NEW MATHEMATICAL METHOD FOR RADIATION FIELD OF MOVING CHARGE

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Abstract

New mathematical method has been developed to compute radiation field from a moving charge in free space. It is not based on the retarded potential and its derivation. It uses the following two facts: (1) once a wave is emitted from a particle, it propagates as a spherical wave. It's wavelet (a part of the wavefront) runs with speed of the light, and does not change its direction, (2) the initial direction of the wavelet is determined by the Lorentz transformation from electron rest frame to the laboratory frame by taking into account the light aberration. 2D radiation simulator has been developed with this method, which simulates synchrotron, undulator and dipole radiation in time domain.

1 INTRODUCTIONS

In various experimental applications of radiation, such as, the synchrotron, undulator and FEL radiations, discussions are made in terms of the angular and frequency spectrum of these radiations. These field properties are historically analysed by solving retarded potential for specified trajectory. Usually only the far-field radiation proportional to r^{-1} is considered, and the Coulomb field is omitted since it decays quickly as r^{-2}. The results from this approximation have been widely used to evaluate the experimental data and well confirmed.

However, today's advanced accelerator uses extreme beam parameters, for example, ultra-short and high-current beams, where both of the space charge field and radiation field affect beam kinematics at the same time.

Back to 1972, R. Y. Tsien[1] firstly computed electric field of a point charge moving at relativistic velocities. He numerically integrated parametric equations for the field lines, using IBM 360/65 computer and visualize the lines by California Computer Products model 665 11-in. drum plotter. His method was based on the retarded potential, and it was very time consuming process.

The method reported in this paper is totally different from these retarded potential methods, which was originally made by the author in 1974 [2]. It is quite simple and suitable to numerical simulation.

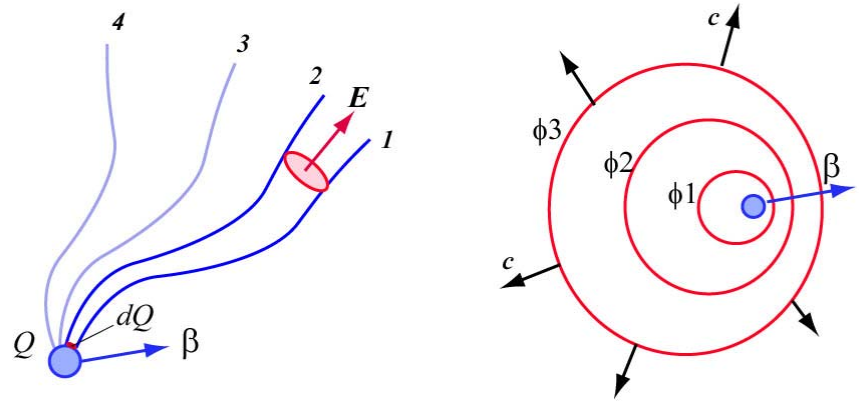
2 MATHMATICAL MODEL

2.1 Basic Equation

The Maxwell equation with field source is

$$\begin{aligned} \nabla \times \boldsymbol{H} &= \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \\ \nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\ \nabla \cdot \boldsymbol{B} &= 0 \\ \nabla \cdot \boldsymbol{D} &= \rho \end{aligned} \qquad (1)$$

Here we consider radiation field from a single charge in free space. In the Maxwell equation, there are two driving terms, ρ and $\boldsymbol{J}$, which are related by the following continues equation,



(a) Electric Field Lines (b) Wavefronts

Fig. 1 Electric field lines of moving charge, and wavefronts.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{J} = 0 \qquad (2)$$

Therefore, we do not need to treat $\boldsymbol{J}$ as explicit manner, which is automatically included when we treat ρ. The magnetic field is derived by the first equation, so that it is enough to calculate only the $\boldsymbol{E}$ with ρ in the case of the single charge field.

Since we know Gauss's theorem is always satisfied for a moving charge, the flux enclosed area $d\boldsymbol{S}$ is kept constant when we follow the "flux pipe" as shown in Fig.1a. Thus

$$dQ = \varepsilon_0 \int \boldsymbol{E} \cdot d\boldsymbol{S} \qquad (3)$$

where dQ is a part of total charge Q of moving charge. If we know the crossing area $d\boldsymbol{S}$, we can simply obtain the field strength E from eq. (3), then the magnetic field from the Maxwell equation (1a).

