NONLINEAR BEHAVIORS IN BUNCH-LENGTHENING BY LOCALIZED IMPEDANCES IN ELECTRON STORAGE RING

EUN-SAN KIM and MOOHYUN YOON, Pohang Accelerator Lab., POSTECH, Pohang, Korea

Abstract

The equilibrium longitudinal particle distribution of a bunch in an electron storage ring is investigated using a localized constant wake function and a localized δ wake function. We also investigated the composite wake effects on the particle distribution when both the constant wake and δ wake simultaneously exist in the ring. When moving around the parameter space, the system can show bifurcation phenomena and transition features between periodic states, which are not always reversible. We study this behavior using the Gaussian approximation and compare the results with multi-particle tracking. The results show qualitative agreement.

1 INTRODUCTION

When a beam circulates around a storage ring, wake force can be generated by an electromagnetic interaction between particles in the beam and the vacuum-chamber environment. The wake force may affect the distribution of particles in the beam. Various approaches to find the particle distribution in a beam have been investigated by many authers. With the Gaussian approximation, Hirata [1] investigated the distribution of particles in a beam by the model using a localized wake, which was assumed to be constant wake. Hirata $et\ al$. [2] then showed that the equilibrium bunch length in electron storage rings could have a cusp-catastrophe behavior, which had never been observed for smooth wake forces. Kim $et\ al$. [3] showed that several stable multi-periodic states in particle distributions of a beam can also exist for the constant wake.

The aim of this paper is to investigate the dynamic features of the beam distribution in the presence of the two localized wake sources in the ring. Here, we consider δ wake as well as the constant wake as the sources of the localized wakes. It seems that it is also interesting to investiagte the dynamical behaviors on the beam distribution for the composite cases of the constant wake and the δ wake in the ring. Characteristic features of the beam distribution will be investigated in terms of damping time, strength of wake force and synchrotron tune. We show that a system in the equilibrium state when two localized wake sources exist may present the different dynamic states from that when individual wake source exists.

2 THE MODEL

2.1 Basic Dynamics

We assume for simplicity that there are two localized wake sources in the ring. More realistic cases in which the wake function varies from position to position in the ring can also be studied by a straightforward extension of the present formalism. It is then convenient to introduce normalized longitudinal variables,

$$x_1 = \frac{\text{longitudinal displacement}}{\sigma_z},\ x_2 = \frac{\text{energy deviation}}{\sigma_E},$$

where σ_z is the natural bunch length and σ_E is the natural energy spread. The center of the bunch is x_1=0 ; $x_1 > 0$ corresponds to the rear part of the bunch. The motion of a particle in a ring can be modeled as follows:

1) Radiation

$$\begin{pmatrix} x_1' = x_1 \\ x_2' = \Lambda x_2 + (1-\Lambda^2)^{1/2}\hat{r} \end{pmatrix} \tag{1}$$

2) Wake

$$\begin{pmatrix} x_1' = x_1 \\ x_2' = x_2 - \phi(x_1) \end{pmatrix} \tag{2}$$

3) Synchrotron oscillation

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = U \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \tag{3}$$

where

$$U = \begin{pmatrix} \cos(2\pi\nu) & \sin(2\pi\nu) \\ -\sin(2\pi\nu) & \cos(2\pi\nu) \end{pmatrix}. \tag{4}$$

In the above equations, $\Lambda = \exp(-2/T_e)$, T_e being the synchrotron damping time divided by the revolution time, ν the synchrotron tune and $\hat{r}$ a Gaussian random variable with zero mean and unit standard deviation.

After one turn in the ring the motion of a particle can be represented by

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = U \begin{pmatrix} x_1 \\ \Lambda x_2 + (1-\Lambda^2)^{1/2}\hat{r} - \phi(x_1) \end{pmatrix}. \tag{5}$$

The wake force is

$$\phi(x_1) = \int_0^\infty \rho(x_1 - u)W(u)du, \tag{6}$$

where $\rho(x)$ is the longitudinal charge density, which is normalized to unity and $W(u)$ is the longitudinal wake function multiplied by eQ/σ_E, where e is the electron charge and Q the total charge in a bunch.

