

# Coherent beam oscillations and transverse impedance in the SPS

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## Abstract

We describe experimental studies of single bunch coherent oscillations in the SPS. The coherent tune shift has been measured as a function of current and growth/decay rates of head-tail modes have been measured over a wide range of chromaticities, providing information on the frequency dependence of the SPS impedance. Simulations of coherent tune shifts and growth/decay rates in the presence of broadband wake fields, space charge, chromaticity and detuning with amplitude are then matched to the observations in order to establish an appropriate model for the SPS transverse impedance.

## 1 INTRODUCTION

There has been a major effort to reduce the SPS impedance in view of the future use of the SPS as LHC injector. The main hardware changes happened during the long shutdown between 2000 and 2001, see [1]. The changes on the hardware side were followed by a series of measurements, both in the transverse and longitudinal [2] planes.

We report here on the measurements in the transverse planes. The principle of impedance measurements based on coherent tune shift and head-tail growth and decay rates is based on Sacherer et al. [3].

Table 1: Relevant SPS parameters

variable	symbol	value
momentum	$p$	26 GeV/c
revolution frequency	$T_0$	43347.3 Hz
betatron tunes	$Q_{x,y}$	$\sim 26.6$
synchrotron tune (at 3 MV)	$Q_s$	$6.9 \times 10^{-3}$
momentum compaction	$\alpha_c$	$1.856 \times 10^{-3}$
bunch population	$N$	$0.5 - 10^{10}$
rms bunch length	$\sigma_t$	$\sim 0.5$ ns

The effect on the betatron motion of the bunch centroid is a complex frequency shift of

$$\Delta\omega_\beta = \frac{N e c}{4\sqrt{\pi} \omega_\beta (E/e) T_0 \sigma_t} i(Z_\perp)_{\text{eff}} \quad (1)$$

where  $N$  is the number of particles in the bunch,  $E$  its energy,  $T_0$  the revolution frequency,  $\sigma_t = \sigma_z/c$  the bunch length and  $\omega_\beta = 2\pi Q f_{\text{rev}}$  the betatron frequency. Eq. (1) applies to both  $x, y$ . For SPS parameters see Table 1.  $(Z_\perp)_{\text{eff}}$  is the effective transverse impedance, which for a round chamber is the impedance convoluted with a weight

function  $h$  representing the longitudinal bunch shape

$$(Z_\perp)_{\text{eff}}(\omega_\xi) = \int_{-\infty}^{\infty} Z_\perp^\perp(\omega) h_m(\omega - \omega_\xi) d\omega. \quad (2)$$

For the 0-mode coherent bunch oscillations analysed here and Gaussian bunches the weight function is

$$h_0(\omega) = \frac{\sigma_t}{\sqrt{\pi}} e^{-(\omega\sigma_t)^2},$$

that is, a Gaussian in  $\omega$  with an rms width of  $1/(\sqrt{2}\sigma_t)$ , or equivalently a Gaussian in frequency  $f = \omega/(2\pi)$  of width  $\sigma_f = 1/(2\pi\sqrt{2}\sigma_t)$ . For  $\sigma_t = 0.5$  ns, the rms width in frequency is 225 MHz and the weight has dropped to 5% at  $f = 0.275/\sigma_t = 551$  MHz. The Gaussian weight function in Eq. (2) is centred at  $\omega_\xi$  which depends on the chromaticity  $\xi = Q'/Q = \frac{\Delta Q}{Q} / \frac{\Delta p}{p}$  and the phase slip factor  $\eta$  according to

$$\omega_\xi = \xi \frac{\omega_\beta}{\eta}. \quad (3)$$

For the SPS at 26 GeV, we have numerically  $f_\xi = \omega_\xi/(2\pi) = \xi \cdot 2.08$  GHz.

The SPS has a rather flat vacuum chamber (with V/H ratios from 1/3 to 1/4 in the bending sections). This asymmetry enhances the tune shift in the vertical, compared to the horizontal plane [4].

