Measurements and Simulations of the Damping Effect of the Harmonic Sextupole on Transverse Instabilities

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Abstract

Measurements at ELETTRA have shown that the harmonic sextupole provides Landau damping capable of suppressing transverse coupled multibunch instabilities. There is strong evidence that the damping is induced by the non-linear tune spread with amplitude among the electrons within the individual bunches together with a change in the electron bunch distribution. Results of measurements are compared to simulations.

1 INTRODUCTION

Landau damping is the phenomenon in which a collective motion performed by a certain number of particles is damped by an increasing spread in the oscillation frequencies of the individual particles. As the coherent oscillations build up a centre of mass motion, the latter starts decreasing as particles go out of phase with respect to each other and decohere. Individual particles may be still oscillating but the centre of mass motion is damped. One possibility to generate Landau damping is non-zero chromaticity and another is non-linear elements in a storage ring. A typical example of non-linear elements is the use of octupole magnets in which tune shifts with amplitude are a first order effect. Sextupoles also induce tune shifts with amplitude, although as a second order effect [1], but may nevertheless give rise to the same damping mechanism. In Elettra where a harmonic sextupole S1 is present, strong correlation between the damping of collective motion and the setting of the above sextupole has been noticed in the past [2,3]. It is possible to change the non-linearities of the machine (hence the tune shifts with amplitude) by acting on its settings, maintaining at the same time the constant values of the chromaticities.

In this paper results are presented of variations of the harmonic sextupole settings that give rise to changes in the Landau damping. Results are given both of measurements performed using the transverse multibunch feedback system (TMBF) [4] as an acquisition tool, as well as of computer simulations. Due to the relatively simpler mechanism, both measurements and simulations were done in the horizontal plane, thus eliminating the coupling among the two planes due to the sextupoles. In this case the tune shift with amplitude is reduced to $\Delta\nu_x = C\cdot 2J_x$, and presents a parabolic behaviour with the settings of S1. The following section deals with damping effects when coupled horizontal multibunch instabilities are present, while section 3 discusses the fast coherent damping when kicking a single bunch.

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2 COUPLED MULTIBUNCH INSTABILITIY

Coupled multibunch instabilities can be viewed as consisting of two collective motions: a macroscopic one among the centre of mass of the individual bunches and a microscopic one among the single particles within the individual bunches. Thus, in the past, when observations could be not be done with the TMBF, an important question was whether the tune spread induced by the harmonic sextupole was among different bunches or within the particles of single bunch. Simulations done in 1999 using a macroscopic model of the bunches could only reproduce the main features of the phenomena.

Although the TMBF allows the visualization in detail of what individual bunches are doing, it is important to realize that it can detect only the centre of mass motion of the bunches. Fortunately the tune shift with amplitude is unidirectional, since amplitudes can be only positive. This means that if there is an increase in tune spread within a bunch during the collective motion, then the centre of mass will result in having a tune shift.

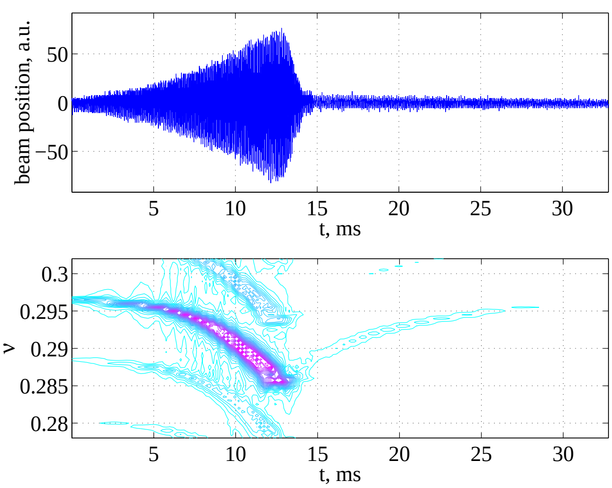

Figure 1. Horizontal instability at S1 = 50 A

Figure 1 shows the centre of mass motion, together with its spectrum (tune resolution is $5\cdot 10^{-4}$), of one of the bunches undergoing a horizontal coupled multibunch instability (driven by a known higher order dipole mode of one of the RF cavities) with S1 set near the minimum value for the tune shift with amplitude.