Here, we consider the constant and δ wake functions for the sake of simplicity: $W(u) = a\Theta(u)$ (Θ being the unit step function) and $W(u) = b\delta$ where a and b signify the strengths of the wake.

2.2 The Gaussian Model

Since the wake is assumed to vanish at a short distance behind the particles which produce it, we can neglect the multi-turn effects. Generally, we know that particles in front of a bunch lose energy due to wake fields. In order to meet this condition we note here that the sign of a and b in the wake functions should be positive.

Since it is not realistic to observe individual particles, we are more interested in some statistical quantities, such as

$$\bar{x}_i =< x_i >, \ \sigma_{ij} =< (x_i - \bar{x}_i)(x_j - \bar{x}_j) >, \tag{7}$$

and so on, where i, j are 1 or 2, which are the moments of the phase-space distribution $\Psi(x_1, x_2)$. In reality, we need all of the higher order moments to reproduce $\Psi(x_1, x_2)$. We always approximate $\Psi(x_1, x_2)$ as

$$\Psi = \frac{1}{2\pi\sqrt{det\sigma}} \exp[-\frac{1}{2}\sum_{i,j}\sigma^{-1}_{i,j}(x_i - \bar{x}_i)(x_j - \bar{x}_j)] \tag{8}$$

However, to make the model simple, we assume here that the distribution function in phase space always remains Gaussian, even under the influence of the wake force. We thus need to consider only the first and second order moments.

With the same treatment used in Ref. [1], each mapping in Eqs.(1)-(3) can be given as follows:
1) Radiation

$$\begin{aligned} \bar{x}'_1 &= \bar{x}_1, \ \bar{x}'_2 = \Lambda\bar{x}_2, & (9)\\ \sigma'_{11} &= \sigma_{11}, \ \sigma'_{12} = \Lambda\sigma_{12}, & \\ \sigma'_{22} &= \Lambda^2\sigma_{22} + (1 - \Lambda^2) & (10) \end{aligned}$$

2) Wake

$$\begin{aligned} \bar{x}'_1 &= \bar{x}_1, \ \bar{x}'_2 = \bar{x}_2 - <\phi>, & (11)\\ \sigma'_{11} &= \sigma_{11}, \ \sigma'_{12} = \sigma_{12} - <(x_1 - \bar{x}_1)\phi>, & \\ \sigma'_{22} &= \sigma_{22} - 2 <(x_2 - \bar{x}_2)\phi> + <\phi^2> - <\phi>^2 . & (12) \end{aligned}$$

By further calculations, we obtain for the constant wake [2]

$$\begin{aligned} \bar{x}'_1 &= \bar{x}_1, \ \bar{x}'_2 = \bar{x}_2 - a/2, & (13)\\ \sigma'_{11} &= \sigma_{11}, \ \sigma'_{12} = \sigma_{12} - a\frac{\sqrt{\sigma_{11}}}{2\sqrt{\pi}}, & \\ \sigma'_{22} &= \sigma_{22} - a\frac{\sqrt{\sigma_{12}}}{\sqrt{\pi\sigma_{11}}} + a^2/12. & (14) \end{aligned}$$

we also obtain for the δ wake [4]

$$\begin{aligned} \bar{x}'_1 &= \bar{x}_1, \ \bar{x}'_2 = \bar{x}_2 - \frac{b}{2\sqrt{\pi\sigma_{11}}}, & (15)\\ \sigma'_{11} &= \sigma_{11}, \ \sigma'_{12} = \sigma_{12}, & \\ \sigma'_{22} &= \sigma_{22} + \frac{b^2}{2\sqrt{3}\pi\sigma_{11}} - \frac{b^2}{4\pi\sigma_{11}}. & (16) \end{aligned}$$

