EFFECT OF LATTICE RANDOM ERRORS ON A SPACE CHARGE DOMINATED BEAM

G. Franchetti*, I. Hofmann, GSI, Darmstadt, Germany

Abstract

The effect of random focusing errors in linear accelerators raises some fundamental questions on error propagation and transformation into emittance growth due to the action of space charge. In this paper we carry out a generic study of this problem by using 2D computer simulation in a periodic quadrupole channel. We find a scaling law for growth of the average value of the rms emittance (over an error ensemble) and relate this to free energy conversion.

1 INTRODUCTION

In high-intensity accelerators for protons or ions mismatch of beam envelopes leads to halo formation, which may be a source of beam loss. The mechanism of halo formation for round beams with initial mismatch has been understood as a parametric 2:1 resonance effect reviewed in [1]. Reiser on the other hand, used a thermodynamic picture to explain that initial mismatch works as a source of free energy in the beam, which nonlinearities would in some way convert into emittance growth [2]. The anisotropic free energy limit of halos has recently been studied in [3] with the finding that the response of the beam on initial mismatch can be - away from equipartition - due to "attraction" of fixed points in the direction of stronger focusing. The halo formation mechanisms considered for initial mismatch, however, cannot be easily transferred to randomly generated mismatch. As in Ref. [3] the generic longitudinal anisotropy of linac bunches is modeled here by the anisotropy between two transverse degrees of freedom of a periodic focusing channel, e.g. different focusing strength and initial emittances. Using equal focusing strength and initial emittances, our study then also models the quadrupole errors of a linac ignoring coupling to the longitudinal degree of freedom.

2 ZERO SPACE CHARGE CASE

When a beam is transported in the 'real' lattice it meets a sequence $\mathbf{K} = \{k_f + \delta k_1, k_d + \delta k_2, k_f + \delta k_3, k_d + \delta k_4, ...\}$ of perturbed focusing/defocusing elements. The periodic sequence $\mathbf{K}_0 = \{k_f, k_d, k_f, k_d, ...\}$ represents the 'ideal' machine gradient settings, and the perturbation $\delta\mathbf{K} = \{\delta k_1, \delta k_2, \delta k_3, \delta k_4, ...\}$ is the *error set*. Since the error sequence $\delta\mathbf{K}$ is not known a priori, a study of the random error induced statistical properties of the beam evolution becomes appropriate. We make the following assumptions for δk_i: 1) it belongs to a Gaussian distribution with $\langle \delta k_i \rangle = 0$ and variance $\sigma_{\delta k_i} = \sigma k_i$, and 2) it is statistically independent. We call σ the error set strength. With this choice of errors, given $\mathbf{K}_0$ and σ, the *ensemble of the error set* $\{\delta\mathbf{K}\}$ is defined. By taking the matched beam for the ideal transport line and propagating it through each 'real' machine $\mathbf{K} = \mathbf{K}_0 + \delta\mathbf{K}$ we obtain the *ensemble of beam evolutions* $\{X(s,\mathbf{K}), Y(s,\mathbf{K})\}$ and from this follow the statistical properties of the beam evolution: average beam envelopes, $\langle X\rangle(s), \langle Y\rangle(s)$, and their variances, $\sigma_X(s), \sigma_Y(s)$. We apply this procedure to perform a numerical study with a periodic FODO cell transport line with structure Q_F, D_1, Q_D, D_2. Here D_1, D_2 are respectively 1 m and 1.5 m long drift sections; Q_F, Q_D are focusing/defocusing quadrupoles each 1 m long. In the simulation we used a KV beam of 2000 macroparticles with no space charge, with $\epsilon_z = \epsilon_x = 50$ mm mrad and set the quadrupoles strength of the ideal FODO cell such that $k_{0z}/k_{0x} = 1$. The estimates of the beam evolution ensemble has been obtained with a sample of 200 error sets. In Fig. 1a,b we plot the evolution of average and variance of envelopes (averaged over the two planes) versus the distance expressed in betatron wavelength. The error set strengths used are $\sigma = 0.5\%, 1.625\%$, and 2.75%. Fig. 1a shows that average envelopes grow almost linearly with distance. Note that during the first betatron oscillations the envelope variance growth (Fig. 1b) follows a ran-

