SYNCHROBETATRON DYNAMICS WITH A RADIO-FREQUENCY QUADRUPOLE

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Abstract

Variation of the betatron tune over the bunch length with a radio frequency (RF) quadrupole is proposed in [1] as a way to increase the threshold of the transverse mode coupling instability in storage rings. A significant effect can be achieved if the betatron tune modulation is comparable with the synchrotron tune. However, the required time-varying field introduces a strong coupling between the transverse and longitudinal degrees of freedom, which imposes a negative impact on the single particle dynamics, leading to the synchrobetatron resonances. In this paper we present the multi-mode analysis of the strong head-tail instability and results of numerical simulation of the single particle motion in a storage ring with an RF quadrupole, and discuss its applicability for suppression of the transverse mode coupling instability.

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1 INTRODUCTION

The transverse mode coupling instability (TMCI) is known as one of the intensity-limiting mechanisms in circular accelerators [2]. Similar effect in linear accelerators where the particles do not perform synchrotron oscillations is named the beam breakup. It was successfully cured by introducing a difference in transverse focusing along the bunch, thus driving the head and the tail particles off resonance [3].

In a circular accelerator the longitudinal positions of particles in a bunch are not fixed, rather they oscillate in both the transverse and longitudinal directions. Nature of the head-tail instability is different from the linac case but still it seems possible to increase the instability threshold by adding a longitudinal gradient of the betatron tune (e.g. with an RF quadrupole). A theoretical analysis of the RF quadrupole action on TMCI using the model of multiple hollow beams to represent the radial modes was presented in [1], and the factor of 3 increase in the instability threshold was predicted for the betatron tune variation over the bunch length equal to the synchrotron tune. A numerical simulation for the Very Large Hadron Collider (VLHC) showing basically the same result was done in [4]. These predictions are consistent with results of an alternative approach [5] using a bunch with a Gaussian longitudinal distribution and the Hermitian modes.

In all these references resonances were disregarded in the analysis of the transverse mode dynamics implying that a small enough synchrotron tune helps to stay off the synchrobetatron resonances and thus to avoid instability in single particle motion. In a conventional electron machine, however, the synchrotron tune much smaller than 0.005 is hard to achieve, which hinders operation with large betatron tune modulation. In effect, the RF quadrupole introduces the betatron tune spread, increasing the beam footprint in the tune diagram and limiting the choice of the operating point.

In this paper we compare the effect of the RF quadrupole on the collective transverse modes with results of particle tracking, done with the full account of the betatron and synchrotron oscillations in a storage ring equipped by an RF quadrupole. One transverse degree of freedom is considered since typically the coupling between the horizontal and vertical oscillations is small. The subject of our study concerns with the fast synchrobetatron instabilities, hence we do not regard the radiative effects.

Section 2 presents the equations of motion. In Section 3 the frequency spectrum of the single particle betatron oscillations is discussed and the spectrum width is estimated analytically. Coherent synchrobetatron oscillations in a system with collective head-tail interaction are considered in Sections 4,5 where the effect of the RF quadrupole on the TMCI threshold is described by an analytical model and compared with the macroparticle tracking results.

2 EQUATIONS OF MOTION

Consider a thin quadrupole with time-varying gradient at some azimuth s_0 of the ring. Then a change of the particle's transverse momentum p after one passage is given by

$$\Delta p = Gx\sin(\Omega t)\,, \tag{1}$$

G is the gradient integrated over the quadrupole length, and frequency Ω is a multiple of the revolution frequency ω_0. For a bunch with the length $\sigma_s \ll c/\Omega$, Eq. (1) can be linearized. Then, we write the Hamiltonian H in the smooth variables: betatron coordinate x, momentum p, longitudinal position z with respect to the synchronous particle, and relative energy deviation $\delta = \Delta E/E_0$:

$$2H = p^2 + k_\beta^2 x^2 + \alpha_p \delta^2 + k_s^2 z^2/\alpha_p + g\delta_\Pi(s-s_0)x^2 z\,. \tag{2}$$

Here $k_\beta = \nu_\beta\omega_0/c$ and $k_s = \nu_s\omega_0/s$ are the betatron and synchrotron oscillation wave numbers, α_p is the momentum compaction, and $g = \Omega G/c$. The last term in Eq. (2) describes the synchrobetatron coupling due to the RF quadrupole. Its form corresponds to the resonant condition $2\nu_\beta + m\nu_s = n$, where m, n are integers, m coming from the betatron tune modulation at ν_s. Even m means that $(m/2)$th sideband of the betatron tune hits a half-integer resonance, and odd m corresponds to coupling of the synchrobetatron modes with the "reflected" modes belonging to the FFT-aliased spectrum.

