NUMERICAL INVESTIGATIONS OF COHERENT SYNCHROTRON RADIATION DRIVEN INSTABILITY IN MAGNETIC BUNCH COMPRESSORS

T. Limberg, Ph. Piot, F. Stulle*, DESY, Hamburg, Germany

Abstract

It was recently recognized that an initial density or energy modulation in an electron bunch upstream of a magnetic bunch compressor chicane can be enhanced during the compression process. This amplification occurs essentially because of the bunch self-interaction via the coherent synchrotron radiation field. In this paper we present numerical results that support the theoretical expectation of the analytical model elaborated by E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov. We especially study the enhancement of the modulation amplitude for various initial density modulation parameters (e.g. frequency, amplitude, ...).

1 INTRODUCTION

The study of coherent synchrotron radiation (CSR) effects is a main aspect of the design of free electron lasers (FELs). Short wavelength FELs based on self amplified spontaneous emission (SASE) require electron bunches with a high phase space density. Therefore bunches with a high peak current but low emittance and energy spread are needed. Since electron guns cannot produce the needed high peak current, the bunches have to be compressed longitudinally by magnetic bunch compressor chicanes. For example in TTF2 [1] the gun produces bunches of a length of around 2.1 mm. In two stages these bunches will be compressed to an rms length of about 50 μm to attain a peak current of 2.5 kA, assuming a longitudinal Gaussian charge density.

On the other hand radiation emitted at a retarded time by the bunch tail in a system of bending magnets can catch-up with electrons in the bunch head. This is the regime of CSR. It induces an energy redistribution along the bunch and, via dispersion coupling, an emittance dilution. Another effect of this bunch self-interaction that was recently identified is the enhancement of small density or energy modulations in the longitudinal beam profile [2, 3, 4, 5]. Energy modulations are converted into density modulations via dispersion in the chicane. The density modulations induce a bunch self-interaction via the CSR field and thus produce energy modulations. These induced modulations can be much larger than the initial ones and might even lead to phase space fragmentation [6].

2 ANALYTICAL GAIN CALCULATION

An analytical study of the amplification of an incoming sinusoidal modulation due to wakefields was performed in [3]. There also the case of a beam passing a 3-dipole bunch compressor chicane was considered. But under certain limitations the results are also valid for a 4-dipole chicane. In the model that was elaborated the beam is assumed to be 1D pencil-like and has no correlated energy spread. Therefore the beam is not compressed in the chicane.

The formula found for the gain in [3] depends strongly on the modulation wavelength λ. For an electron beam without uncorrelated energy spread the gain

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}} \left(\frac{I_0}{\gamma_0 I_A}\right)^2 \frac{k^{8/3}\left|R_{56}\right|^2 L_d^2}{R^{4/3}}$$

grows very fast for larger wave vectors $k = 2\pi/\lambda$, that is for smaller wavelengths λ. The gain also depends on the peak current I_0 of the unmodulated profile, the relativistic factor γ_0, the Alven current I_A, the momentum compaction R_{56}[1] of the chicane, the length of the dipoles L_d and their bending radius R.

If an uncorrelated energy spread σ_γ is added to the beam, the amplification is suppressed especially for short modulation wavelengths. Then the gain is

$$G = \frac{2\Gamma^2(2/3)}{3^{5/3}} g_0^2\, f(\hat{k}) \ .$$

Here the factor

$$g_0 = \frac{I_0}{\sigma_\gamma I_A} \left(\frac{\gamma_0}{\sigma_\gamma}\right)^{1/3} \frac{L_d}{\left(R^2\left|R_{56}\right|\right)^{1/3}}$$

is independent of the wavelength. The wavelength dependence is described by

$$\begin{aligned} f(\hat{k}) \quad &= \quad 3\hat{k}^{2/3}\exp\left(-\hat{k}^2/2\right) \\ &\quad \left[1 + \frac{\sqrt{\pi}}{2}\frac{\hat{k}^2-2}{\hat{k}}\exp\left(\hat{k}^2/4\right)\,\mathrm{erf}\left(\hat{k}/2\right)\right] \end{aligned}$$

and $\hat{k} = \frac{\sigma_\gamma}{\gamma_0}\left|R_{56}\right| k$.

The gain curve of a beam with uncorrelated energy spread has a maximum of $G_{max} = 1.16{g_0}^2$ at $\hat{k}_{opt} = 2.15$. For $\hat{k} \to 0$ we get $f(\hat{k}) \approx \hat{k}^{8/3}$ and the formula is reduced to the one for a beam without energy spread.

---

[1] The momentum compaction R_{56} describes the linear dependence of the path length on $\delta\gamma/\gamma_0$, $R_{56} = \frac{\partial s}{\partial(\delta\gamma/\gamma_0)}$

* frank.stulle@desy.de

3 NUMERICAL SIMULATIONS

The simulation code TraFiC^4 [7] was used to simulate the beam dynamics in the bunch compressor chicane that is sketched in Figure 1. This code uses a 3D self-consistent algorithm to track a beam consisting of so called sub bunches through a beamline. The sub bunches are 3D Gaussian charge distributions of macroscopic size.