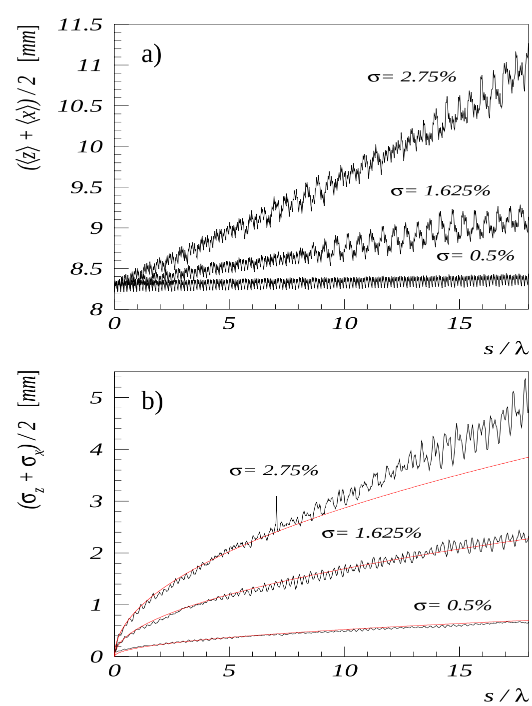

Figure 1: Evolution of average a) and variance b) of the rms envelope in a periodic channel with no space charge.

* g.franchetti@gsi.de

dom walk like curve as shown comparing with the function $\sigma_X = \alpha_m \sigma \sqrt{s\,N/L}$ (dotted curves), with $\alpha_m = 49.5$ best fitting parameter. L is the length of the periodic cell, and N is the number of focusing/defocusing elements per cell. When the envelope variance becomes of the order of a third of the average beam size, higher order terms in the envelope transport break down the random walk path. This effect is shown in Fig. 1b by the deviation of the variance for $\sigma = 2.75\%$ from the respective random walk curve. Since beam emittances are constant, envelope growth can be associated with a beam mismatch. Variance of envelopes incorporates in statistical sense the same feature as well.

3 SPACE CHARGE EFFECTS

As it was shown in [3], initial mismatch disappears almost completely for initial Gaussian beams with saturation of the emittance growth. By contrast we found in Sect. 2 that random errors excite a *statistical beam mismatch* growth according to $M = 1 + (\alpha_m \sigma / X)\sqrt{sN/L}$. When a space charge dominated beam is transported through a lattice with gradient errors presumably the two phenomena happen simultaneously: gradient errors try to build up a mismatch, but some appropriate mechanism (possibly the 2:1 parametric and higher order resonances) moves particles into the halo, which damps the mismatch. We use again the same lattice described in Sect. 2 and an rms matched Gaussian distribution of 50000 macroparticles, $\epsilon_x = \epsilon_y = 50$ mm mrad, 20 integration steps per cell and track the beam through 130 cells by employing a total of 200 error sets. The space charge simulation settings are as in [3]. We consider a tune depression of $k_x/k_{0x} = 0.6$ and first keep the depressed tune ratio at $k_z/k_x = 1$ shown in Fig. 2. Note in Fig. 2c the steady growth of the average emittance as result of a continuous energy transfer from beam mismatch into beam halo. Consistently, in Fig. 2b, after few betatron oscillations the envelope spread stops growing as indication of the statistical balance of mismatch creation and conversion rates. Note that the initial envelope spread exhibits a faster growth than the analytic estimates. We attribute this effect to the reduced effective focusing strength of the FODO cell due to space charge. In fact the error sequence $\delta\mathbf{K}$ determined by σ and $\mathbf{K}_0$ is felt as stronger by a weaker focusing transport line. This leads to an effective error strength $\sigma_{eff} > \sigma$. We can use the overlap close to the origin (Fig. 2b) of the simulation curve for $\sigma = 2.75\%$ with the analytical fit curve for $\sigma = 1.625\%$ to draw the conclusion that the effective error in this case is 1.7 time large than σ. Since $\sigma_{eff}/\sigma \rightarrow 1$ for $k_x/k_{0x} \rightarrow 1$ we attempt to linearly extrapolate the dependence of σ_{eff}/σ on the tune depression as $\sigma_{eff}/\sigma = 2.75 - 1.75\,k_x/k_{0x}$ in the region $0.6 \leq k_x/k_{0x} \leq 1$. The linear emittance growth can be explained as follows: from [3] the conversion of free energy created by an initial mismatch M_i in emittance growth (halo) is given by $\epsilon = \epsilon_0 + \epsilon_0\alpha(M_i - 1)^2$, with a fitting parameter α. On the other hand, the mismatch built up by