3 SINGLE-PARTICLE DYNAMICS

For small g, we apply the first-order perturbation analysis to Eq. (2) and consider, as usual, only the transverse motion, while z is left unperturbed, $z = \sigma_s \cos(k_s s + \phi_s)$. Then

$$x \propto \cos(k_\beta s + \Delta(\sin(k_s s + \phi_s) - \sin\phi_s) + \phi_\beta), \quad (3)$$

where we defined the index of phase modulation caused by the RF quadrupole,

$$\Delta = \frac{\Delta\nu}{\nu_s}, \qquad \Delta\nu = \frac{g\sigma_s}{4\pi k_\beta}, \quad (4)$$

$\pm\Delta\nu$ being the tuneshift for a particle at the endpoints of the bunch. The spectral amplitude of the mth sideband of the betatron tune is known to be determined by the Bessel function, $J_m(\Delta)$.

For $\Delta \gg 1$, the higher-order sidebands become important in the oscillation spectrum. A rough estimate of the upper bound of m is $\sim 2.718\Delta/2$. Thus, the single line of the synchrobetatron mode is replaced with a cluster of the total bandwidth of $\sim 2.7\Delta\nu$. This conclusion is well confirmed by our particle tracking data. The finite size of the beam footprint caused by the RF quadrupole complicates the choice of the operating point in the betatron tune space.

4 SYNCHROBETATRON MAPPING

Following [6], we outline the multi-mode TMC formalism based on one-turn synchrobetatron mapping of the bunch using the circulant matrices [7].

We use here the so-called "hollow beam" model. It assumes that all particles of the bunch are evenly spread over the synchrotron phase with equal synchrotron amplitudes. The bunch is divided into N mesh elements, each characterized by its transverse dipole moment and its synchrotron phase. The dipole moment of the ith mesh, $1 \le i \le N$, is proportional to the transverse displacement x_i of the centroid of the particles populating this mesh, times the portion N_b/N of the bunch intensity, N_b, per mesh. The betatron motion will be described in terms of the normalized betatron variables, x_i and p_i, where p_i is the respective momentum. Thus the synchrobetatron motion in the bunch is characterized by a $2N$-vector X, where x_i and p_i are listed in the order corresponding to the mesh number, according to its synchrotron phase.

The synchrobetatron transformation of the above vector over the collider arc is done by $2N \times 2N$ matrix M,

$$M = C \otimes B, \quad B = \begin{pmatrix} \cos\mu_\beta & \sin\mu_\beta \\ -\sin\mu_\beta & \cos\mu_\beta \end{pmatrix},$$

where $\otimes$ denotes the outer product, B is the betatron oscillation matrix, C is the circulant matrix [7] with elements

$$C_{ij} = \frac{\sin N\varphi_{ij}}{N\sin\varphi_{ij}}, \quad \varphi_{ij} = \frac{1}{2}\Big(\mu_s - (N - i + j)\frac{2\pi}{N}\Big),$$

$1 \le i, j \le N$, and μ_β, μ_s are the betatron and synchrotron phase advances. With $N = 2m + 1$, the eigenvectors and eigenvalues of matrix M exactly correspond to the first $-m, \ldots, m$ synchrobetatron sidebands with the tunes $\nu_\beta - m\nu_s, \ldots, \nu_\beta + m\nu_s$, $\nu_{\beta,s} = \mu_{\beta,s}/2\pi$.

Note that the synchrotron oscillation in the circulant matrix formalism transports the dipole moment values around the circle formed by the mesh elements with fixed synchrotron phases (i.e., fixed longitudinal positions in the bunch), rather than performing a permutation of the meshes themselves.

The impedance element is introduced once on the revolution, in the same manner as in [6]. The action of RF quadrupole is modelled with thin-lens kicks proportional to the longitudinal position z of a mesh element and applied to each of them past the impedance. These two transformations, together with the synchrobetatron mapping above described, form the total one-turn mapping, whose eigenvalues are then computed to find the complex tunes of head-tail modes.

First we take the case of vanishing wake and consider the mode tunes in an 11-mode system as functions of the tune modulation Δ, see Fig. 1. As Δ grows, we can see the mode -4 merge with the "reflected" mode -5, then these two modes decouple, then the mode merging occurs to other modes, some of them hit the half-integer resonance, etc., and at $\Delta \sim 2$ the mode coupling is strong and complicated.

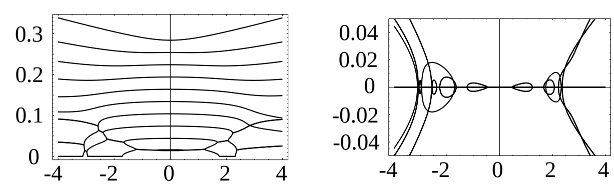

Figure 1: Real (left) and imaginary (right) parts of the synchrobetatron mode tunes vs the tune modulation Δ at $\nu_\beta = 0.134$, $\nu_s = 0.03$, and vanishing wake.

Next we take a constant wake as an illustration, and normalize its amplitude W times the bunch intensity so as the coherent tuneshift at the threshold of the mode 0 and -1 merge be equal to the standard value of $0.57\nu_s$ without the RF quadrupole. With the tune modulation $\Delta = 1$, the wake dependence is plotted in Fig. 2, showing a 3-fold increase in the threshold of the mode 0 and -1 merge due to RF quadrupole. However, on the whole, the system is unstable from the very beginning because of increment caused by the mode -4 merge with the "reflected" mode -5. Such a situation is typical for strong modulation, $\Delta \ge 1$, except for narrow gaps of stability, e.g. $1.25 \le \Delta \le 1.65$, Fig. 1.