## 2 BEAM PARAMETERS

Impedance measurements are ideally done at identical beam conditions, varying only one parameter like the intensity in case of the coherent tune shift or chromaticity in case of the growth/decay rate measurements. To the extent, that intensity variations were provoked by vertical scraping, we also have a variation of the vertical beam size with intensity. Typical normalized emittances were  $3 \mu\text{m}$  horizontally and between 0.3 to  $3.0 \mu\text{m}$  vertically depending on intensity. We do not expect (to first order) a bias of our results on the 0-mode tune shift and growth and decay rates on the transverse beam sizes. Transverse beam sizes were recorded for all conditions. They were used to estimate incoherent tune shifts from space-charge and as input to detailed simulations, confirming the smallness of second order effects.

The bunch length directly enters in the relation between coherent tune shift and impedance, see Eq. (1). The 200 MHz rf-voltage was matched at injection to values of about 0.7 – 0.8 MV to minimize bunch length oscillations. Between injection/capture and the measurements, the rf-voltage is ramped adiabatically in 100 ms to a constant measurement level. Our standard choice was 3 MV, resulting in shortened bunches (of about  $\sigma_t = \sigma_z/c \approx 0.5$  ns

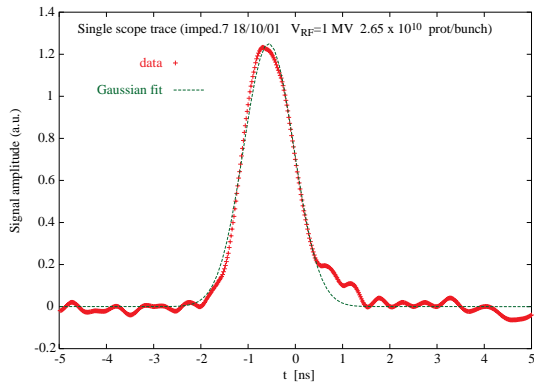


Figure 1: Example of a bunch shape with a Gaussian fit.

rms length) with enhanced tune shifts with intensity. Bunch length data have been acquired using a digital scope. Each scope signal consists of 50 bunch traces, recorded every 200 turns during the time of the measurements of the transverse oscillations. In the off-line analysis, each trace has been fitted by a Gaussian shape, as illustrated in Fig. 1. The average of the 50 numbers is used as bunch length  $\sigma_t$  in the further analysis. Bunch length oscillations of up to about  $\pm 10\%$  have been observed. A small increase in bunch length with intensity was observed (compatible with a linear rise of roughly 0.1 ns for  $1 \times 10^{11}$  protons).

### 3 TUNE SHIFT AND GROWTH RATE

Fig. 2 shows a comparison of the observed tune shifts with intensity in the vertical plane, measured in the year 2000 before [5] and in 2001 after the major part of the hardware changes. The beam conditions were very similar in both cases.

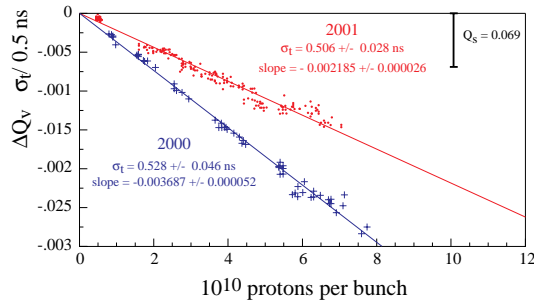


Figure 2: Comparison of measured coherent tune shifts (normalized to bunch length) with intensity in the vertical plane in the years 2000 (before) and 2001 (after the major part of the impedance reduction program).

The observed tune shifts with current in the horizontal plane are much smaller, see Fig. 3. Table 2 summarizes results on the imaginary part of the effective horizontal and vertical impedances, as calculated from Eq. (1) for several series of measurements in the years 2000 and 2001. The errors quoted were obtained from the spread of results within one year. Details are given in [6, 7].

Fig. 4 shows the principle of the determination of growth and decay rates. The growth and decay rates are obtained

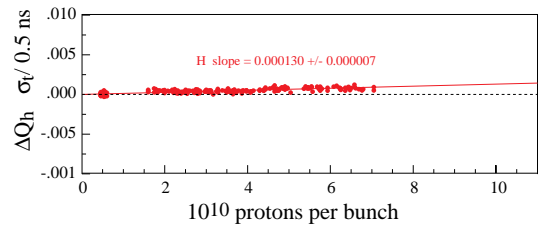


Figure 3: Coherent tune shift (normalized to bunch length) with intensity in the horizontal plane recorded in 2001.