The damping of the centre of mass motion is not due to a simple detuning of the mode. In fact, observing figure 2 where the same analysis is shown for the same bunch but with the sextupole set to a stronger tune shift with amplitude, one can note that the damping occurs at a much smaller tune shift of the centre of mass.

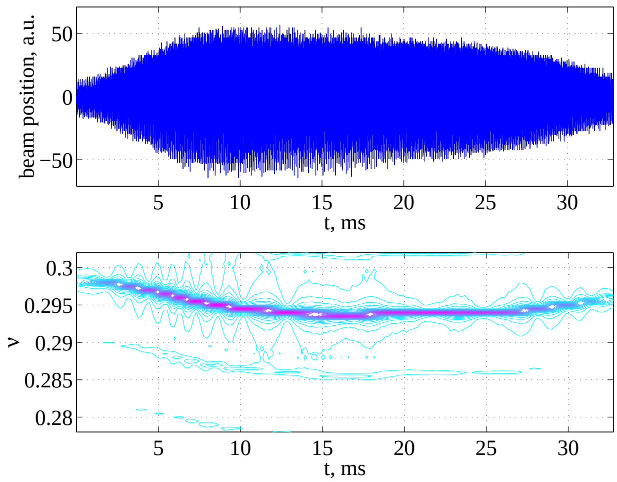

Figure 2. Horizontal instability at S1 = 40 A

Although the two motions are qualitatively different they have one common feature: during growth of the instability there is a broadening of the tune width. This broadening cannot be due to the spread in amplitudes of the center of mass, because during damping the spread in amplitudes of the center of mass is much larger, but the width is noticeably smaller. The only interpretation can be that there is an increasing tune spread within the coherent particles of the bunch, resulting as a tune shift of the center of mass together with a broadening. The same analysis has also been performed on all the other bunches for the same two sextupole settings and they all simultaneously present similar features.

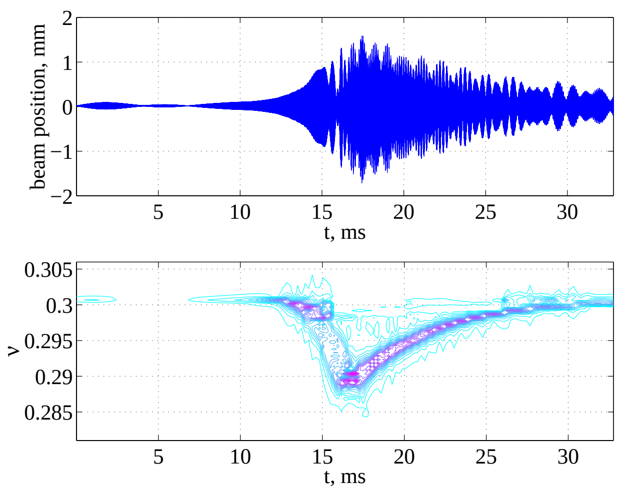

Figure 3. Simulation of the instability

To check the above-mentioned conclusions, a simulation of the instability has also been done using multi-bunch multi-particle tracking with an interaction between the beam and the single RF cavity mode. The results are shown in figure 3. Although the details of the centre of mass motion may differ from reality, the tune behavior is similar to the one of the measured data shown in figure 1 with a broadening of the spectrum while the instability rises.

Knowing the measured rise time (0.45 ms) given by the horizontal higher order mode of the cavity, the radiation damping time (10.21 ms) and extracting the rise time of the bunch mode (6 ms) from figure 1, the Landau damping required to kill the instability is τ_L = 0.512 ms, with a tune spread of $\Delta\nu = (\pi f_{rev}\tau_L)^{-1} = 5.37\cdot 10^{-4}$. Assuming a constant chromatic tune spread, estimated for a stable beam and knowing the beam size at rest, the required amplitude for Landau damping results to be 1.8 times the beam size at rest, consistent with measurements from the synchrotron radiation beam profile monitor.