Along with the motion of charge, each event information propagates outward with speed of the light as a spherical wave. Because of the causality, we can make numbering on each wave-front as shown in Fig. 1b. The shape of the wave-front is always perfect sphere in free space, and continuously expands with speed of the light, only the origin (start point) differs for each event.

In the numerical calculation, the electric field lines and wave-fronts form a grid space as seen in Fig. 2, and motion of the node points (crossing point) is tracked in real time manner. It should be noted that, this grid space is not always orthogonal. For a rest particle the grid space becomes orthogonal, but for a moving charge the electric field line and wave-front becomes non orthogonal. This is due to the light aberration effect, which discussed in the next section.

2.2 Initial Conditions

When a wave-front is emitted from a moving charge, direction of the wavelet (a part of the wave-front) is tilted toward the velocity of motion. This is due to the light-aberration effect. When a wavelet is emitted along unit wave-vector **k**' on the electron rest frame, the observed wavelet on the laboratory frame propagates along vector **k**,

$$\mathbf{k} = \begin{bmatrix} k_{//} \\ k_{\perp} \end{bmatrix} = k \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \frac{1}{1+\beta \cdot k_{//}'} \begin{bmatrix} k_{//}' + \beta \\ \dfrac{k_{\perp}'}{\gamma} \end{bmatrix} \qquad \text{.....(4)}$$

When a particle is running at relativistic speed, the direction of the wavelet is focused in the direction of motion. This is the physical origin of the radiation power of the synchrotron or undulator radiation being focused in a cone angle of 1/γ along the direction of particle velocity.

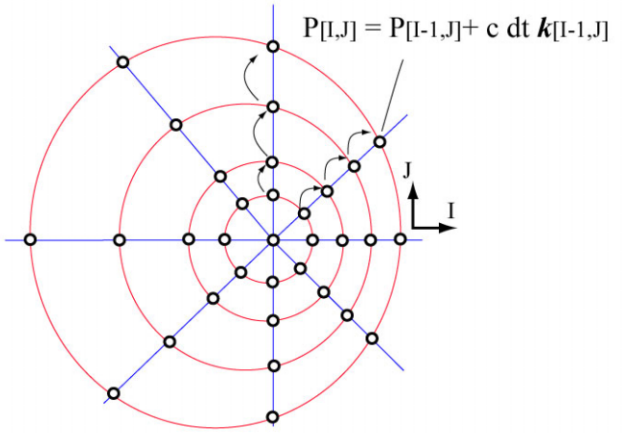


Fig. 2 Series of wave-fronts and field lines form 2D grid space. The positions of the node points P are computed in time step.

3 THE 2D RADIATION SIMULATOR

The windows application has been created, which simulates 2D radiation field. It shows electric field line motion in real-time, and wavefront propagation. The code is available from our Web-site http://www-xfel.spring8.or.jp, which runs on Windows 98, 2000, XP. No Linux, nor Machintosh version is supported.

3.1 Numerical Model

To visualize radiation field in real time, 2D radiation simulator was developed. In this simulation, particle runs along 2D trajectory, such as, circle, sinuous wiggle or line trajectory, whose radiation becomes the synchrotron, the undulator and dipole radiation, respectively. Extension to 3D field will be straight forward, but it will still need a painful work on coding and debugging.

In the 2D radiation simulator, the positions of the node points of the electric field lines and wave-fronts are recorded in 2D matrix. One step of the computation is

(1) move the particle in one step: $c\beta dt$.
(2) compute the direction of wavelet by eq. (4).
(3) move node point one step with speed of light.
(4) shift the address one step along electric field line.

In each time steps, new wave-front is generated from the particle, and propagates outward by the following equations:

$$\begin{aligned} \boldsymbol{P}[I,J] &= \boldsymbol{P}[I-1,J] + cdt \cdot \boldsymbol{k}[I-1,J] \\ \boldsymbol{k}[I,J] &= \boldsymbol{k}[I-1,J] \end{aligned} \qquad (5)$$

Followings are snapshots from the simulator.

3.2 Static Field

One important example is the static field of a rest particle. Even when the particle rests, wavefronts are generated by the particle, and propagate outward. Since time derivation of E is zero and magnetic field is zero, the pointing vector becomes zero. Therefore, there is no energy loss, and only the information is transferred.