We checked that splitting and distributing the RF quadrupole over the circumference practically does not affect the

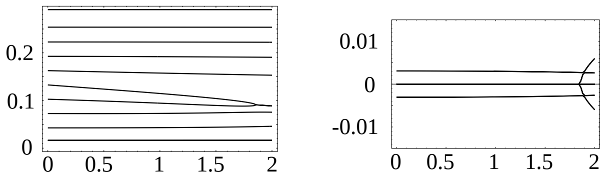

Figure 2: Real (left) and imaginary (right) parts of the synchrobetatron mode tunes vs the wake (times the bunch intensity) at $\Delta = 1$, $\nu_\beta = 0.134$, $\nu_s = 0.03$.

strengths of these synchrobetatron resonances. The reason is that this localized perturbation of focusing with $\Delta\nu \sim \nu_s \ll 1$ is actually weak.

5 COMPARISON WITH TRACKING

The results of the analytical model of the previous section are compared with the mode tunes and increments obtained by tracking of 100 macroparticles uniformly populating the "hollow beam", see an example in Fig. 3. While the predictions of the increased TMCI threshold are confirmed by tracking, at least with moderate tune modulation, the simulated mode increments show a more complicated behavior than those in the analytical model.

Similar results were obtained for the case of Gaussian bunch (Figs. 4,5).

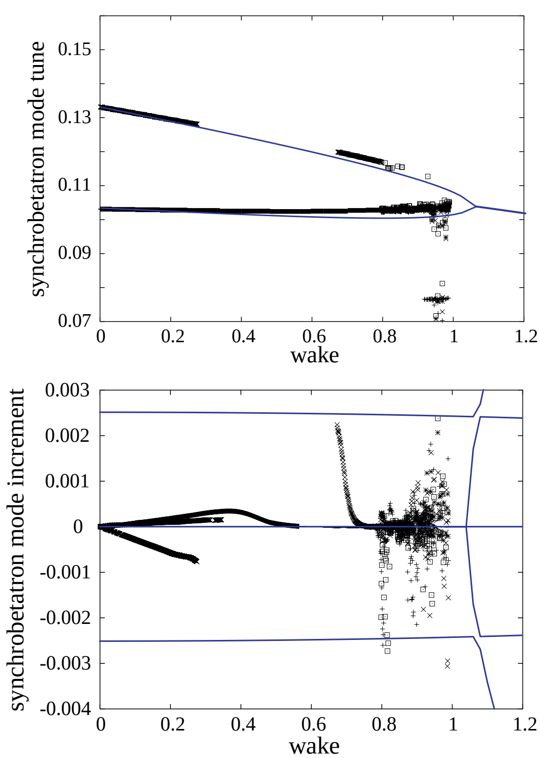

Figure 3: Comparison of the mode tunes (top) and increments (bottom) from the analytical model (lines) with tracking of 100 macroparticles (points) at $\nu_\beta = 0.134$, $\nu_s = 0.03$, $\Delta\nu = 0.8\nu_s$. For $\Delta\nu = 0$ the 0 and -1 modes would merge at wake = 0.57.

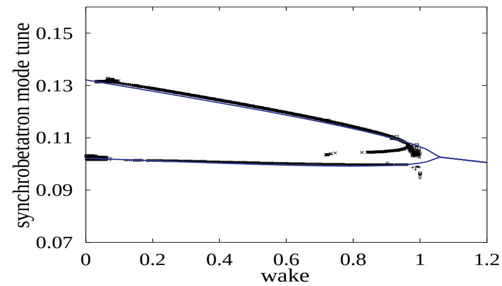

Figure 4: Comparison of the mode tunes from the analytical model (lines) with tracking of Gaussian bunch (points) at $\nu_\beta = 0.134$, $\nu_s = 0.03$, $\Delta\nu = 0.8\nu_s$.

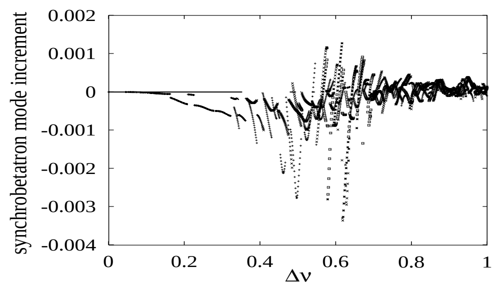

Figure 5: Mode increments per turn from tracking of Gaussian bunch vs Δ for zero wake. $\nu_\beta = 0.134$, $\nu_s = 0.03$.

6 CONCLUSION

Both the analytical model of Section 4 and macroparticle tracking confirm in principle the prediction [1] on increased threshold of the transverse mode coupling instability resulting from the betatron tune modulation $\Delta\nu$ along the bunch which is introduced by an RF quadrupole.

However, using the tune modulation index $\Delta\nu/\nu_s$ in excess of unity may be limited in practice by the synchrobetatron resonances, hardly avoidable because of widening of the beam footprint in the betatron tune diagram.

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