3 LANDAU DAMPING

Effects of non-linearities induced by the harmonic sextupole on horizontal betatron motion have been measured in single bunch and compared with computer simulation. Betatron motion was excited using the injection kickers and turn-by-turn data was taken.

There is a theoretical analysis [5] of non-linear beam dynamics in the case of free betatron oscillation excited by a short kick and observed using a photomultiplier tube with a blind placed in the image plane. This analysis can be also applied to a BPM. A single bunch in the absence of coherent betatron and synchrotron oscillations will have a particle distribution function in betatron phase space represented in the action-phase variables as:

$$f(J,\varphi,t) = \sum_{n=-\infty}^{\infty} f_n(J,t)\cdot e^{-in(\omega t-\varphi)} \tag{1}$$

where $J = a^2/2$ is the perturbed action (a is the oscillation amplitude), φ is the phase and $\omega = 2\pi\nu f_{rev}$ is the betatron frequency. If $G(x, \omega)$ is the transfer function of a diagnostic device, the output signal is proportional to

$$X = \sum_{k=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} e^{i(k\omega_0 - n\omega)t}\int_0^{\infty} f_n(J,t)K_{kn}\,dJ\,, \tag{2}$$

where $\omega_0 = 2\pi f_{rev}$ and $K_{kn} = \int_{-\pi}^{\pi} G(\sqrt{2J}\cos\varphi, k\omega_0)e^{in\varphi}d\varphi$.

Supposing that the transfer function of the BPM electronics is linear and the amplitude-phase characteristics is constant at frequencies where the betatron modes have considerable amplitudes (usually $n < 5$ to 10), then $K_{kn} = K = const$, and the signal of the n-th betatron harmonic is proportional to

$$X_n = Ke^{-i(\omega_0 - n\omega)}\int_0^{\infty} f_n(J,t)\,dJ + c.c.\,, \tag{3}$$

and its envelope is

$$A_n = K\left|\int_0^{\infty} f_n(J,t)\,dJ\right|. \tag{4}$$

If the beam current is small and coherent effects are negligible, as in the case for zero chromaticity, a first-order approximation gives:

$$f_n(J,t) = f_n(J,0)e^{-in2\pi CJt}\,, \tag{5}$$

where $C = \partial\nu/\partial a^2$ is the non-linearity. Placing (5) into (4) gives:

$$A_n = K\left|\int_0^{\infty} f_n(J,0)e^{-in2\pi CJt}\,dJ\right|. \tag{6}$$

In the case of non-zero chromaticity, if $f_n(J, \varepsilon) = f_n(J)\phi(\varepsilon)$, then, to take the chromaticity into account, $f_n(J, t)$ in (5) should be multiplied by

$$M_n = 2\pi \int_0^\infty \phi(\varepsilon) J_0\left(\frac{2n}{\Omega}\frac{\partial \omega}{\partial E}\varepsilon \sin\frac{\Omega t}{2}\right)\varepsilon d\varepsilon , \quad (7)$$

where Ω and ε are the frequency and amplitude of the synchrotron oscillation. In this case $A_n(t)$ is modulated with the modulation parameter $M_n(t)$.

In our experiments a coherent betatron oscillation was excited by a pulse kick. If $F(J)$ is the distribution function of oscillation amplitudes before the kick, then after the kick the distribution function becomes:

$$f(J,\varphi,\varepsilon) = F\left((\sqrt{2J}\sin\varphi - \sqrt{2\delta J})^2 + 2\delta J\cos\varphi\right)\cdot\phi(\varepsilon),$$

where $\delta J = \delta a^2/2$ is the action perturbation due to the kick with δa amplitude. Calculating $A_1(t)$ with the conditions $\delta J >> \sigma_\perp^2$ and $t << (C\sigma_\perp^2)^{-1}$ where $\sigma_\perp$ is the transverse beam size gives:

$$A_1(t) = M_1(t)\left|\int_0^\infty F(J) J_0(8\pi C\sqrt{J\delta J}\; t)dJ\right|, \quad (8)$$

with J_0 the Bessel function. For a stable bunch the distribution functions $F(J)$ and $\phi(\varepsilon)$ are Gaussian and, according to (8), the envelope is:

$$A_1(t) \propto e^{-\frac{t^2}{2\tau^2}} \cdot e^{-\left(\frac{\partial\omega}{\partial E}\frac{\sigma_E}{\Omega}\right)^2(1-\cos\Omega t)}, \quad (9)$$

where $\tau = (4\pi C\sqrt{2\delta J}\sigma_\perp)^{-1}$ and σ_E is the rms energy spread. Thus, if the chromaticity is zero, the envelope of betatron oscillation is Gaussian.

