## STUDY OF RESISTIVE-WALL INSTABILITIES

# WITH A MULTI-BUNCH TRACKING\*

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#### Abstract

A tracking code to simulate the resistive-wall instability is developed and applied to the ESRF machine. Starting from tracking a single bunch with a broad-band impedance, the internal head-tail motion of a bunch is followed and re-modelled in terms of a fewer number macro particles, which are then used for the multibunch tracking. Both short and long range wake forces are taken into account. The scheme allows inclusion of optical nonlinearities that may induce decoherence of the motion as well as a feedback system. Preliminary results obtained are presented and discussed.

### **1 INTRODUCTION**

Due to the presence of a number of low-gap vacuum chambers for insertion devices, the machine operation is severely limited by the resistive-wall (RW) instabilities in synchrotron light sources such as the ESRF [1]. The means usually taken to combat the instability is to increase the chromaticity to positive values, which however reduces lifetime of a Touschek scattering dominated beam, thus being critical to light sources. Since multiple coupled-bunch modes are usually excited, a simple feedback acting on a single mode is far from being sufficient at the ESRF.



Figures 1: Left: Measured vertical threshold versus chromaticity for different fillings. Right: Threshold dependence on the head-tail mode, in uniform filling.

It was also noticed that a tiny residual beam excitation prevents from achieving the ultimately small vertical beam size. In the aim of understanding the underlying physics, studies were made both theoretically and experimentally. Despite being basically a multibunch instability, the RW instability was found to exhibit certain dependence on bunch current, due to the strong short range part of the RW wake field. This can be seen on different thresholds observed according to different beam fillings (Fig. 1a), as well as in different head-tail modes that drive the coupled-bunch instability, which depends on chromaticity and beam current (Fig. 1b). Analysis was made in the frequency domain solving the Sacherer's equation, finding that current threshold can be reproduced with the expected pipe radius of the RW, as well as an important role of the broad-band impedance in raising the zero head-tail mode threshold.

However, the frequency domain calculation performed is only applicable for the uniform filling and is not capable of explaining the large filling dependence observed. Furthermore, it is neither suited to describe the decoherence due to tune spreads that may critically influence the threshold behaviour. In view of the above difficulties, a multibunch tracking in the time domain has been attempted. A principal effort was made to render the computation feasible without losing the essence of the involved physics.

The paper is organised as follows. In the next section, the developed tracking scheme is described. Application made to the ESRF machine is presented in Sec. 3. A conclusion is given in Sec. 4.

#### **2 DEVELOPED SCHEME**

From the point of view of simulation, the best would be to directly extend the single bunch tracking to many bunches, by taking into account the inter-bunch forces. This, however, becomes obviously too time consuming. What then must be sought for is a way to model macro particles within a bunch, with a significantly reduced number as compared to the original particles used in the single bunch tracking, simultaneously keeping the degree of freedom to describe various head-tail mode oscillations excited in a bunch.



Figures 2: Vertical coherent amplitude in the normalised longitudinal phase space, obtained from a single bunch tracking. Left:  $\xi_z$ =0.1. Right:  $\xi_z$ =0.3.

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The developed scheme thus begins from performing a single bunch tracking with the bunch current and the chromaticity assumed for the subsequent multibunch tracking. Here, only short-range wake fields (or equivalently the broad band impedance) that determine the coherence in single bunch are considered (Figs. 2). When a stationary distribution is reached, the initial values of the transverse coordinates for *weighted macro particles* in a bunch are defined by binning the longitudinal phase space into  $N \times N$  pieces and summing up the transverse coordinates and the population of the original particles.

In the multibunch tracking, a freedom is left for each bunch to represent either a single bunch in the original sense or that of macro particles. Despite becoming more time consuming, keeping single bunches leaves the possibility of treating intra-bunch tune spreads and thereby redefining macro particle amplitudes in course of tracking. Inter-bunch tune spreads, on the other hand, can be straightforwardly incorporated. Both short (BBR) and long range (RW) wake fields are considered for single and/or macro-particle bunches. In the current version, RW wake fields longer than one revolution time were truncated upon the observation that a single bunch is not affected by the concerned instability.

As diagnostics of the tracked beam, statistics of the betatron invariant is computed at given intervals in time, over each bunch and the beam. Averaged invariant is also recorded as a function of turn number to deduce the growth rate of instability. A simulated pickup records the passage time of all bunches, by slicing each bunch longitudinally to a given number (default = 13), to be able to deduce the frequency of coherent bunch motions. The accumulated signal is Fourier transformed to get the tune spectrum.