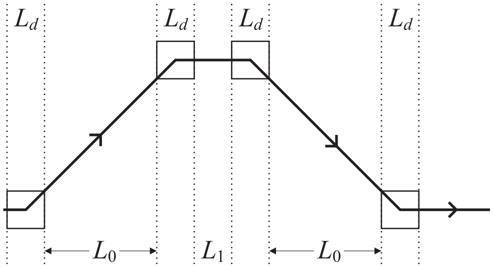

Figure 1: The bunch compressor chicane that was simulated is a standard 4-bend chicane. To include the effect of the radiation traveling in the drift spaces, a drift of 2 m length is taken into account after the chicane.

For the simulations 1D charge distributions with no initial transverse emittance were used. The initial longitudinal beam profile has a flat top with a length of 20 modulation wavelengths λ and Gaussian head and tail with $\sigma = 30\ \mu$m. Only the flat top has a sinusoidal density modulation with a relative depth η . Also the uncorrelated energy spread σ_E is only modeled by adding sub bunches to the flat top, since we are only interested in the dynamics on the flat top. Comparisons to simulations done with beam profiles that have energy spread also added to the head and the tail showed that this can safely be done. General beam and chicane parameters are listed in Table 1.

Table 1: The chicane and beam parameters listed here have been used throughout all of the simulations. The values of the beam parameters are the values at the entrance of the first dipole.

dipole length	L_d	0.5	mm
length of 1st and 3rd drift	L_0	5.0	m
length of 2nd drift	L_1	1.0	m
bend radius	R	10.35	m
bending angle	Φ	2.77	deg
momentum compaction	R_{56}	-25	mm
nominal energy	E_0	5.0	GeV
flat top current	I_0	6	kA
rel. modulation depth	η	10^{-4}	
uncorr. rms energy spread	σ_E	$3 \cdot 10^{-5}$	
norm. emittance	$\epsilon_{x,y}$	0	mm mrad

Of main interest was the dependence of the gain on the modulation wavelength, since this can be directly compared to the analytical results. Other parameter scans (e.g. number of sub bunches per modulation wavelength, number of sub bunches for modeling of the energy spread) were done to make sure that the results have converged, since the gain showed a strong dependence on the number of sub bunches when using too small numbers of sub bunches for the modeling of the modulation or the uncorrelated energy spread.

It came out that it is reasonable to use at least 13 sub bunches per modulation wavelength each spaced at most by 1σ of their width. Seven sub bunches are used for each slice of a bunch to model the energy spread. To avoid noise they are not distributed randomly, but have an energy offset of 0, $\pm1\sigma_E$, $\pm2\sigma_E$ and $\pm3\sigma_E$. Their charge is weighted with a Gauss function. The relative modulation amplitude of $\eta = 10^{-4}$ is chosen to be small enough to avoid saturation at high gain, but on the other hand is high enough to surpass noise even at low gain.

3.1 Simulation Results

The simulations with uncorrelated energy spread added to the beam were done for different modulations with wavelengths of $\lambda = 1.8, 2.0, 2.5, 3.6\ \mu$m. These values are chosen to resolve the peak that is expected in the gain curve obtained for a beam with uncorrelated energy spread. Using the parameters from Table 1 the analytical formula shows a peak around $\lambda_{max} = 2.2\ \mu$m.

For the simulations of a beam without energy spread modulation wavelengths of $\lambda = 2.0, 4.0, 5.0, 6.0, 10.0$ μm have been chosen, since no local characteristics are expected in the gain curve. In this case we wanted to find the global shape of the gain curve. Figure 2 compares the analytical curves with and without energy spread to the corresponding simulations.

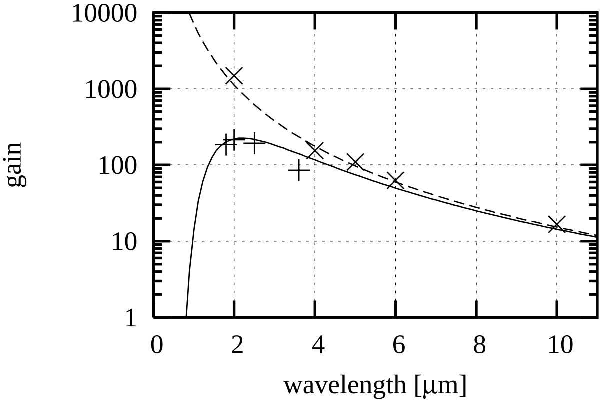

Figure 2: The analytical gain curves with energy spread (solid) and without energy spread (dashed) are in very good agreement with the simulations with energy spread (+) and without energy spread (×).

One can see that there is a very good agreement between the simulations and the analytical model. As expected from the analytical model we find a peak in the gain curve of the beam with energy spread around $\lambda = 2.2\ \mu$m. The gain curve of the beam without energy spread is growing very fast for smaller wavelengths. Deviations of the simulations from the analytical curves seem to be mainly due to noise. The only indication of a systematic difference is the too low gain of the 3.6 μm modulation with energy spread.