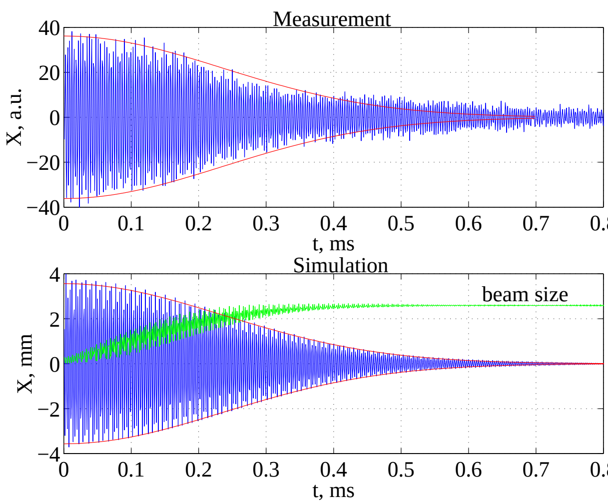

Figure 4. Gaussian damping of the coherent oscillation

Figure 4 shows an example of measured data in comparison with simulation. The measurement was performed on a single bunch mode at 0.9 GeV energy with a beam current of 1 mA and a chromaticity of 0.1. The simulation was carried out for 1000 particles and the initial amplitude was fit to get the correct damping time close to the measured one because the measurement system is not calibrated. In the simulation plot one can see that the coherent oscillation damps and the incoherent one (i.e. beam size) grows. This effect is due to the mismatching of the particles' phases caused by the non-linear tune spread.

The damping time was measured with the various harmonic sextupole settings in the 13 to 35 A range. Figure 5 shows the measured damping rate vs the harmonic sextupole current in comparison with the simulation data and with the theoretical curve calculated using the $\tau = (4\pi C\sqrt{2\delta J}\sigma_\perp)^{-1}$ formula. The parabolic shape of the damping rate graph repeats the behavior of a cubic non-linearity parameter and namely the harmonic sextupole acts as an octupole. One can see that the non-linear damping of coherent betatron oscillation is rather strong, τ = 0.2 to 1 ms with the nominal chromaticity of only 0.1.

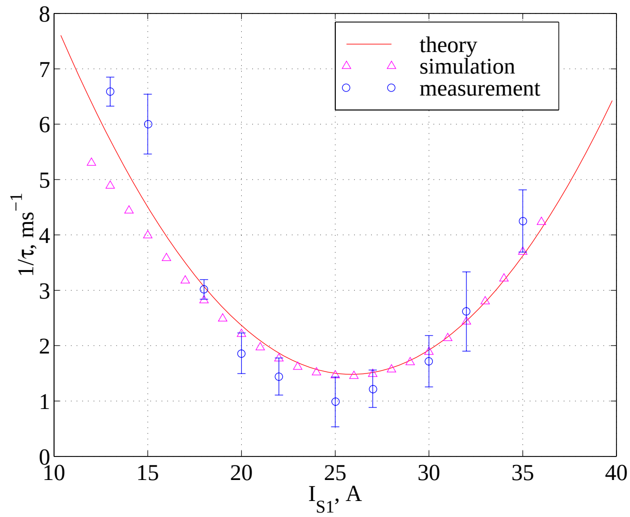

Figure 5. Damping rate vs harmonic sextupole current

4 CONCLUSIONS

The harmonic sextupole provides a non-linear tune spread among the particles in the individual bunches which gives rise to Landau damping and decoheres collective motion. The harmonic sextupole provides also a fast damping of the coherent betatron motion in single bunch when a single bunch is kicked with a pulse. The damping rates obtained by measurements, simulations and theory agree all very well.

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