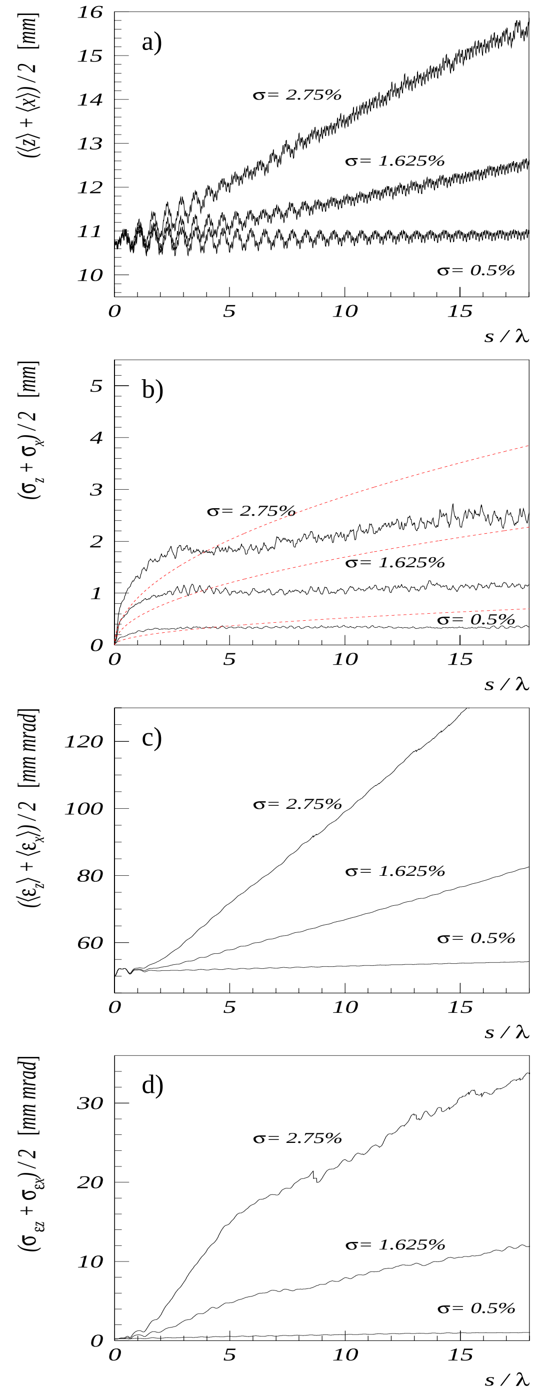

Figure 2: Evolution of ensemble averages and variances for rms envelopes [a), respectively b)] and for rms emittance [c), respectively d)]. The tune depression is $k_x/k_{0x} = 0.6$ and λ is the betatron wavelength without space charge.

the gradient errors for a beam without space charge can be described as $(M-1)^2 = (\alpha_m\sigma/X)^2 sN/L$. The total free energy created into the beam when transported over a distance s is then $\epsilon_0\alpha(\alpha_m\sigma/X)^2 sN/L$. If all the free energy is converted into emittance growth the actual emittance scaling is found as $\epsilon = \epsilon_0 + \epsilon_0\alpha(\alpha_m\sigma/X)^2 sN/L$. This expression suggests an ansatz for the *random er-*

ror induced emittance growth rate (per unit length), e.g. $(\Delta\epsilon/\Delta s)/\epsilon_0 \leq \lambda\sigma^2$. We tested this prediction by comput-

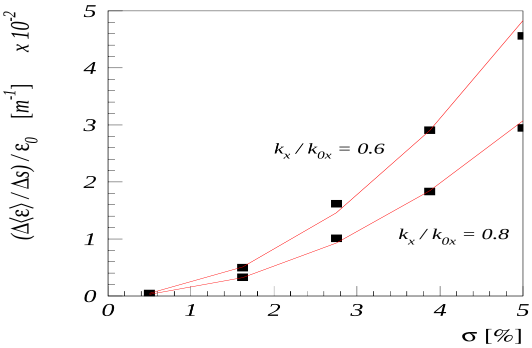

Figure 3: Numerical and analytical fit of the emittance growth slope

ing the slope $(\Delta\epsilon/\Delta s)/\epsilon_0$. In Fig. 3 we plot with markers the slope for the simulation run and compare it with the emittance growth rate with $\lambda = 12$ and 19 as best fitting parameters for $k_x/k_{0x} = 0.8$ and 0.6 respectively. By using the effective error set strength the emittance growth rate becomes $(\Delta\epsilon/\Delta s)/\epsilon_0 \simeq 50(1 - 0.63\,k_x/k_{0x})^2\sigma^2$. This expression fits all our numerical findings in Fig. 3 and justifies the linear interpolation used to estimate σ_{eff}.

