gas bremsstrahlung	461 hours
Touschek	18 hours
Combined effect	16.5 hours