## 3 APPLICATION TO THE ESRF MACHINE

First applications were made to the ESRF machine in the uniform filling, for which the results are known both experimentally and theoretically [1,2].



Figure 3: Averaged betatron invariant  $W_{cz}$  versus number of turns, obtained for the uniform filling

The broad band resonator model  $(R_T \beta=13 \text{ M}\Omega, f_{res}=22 \text{ GHz}, Q=1)$  found in the single bunch study was employed for the single bunch tracking, while b=8 mm for the RW pipe radius and a reduced BBR shunt impedance of  $R_T \beta=6.8 \text{ M}\Omega$  found in the frequency domain analysis were used for the multibunch tracking. A 200 mA beam was tracked for different vertical chromaticities over 400 turns, by taking 13<sup>2</sup> macro particles per bunch. The main output looked at is the betatron invariant  $W_{cz}$  averaged over both macro particles and bunches (Fig. 3). The steep increase of  $W_{cz}$  seen initially in the figure is due to artificially reduced *b* parameter (up to 40 turns) to promote the instability.



Figure 4: Current threshold versus chromaticity.

Current thresholds were then deduced by assuming a linear dependence of the growth rate on the beam current, which was numerically confirmed to hold for assessed cases, and equating it with the rate of the radiation damping (Fig. 4). The threshold current of 20 mA obtained at zero chromaticity as well as its dependence on which agrees well with the expectation. The correctness of the obtained dynamics is further confirmed in the tune spectrum, in which the excited coupled-bunch modes at the *mirror* betatron frequencies are descending in the expected order (Fig. 5). It was also verified that the instability is sensitive to the fractional part of the betatron tune in the way as expected from the theory.



Figure 5: Calculated tune spectrum (uniform and  $\xi_z=0$ ).

The computation was now repeated for 1/3 filling at 200 mA, for which no analysis is made with the frequency domain approach as already stated. While the threshold current remained low as for the uniform filling at low  $\xi_z$ 's, it rose faster as  $\xi_z$  is increased, in agreement with the observed trend (Figs. 4 and 1). Threshold currents at higher  $\xi_z$ 's grow to infinity in both fillings with the assumed definition. In both cases, however, the invariant  $W_{cz}$ 's exhibited a beating in time, which was more pronounced for 1/3 filling (Fig. 6). This behaviour matches with the observation that at higher  $\xi_z$ 's, the instability threshold becomes more obscure, being able to ramp the current in the presence of instability.



Figure 6: Beating of  $W_{cz}$  seen in 1/3 filling.

Unlike the uniform filling, a large increase of the betatron invariant  $W_{cz}$  across the bunch train towards the tail appeared in 1/3 filling (Fig; 7), which again matches the reality where a part of the transverse beam profile can be seen to be blown up.



Figure 7:  $W_{cz}$  across the bunch train in 1/3 filling.

A point of particular interest in the present study was to verify if the developed code, with its macro-particle model, is capable of reproducing a coupled-bunch instability driven by a higher-order head-tail mode. This is more prominently observed in partial fillings at high chromaticities, due to their increased current per bunch. Analysing the maximum peak in the tune spectrum, it is found, as anticipated, that the instability at  $\xi_z = 0.4$  in 1/3 filling (i.e. what corresponds to the beating  $W_{cz}$  shown in Fig. 6) is driven by m=-1.



Figures 8: Appearance of different head-tail modes in the peak of the calculated tune spectra. Upper: at  $\xi_z=0$  in the uniform filling. Lower: at  $\xi_z=0.4$  in 1/3 filling

#### **4 CONCLUSION**

A multibunch tracking code was developed to study the resistive-wall instability that seriously disturbs some of the light sources such as the ESRF. The code is capable of simulating, with a reduced number of macro particles, the internal head-tail motion of a bunch that varies with the chromaticity and critically influences the threshold. Thanks to which, all computations could be made on a standard PC within reasonable cpu times. First results obtained for the ESRF were in good agreement with those obtained with the frequency domain approach for the uniform filling, and those for the 1/3 filling were in basic accordance with the experiments. With the obtained encouraging results, systematic studies are to be made with extensions to include intra and inter bunch tune spreads, as well as transverse feedback.

#### **5 REFERENCES**

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