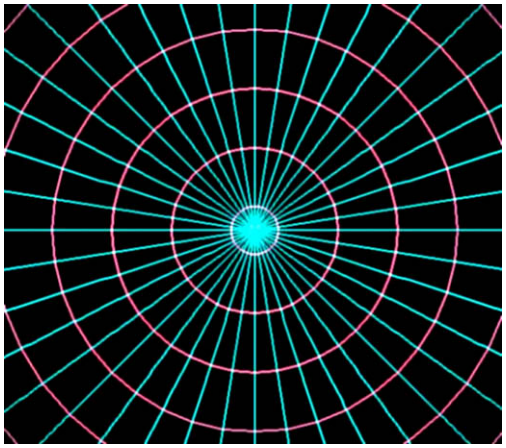


Fig. 3 Static field. Snapshot from the Radiation 2D.

3.3 Synchrotron Radiation

When a charge particle runs along a circular trajectory, it generates spiral shape electric field as shown in Fig. 4. The field lines are condensed in bright spiral zone, where the electric field becomes high. Increasing particle velocity, the bright zone becomes narrower, which corresponds to short impulse field. This is the synchrotron radiation.

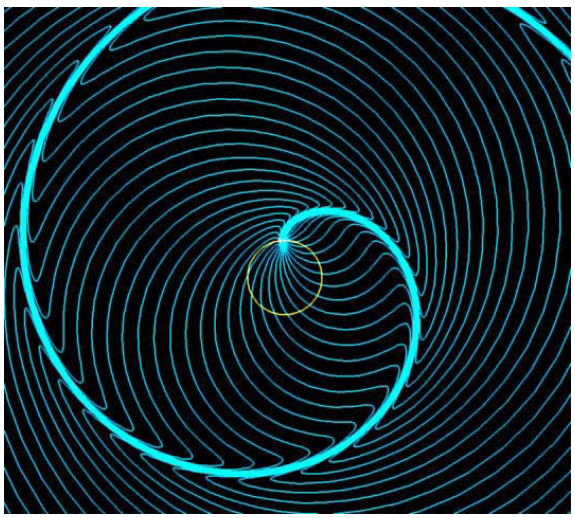


Fig. 4 Synchrotron radiation at v = 0.9c. Snapshot from the Radiation 2D.

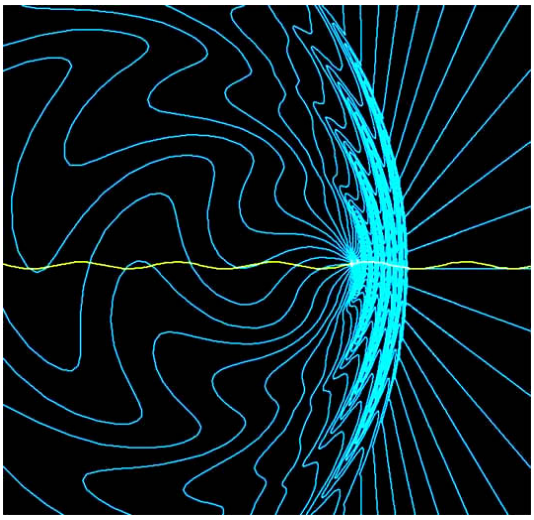


Fig. 5 Undulator radiation, v = 0.9c, K = 1. Snapshot from the Radiation 2D.

3.4 Undulator Radiation

When a charged particle runs through an undulator, it is periodically deflected due to series of transverse magnetic field. In each bending, particle generates radiation in the direction of motion. Since the particle velocity is slightly lower than the speed of the light, wavelength of the accumulated radiation becomes very short due to Doppler effect. This phenomenons can be clearly understood by the Radiation 2D simulator.

4 DISCUSSION

This mathematical method is quite suitable to numerical procedure, and will also be good for educational purpose to study the radiation field.

This method can be applied to analyse the CSR effect in the magnetic chicane bunch compressor. To do this,

(1) 3D field treatment is required.
(2) The trajectory of a large number of the particle has to be computed.

The extension to 3D is straight forward. Problem will be extension to multi-particle problem. However, as seen in the snapshot of the undulator radiation, the radiation power is concentrated in front of the particle, where the data point (node point) are also condensed, which provide enough spatial resolution. This is a kind of auto-zooming function, that is, at a place where high field is generated, the field accuracy is high. This is quite unique feature of this technique, and suitable to particle tracking of short bunch and high frequency field problem, like CSR.

REFERENCES

[1] R. Y. TSIEN, “Picture of Dynamic Electric Fields”, AJP Vol. 40, January 1972
[2] T. Shintake, “Simulation of field lines generated by a moving charge”, private note 1984 March 19 at KEK, not published.