 Table 2:  $\text{Im } Z_{\perp}$  from the coherent tune shift with intensity.

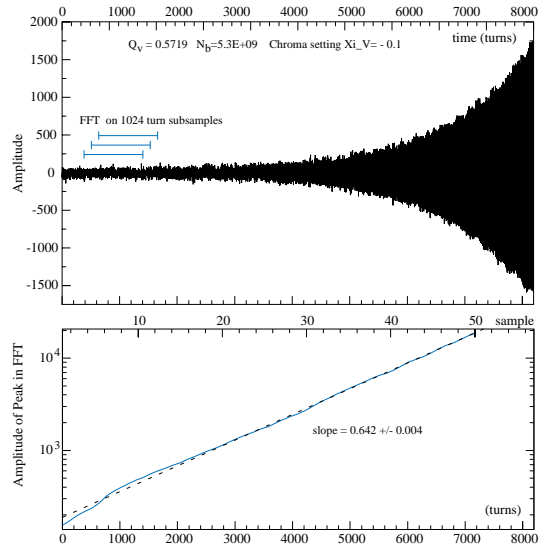
year	$\text{Im } Z_x$ M $\Omega$ /m	$\text{Im } Z_y$ M $\Omega$ /m
2000	$-0.9 \pm 1.8$	$26 \pm 3$
2001	$-0.35 \pm 0.53$	$18.4 \pm 0.5$

from the evolution of the tune peak heights of sub-samples.

Positive chromaticities result in negative growth ( $\tau < 0$ ) or damping (as expected for above transition). A linear behaviour on chromaticity can be expressed in terms of a constant, effective  $Z_{re}$  according to

$$-\frac{\sigma_t}{\tau N} \propto \text{Re}(Z_{\perp})_{\text{eff}} = \frac{\omega_{\xi}}{\omega_r} Z_{re} \quad (4)$$

where  $Z_{re} = R_{\perp}/Q^2$  in case of a broad band resonator. The effective transverse impedance is sampled at frequencies around  $\omega_{\xi} = \xi \frac{\omega_{\beta}}{\eta}$ . The growth/decay measurements were performed at low intensity  $N \sim 5 \times 10^9$  to allow to cover a broad range of chromaticities with reasonable growth/decay times. Fig. 5 shows measured vertical growth


 Figure 4: Growth/decay determination. The vertical chromaticity was set slightly negative at the start of the measurement. Vertical turn to turn amplitudes recorded with the tune meter on successive turns are shown on the top and the height of the tune peak of sub-samples is displayed with logarithmic y-scale in the bottom of the picture. The time constant  $\tau$  of the exponential growth rate is obtained from the slope of the straight dashed line.

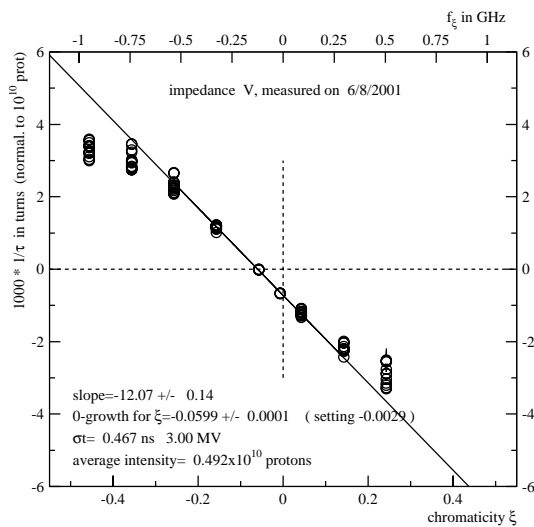


Figure 5: Growth rate of the vertical head-tail 0-mode (in units of  $10^{-3}$  turns $^{-1}$ ) measured as function of chromaticity.

and decay rates. A straight line was fitted to the data at  $|\xi_y| < 0.25$  to extract  $Z_{re}$ . From the 2001 data, we obtain  $Z_{re,x} = 5.1$  M $\Omega$ /m and  $Z_{re,y} = 9.2$  M $\Omega$ /m.