Another interesting characteristic of the amplification of a beam modulation in a bunch compressor chicane is that most of the amplification occurs in the last dipole. This can be seen when plotting the gain against the position in the chicane (Figure 3). Throughout the drift spaces the gain does not change.

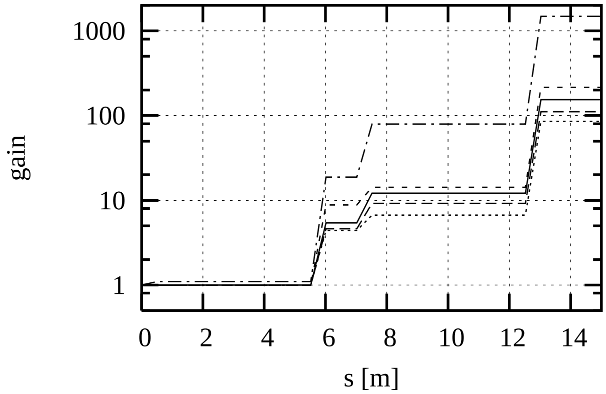

Figure 3: The amplification of the modulations occurs almost only in the last dipole. The plot shows the development of the gain vs. position in the bunch compressor for a 2 μm (short dash) and a 3.6 μm (dotted) modulation with energy spread and for different modulations without energy spread of 2 μm (dash dot), 4 μm (solid) and 5 μm (long dash) wavelength. The gain is calculated before and after each dipole.

When plotting the development of the gain inside the last dipole of the chicane, one sees that the gain is increasing linearly over the dipole (Figure 4). Also this is expected in the analytical model elaborated in [3]. The curve of the 2 μm modulation with uncorrelated energy spread shows a saturation effect.

4 CONCLUSION

Initial density or energy modulation in an electron bunch can be amplified in a bunch compressor chicane due to bunch self-interaction via the coherent synchrotron radiation field. This process dilutes the emittance and might even lead to phase space fragmentation.

In this paper we studied numerically the enhancement of a small sinusoidal modulation in the beam profile due to the CSR field inside a bunch compressor chicane. We compared simulations differing in the wavelength of a longitudinal density modulation in the electron bunch to the analytical gain curve calculated from the formulas developed by E.L. Saldin, E.A. Schneidmiller and M.V. Yurkov. The simulations were done with the 3D tracking code TraFiC^4, but only 1D pencil-like electron beams without transverse emittance were tracked. Despite the different assumptions and simplifications used in the analytical approach and the simulation code TraFiC^4 the results that were obtained are in reasonable agreement. Variations seem to be mainly due to noise. To discover systematic deviations more simulations over a larger range of modulation wavelengths with smaller steps are needed. This also includes cross-checks with different numbers of sub bunches per modulation wavelength and for the modeling of the uncorrelated energy spread as well as simulations with a different size of uncorrelated energy spread.

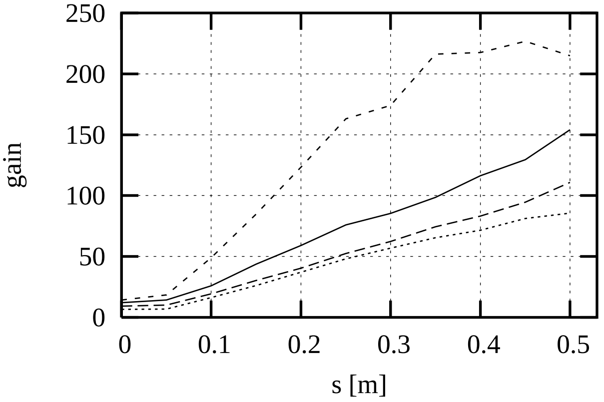

Figure 4: Along the last dipole the gain grows linearly. The same simulations as above were used. The gain is plotted for a 2 μm (short dash) and a 3.6 μm (dotted) modulation with energy spread and for a 4 μm (solid) and a 5 μm (long dash) modulation without energy spread.

REFERENCES

[1] T. Limberg, Ph. Piot, F. Stulle, Design and Performance Simulations of the TTF-FEL II Bunch Compression System, these proceedings.

[2] M. Borland, Start-To-End Simulation of SASE FELs from the Gun through the Undulator, presented at the FEL conference, Darmstadt, Germany, August 2001.

[3] E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Klystron Instability of a Relativistic Electron Beam in a Bunch Compressor, TESLA-FEL 2002-02.

[4] P. Emma, LCLS Tracking Studies, presented at the ICFA Beam Dynamics Mini Workshop on Coherent Synchrotron Radiation and its impact on the dynamics of high brightness electron beams, Zeuthen, Germany, January 2002, http://www.desy.de/csr/ .

[5] S. Heifets, S. Krinsky, G. Stupakov, CSR instabilities in a Bunch Compressor, SLAC-PUB-9165, March 2002.

[6] T. Limberg, Ph. Piot, E.A. Schneidmiller, An analysis of longitudinal phase space fragmentation at the TESLA test facility, Nucl. Instr. and Meth. A 475 (2001) 353.

[7] M. Dohlus, A. Kabel, T. Limberg, Efficient field calculation of 3D bunches on general trajectories, Nucl. Instr. and Meth. A 445 (2000) 338.