3) Synchrotron oscillation

$$\begin{aligned} \bar{x}'_i &= \sum_j U_{ij}\bar{x}_j, & (17)\\ \sigma'_{ij} &= \sum_{h,k=1}^{2} U_{ij}\sigma_{hk}U_{jk}. & (18) \end{aligned}$$

As it can be easily shown, $\bar{x}_i$ falls to a period-1 fixed point for the constant wake as follows:

$$\bar{x}^{\infty}_1 = \frac{-a}{2\tan(2\pi\nu)}\frac{1}{1+\Lambda}, \ \bar{x}^{\infty}_2 = \frac{a}{2}\frac{1}{1+\Lambda}. \tag{19}$$

$\bar{x}_i$ also falls to a period-1 fixed point for the δ wake as follows:

$$\bar{x}^{\infty}_1 = \frac{-b}{2\sqrt{\pi\sigma_{11}}\tan(2\pi\nu)}\frac{1}{1+\Lambda}, \ \bar{x}^{\infty}_2 = \frac{b}{2\sqrt{\pi\sigma_{11}}}\frac{1}{1+\Lambda}. \tag{20}$$

3 DYNAMICAL BEHAVIOR OF THE GAUSSIAN MODEL

First, let us consider the dynamical behaviors of the system for the constant wake function. In the previous paper [3] we studied the dynamic states in the parameter space set: $0.01 \leq \nu \leq 0.3$, $1 \leq T_e \leq 1500$ and $0 < a \leq 45$, which shows four different behaviors, depending on the synchrotron tune. The parameter space shows that several types of equilibrium states such as period-1, period-2, period-3, period-4 and coexistences of these states can exist stably depending on the parameters space.

Second, let us consider the dynamical behaviors of the system for the δ wake function. It is observed that the only equilibrium state is the period-1 state in the parameter ranges of synchrotron tunes from 0.01 to 0.3, T_e from 1 to 1000 and b from 0 to 40, irrespective of the initial conditions.

Third, let us consider the dynamical behaviors of the system for the composite wake effects $(a\Theta + b\delta)$ of the constant wake and the δ wake. Here let us consider the case of $\nu = 0.1866$. When we slowly move the system along the path from $T_2 = 22$ to $T_e = 1$ in Fig. 1.(a), for the case of $a \neq 0$, $b = 0$ and $\nu = 0.01$, it shows period-3 state until $T_e = 7$. It then becomes the period-2 state at $T_e = 6$ and remains so until $T_e = 2$. It then becomes the period-1 state at $T_e = 1$. When we slowly move the system along the path from $T_e = 22$ to $T_e = 1$, for the case of $a \neq 0$, $b \neq 0$ and $\nu = 0.01$, it shows transitions of period-3 state to period-2 state or period-1 state, as shown in Fig. 1.(b).

In conclusion, when two different kinds of wakes exist the dynamical behavior of the system can be changed depending on the relative magnitude of wake force strengths in the two wakes: they may show different stability features in the parameter space from the case of a single wake while slowly changing the parameters of the system.

4 THE MULTI-PARTICLE TRACKING

In this section, we discuss the reliability of the results of the model presented above. The model is based on the Gaussian approximation for particle distribution, whereas the real distribution can be far from a Gaussian shape. We thus need a comparison with the multi-particle tracking in order to see whether the results obtained by the model are merely those coming from too many simplifications in the model. We apply Eq. (5) to the phase-space coordinates of an ensemble of 10000 particles.

First, we investiagte the equilibrium states of the particle distribution for the case of the constant wake function and $\nu = 0.1866$. It is observed that the parameter space (T_e, a) is divided into three parts: the period-1 region, where only the period-1 state is stable; the period-2 region, where only the period-2 is stable and the period-1-2 region, where the system chooses either the period-1 or period-2 state according to the initial condition. These properties was in accordance with the Gaussian model.