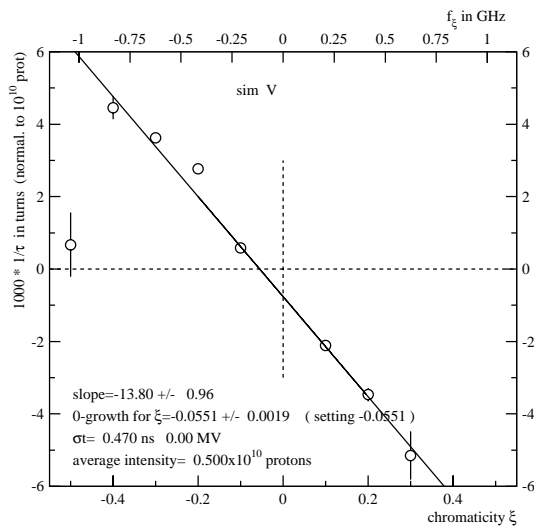


Figure 6: Growth rate of the vertical head-tail 0-mode from simulation for a 1.3 GHz wide band oscillator.

The same analysis can also be applied to our detailed simulation. Results obtained in simulation for a simple broad band oscillator model are shown in Fig. 6. Some of the main features are already quite well reproduced and a more detailed modelling is planned.

For larger positive chromaticity, higher order head-tail modes become visible as extra peaks in the tune spectrum, see Fig. 7. Their frequency, initial amplitude and damping rates can be determined separately. We have started to use this information as additional input for our modelling and plan further analysis and measurements on higher modes including a search for mode-coupling.

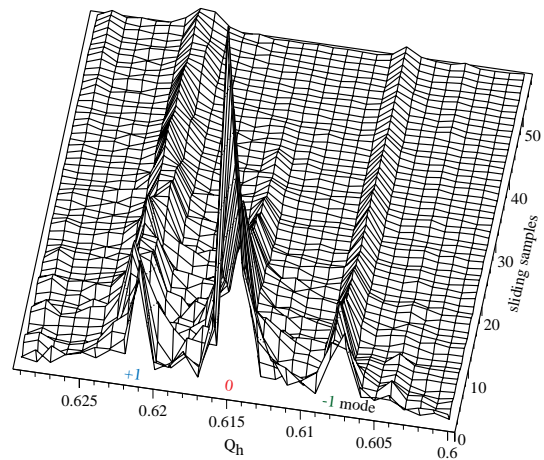


Figure 7: Tune evolution in the horizontal plane for positive chromaticity ( $\xi_x = 0.4$ ), following a single kick. Three modes are clearly visible. The decay of the 0-mode is much faster than for the +1 and -1 modes.

## 4 SUMMARY

The comparison of coherent tune shift measurements with intensity clearly demonstrate a reduction of the effective vertical impedance by about 40%. Additional information was obtained from measurements at low intensity over a wide range of chromaticities. This is being compared to detailed simulations to extract the frequency dependence of the impedance and as basis for a consistent modelling of single bunch and multi-bunch effects in the SPS.

## 5 REFERENCES

- [1] P. Collier, M. Ainoux, *et al.*, “Reducing the SPS Machine Impedance.” this conference.
- [2] T. Bohl, T. Linnecar, and E. Shaposhnikova, “Impedance Reduction in the CERN SPS as seen from Longitudinal Beam Measurements.” this conference.
- [3] F. Sacherer, “Single-beam collective phenomena, Transverse, Bunched Beams.” Proceedings of the 1976 Erice school on accelerators, M.H. Blewett (Ed.), CERN yellow report 77-13, (1977) p. 198 ff.
- [4] J. Gareyte, “Impedances: Measurements and Calculations for Non-symmetric Structures.” this conference.
- [5] G. Arduini *et al.*, “Measurements of the SPS transverse impedance in 2000.” Prepared for IEEE Particle Accelerator Conference (PAC 2001), Chicago, Illinois, 18-22 Jun 2001.
- [6] G. Arduini, H. Burkhardt, F. Zimmermann, and M. Zorzano, “Measurements of the SPS single bunch coherent tune shifts and head-tail growth rates in the Year 2000.” CERN, SL-Note-2001-002 MD.
- [7] H. Burkhardt, G. Rumolo, and F. Zimmermann, “Measurements of SPS Single-Bunch Coherent Tune Shifts and Head-Tail Growth Rates in the Year 2001.” CERN, SL Note 2001-043 (MD), 7-12-2001.