4 ASYMMETRIC FOCUSING

In our preceding study [3] the emittance growth in response of initial mismatch has been systematically explored for split focusing strength, i.e. scanning over k_z/k_x with fixed tune depression k_x/k_{0x}. The simulations showed that the free energy can be transferred into the initially hotter plane: the initially equal emittances evolve into anisotropic emittances in an anti-thermodynamic way. However, surprisingly, the averaged (over z, x) final rms emittance growth was found almost independent of the tune ratio and in good agreement with the analytical expression in Reiser's free energy conversion model for axi-symmetric beams [2]. We study here the case when the beam is matched initially, but the transport line is affected by random gradient errors. Since in Sect. 3 we find that the emittance growth rises linearly without reaching saturation, we compute then the slope $(\Delta\langle\epsilon\rangle/\Delta s)/\epsilon_0$ in the $x - y$ planes for several tune ratios k_z/k_x. This quantity is evaluated at the beginning of the average emittance growth. In all the simulations the settings are the same as Sect. 3. In Fig. 4a we show the scan for $\sigma = 0.5\%$, and in Fig. 4b for $\sigma = 1.625\%$. Using that the energy anisotropy is given by $(\epsilon_z k_z)/(\epsilon_x k_x)$, we find the following: in contrast with Ref. [3] the growth is reduced in the "hotter" plane, e.g. in z for $k_z/k_x > 1$ and in x for $k_z/k_x < 1$. We interpret this result in terms of a change in the mismatch creation rate: for $k_z/k_x < 1$ the z focusing strength is the weaker one, enhancing the effect of random errors in the z plane (higher effective error set strength). Equipartition cannot be claimed to play a role here, since the beam over-

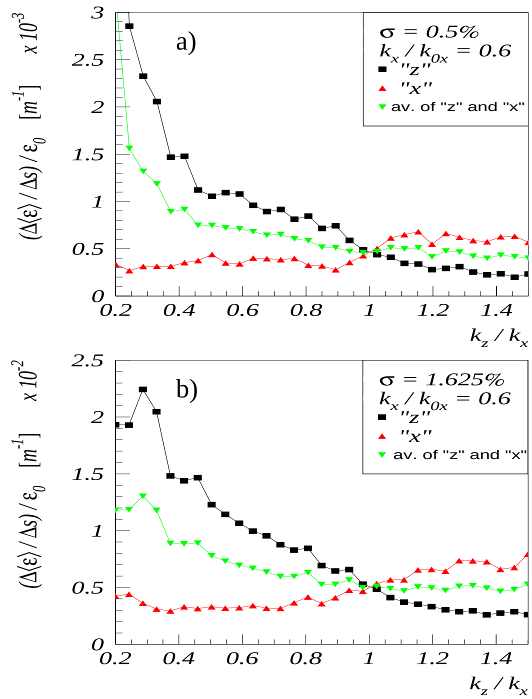

Figure 4: Emittance growth rate (per unit length) and averages (over z and x).

shoots equipartition as we found for the larger error sets. We also note that the average of the growth over x and y is only weakly dependent on k_z/k_x over a large range, but increasing for small k_z/k_x. Using this observation we can still use the empirical formula derived in Sect. 3 for $k_z/k_x = 1$ to predict the average $(\Delta\epsilon/\Delta s)/\epsilon_0$ in the range $0.5 < k_z/k_x < 1.5$ with reasonable accuracy.

5 CONCLUSION

In this paper we have shown that random focusing errors in linear accelerators induce an rms emittance growth on space charge dominated beams rising linearly with distance. It is not obvious from our work what the role of the parametric 2:1 resonance really is in this process. The weak dependence of the average (over z, x) emittance growth rate from k_z/k_x allows use of scaling laws based on the free energy conversion. Future work will have to explore a larger range of parameters and address the growth of halos size beyond rms.

Acknowledgment The support of CDIC at Brookhaven National Laboratories is recognized.

6 REFERENCES

[1] T. P. Wangler, K. R. Crandall, R. Ryne, and T. S. Wang, Phys. Rev. ST Accel. Beams **1**, 084201 (1998).

[2] M. Reiser, J. Applied Phys. **70**, 1919 (1991).

[3] G. Franchetti, I. Hofmann, and D. Jeon, Phys. Rev. Lett., to be published.