Second, we observe the equilibrium states of the particle distribution for the case of the δ wake function. It is shown that the equilibrium state in the multi-particle tracking presents the period-1 state. It is observed that the only equilibrium state is the period-1 state in the parameter ranges of synchrotron tunes from 0.01 to 0.3, T_e from 1 to 1000 and b from 0 to 40, irrespective of the initial conditions. These properties are also in accordance with the Gaussian model.

Third, we observe the equilibrium states of the particle distribution for the $\nu = 0.1866$ due to the composite effects of the constant wake and the δ wake. It is observed that the equilibrium states in the multi-particle tracking present the period-2 or period-1 states. We see that periodic states which occur in the Gaussian model also appear in multi-particle tracking.

As a result, the comparison of results of multi-particle tracking with those of the Gaussian model show qualitative agreement in the existence of the period-doubling bifurcation and in the transition between the periodic states in the presence of δ wake. When moving around parameter space, the system shows a transition which is not always reversible. This feature is also shown in analysis of the Gaussian model and the multi-particle tracking.

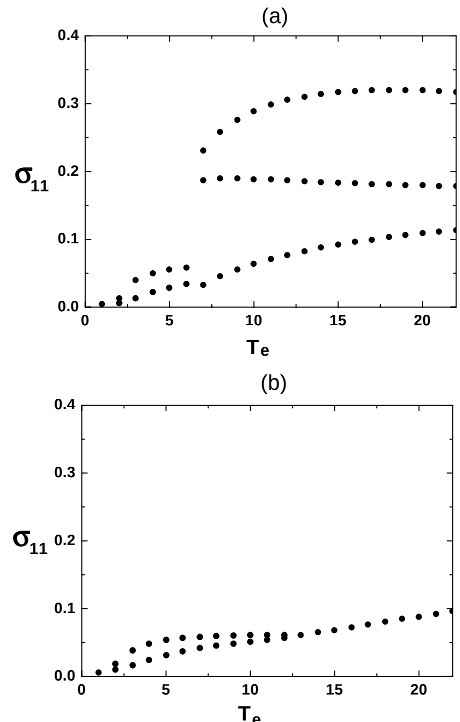

Figure 1: Equilibrium value of σ_{11} when we slowly move the system from T_e=22 to T_e=1. ν=0.01, a=30 and [b=0 for (a) and b=3 for (b)].

5 CONCLUSION

We have found nonlinear behaviors and stable periodic states on the beam distribution in both the Gaussian model and multi-particle tracking. The model calculations showed the multi-periodic states in the dynamic states of a distribution of particles in a beam for the constant wake function. The model calculations also showed the period-1 state in the dynamic state of a distribution of particles in a beam for the δ wake function. These facts are also confirmed by multi-particle trackings. When both the constant wake and δ wake simultaneously exist, the parameter space that the periodic states exist can be changed the under the effect of the δ wake. Overall, the Gaussian model seems to be useful in describing qualitatively the particle distribution in the longitudinal phase space. We showed dynamical states of the equilibrium bunch length and their stability for a localized constant wake force and a localized δ wake with the Gaussian model in electron storage rings. Despite the simplification of the Gaussian model, the model showed good agreement with the multi-particle tracking results. When the constant wake and the δ wake exist, it is shown that the transition between the periodic states can occur depending on the relative strength of the two wakes. In order to compare with the more realistic ring parameters, we need to extend our calculation to the case of many different localized impedances in the ring using our model.

6 REFERENCES

[1] K. Hirata, Part. Accel. 22, 57 (1987).

[2] K. Hirata, S. Petracca and F. Ruggiero, Phys. Rev. lett. 66, 1693 (1991).

[3] Eun-San Kim and M. Yoon, Multi-periodic States and Bifurcations in Bunch Lengthening by a Localized Impedance in Electron Storage Ring, IEEE Trans. Nucl. Sci. Vol.48, No. 2 (2001) 181-188.

[4] Eun-San Kim and K. Hirata, KEK Report 95-5 (1995).