

# SOME FEATURES OF BEAM DYNAMICS IN SUPER-CONDUCTING LINAC BASED ON QUARTER- AND HALF-WAVE CAVITIES

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## Abstract

Super-conducting linear accelerator as injector for COSY has to accelerate  $H^-$  and  $D^-$  particles that differ in ratio of mass to charge by a factor of 2. There are two preferable types of linear accelerator structures appropriate for this purpose: multi-gap structure with internal synchronization of gaps working on first and second harmonics, and system with external synchronizing of the different groups of structures with few gaps. For the second type of structure the super-conducting cavity is more appropriate due to lot of reasons. We consider two options of linear accelerator based on the super-conducting quarter-wave (QWC) and half-wave cavities (HWC). In both options the accelerator consists of two parts with 160 MHz and 320 MHz radio frequencies correspondingly. Simultaneously the cavities are subdivided into the families with the fixed geometry (or with the constant structure velocity  $\beta$ ). We optimize the number of cavities in each group and each family. We discuss how to optimize for protons and deuterium particles simultaneously. We analyze the 6-dimensional beam dynamics in the real field calculated by MAFIA. The quarter-wave cavity technology is quite developed, but due to the dipole component of magnetic and electrical fields the beam is unstable in the transverse plane. We have developed the special method to compensate this effect. In the half-wave cavities such problem does not exist due to the field symmetry. For the transverse plane we have examined the single, doublet and triplet systems. We have investigated the parametric resonance arising in the longitudinal plane due to the drift space needed for the focusing elements and determine the space limitation.

## 1 QUARTER-WAVE CAVITY

To design the linear accelerator for both options quarter-wave and half-wave cavities we integrate numerically the full system of motion equations in 6D phase space with the real field components taken from MAFIA calculations. In common case the transverse components of electrical and magnetic fields can be represented in the region of beam as  $E_r = E_0 + \partial E_r / \partial r|_{r=r_0} \cdot (r - r_0)$   
 $B_r = B_0 + \partial B_r / \partial r|_{r=r_0} \cdot (r - r_0)$ . Thus, the beam has the dipole mode of motion due to  $H_0, E_0$  and the quadrupole mode because  $\partial E_r / \partial r|_{r=r_0}, \partial B_r / \partial r|_{r=r_0}$ . Due to non-symmetry in the quarter-wave cavity the dipole compo-

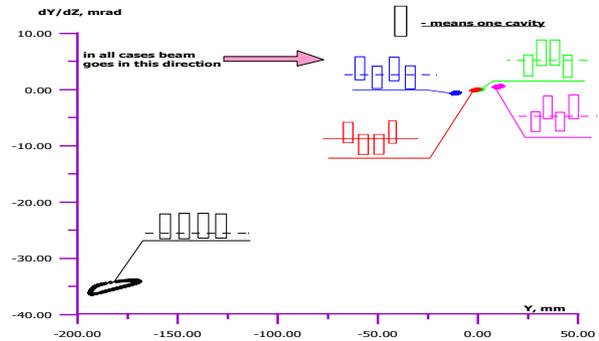


Figure 1: X,  $dx/dz$  position of beam for different schemes of compensation.

nents  $H_0, E_0$  are not negligible. Simultaneously we should note that in space the dipole component  $H_0$  acts to the beam like a  $\pi$ -mode, and  $E_0$  action is like a 0-mode. In time the H field is shifted relatively of E field for 90 degrees. All together give the uncompensated transverse kick to the particles. The total effect for all cavities is dramatically bad for the H and D beams and without compensation the bunch is shifted by 200 mm from axis (see fig.1). The method, which one could help to compensate the dipole components, has to be based of RF field, since  $E_0$  and  $H_0$  depend on RF phase. Therefore we have analysed the different schemes of the cavities placement [1]. Figure 1 shows the results of the particles tracking in the real field for the option “4 groups of cavities with drift between cryomodules 0.7 m of length”. We launch the bunch represented by the empty phase ellipse in the longitudinal plane with the zero initial transverse deviation for x, y,  $dx/dz$ ,  $dy/dz$  and with energy spread  $\pm 1.5\%$  and phase length of bunch  $\pm 15^\circ$ . The rotation of the cavities ( $-90^\circ, 90^\circ, 90^\circ, -90^\circ$ ) or inversely gives the minimum deviation of the bunch from the axis. The effective emittance growth due to this effect is 10% only. However, during the commissioning we expect the problems with the tuning the quadrupoles and RF fields separately. Therefore we have taken decision to use the half-wave cavities.

## 2 HALF-WAVE CAVITY

Since the half-wave cavity has the full symmetry for E and B fields relatively of beam port ( $r = r_0$ ) the dipole components  $E_0$  and  $H_0$  equal zero. It is biggest advantage in comparison with the quarter-wave cavity.

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However, the derivatives  $\partial E_r / \partial r|_{r=r_0}, \partial B_r / \partial r|_{r=r_0}$  (RF electrical and magnetic gradients) can be even stronger in the half-wave cavity due to the same symmetry.

### 2.1 Transverse motion

The optimum number of cavities in one cryo-module depends on the ratio between the quadrupole focusing and RF defocusing strengths and the desirable beam envelope value. In our case it is four. Figure 2 shows one unit cell of linear accelerator. Four super-conducting cavities are placed in one cryomodule. The focusing quadrupoles are localized between cryomodules. We have analysed the different lattices with single, doublet and triplet quadrupoles. The doublet system with space for diagnostic between quadrupoles is appeared to be the most appropriate. To optimise the ratio between the quadrupole gradient and the envelope value we use the 120 cm drift between quadrupoles in doublet.

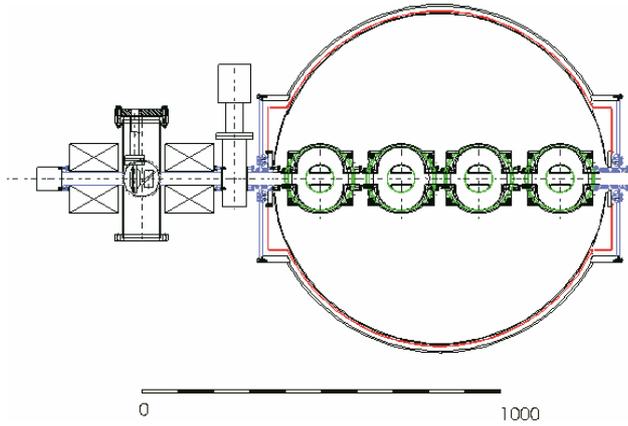


Figure 2: One unit cell of linear accelerator

The defocusing RF field depends on RF phase and it is quite strong factor. Therefore the transverse phase portrait is different for the different cross-section of bunch, and the effective emittance should be strongly determined by the longitudinal phase portrait. From this point of view we consider the lattices with the phase advance around  $80^\circ$ . Figure 3 shows the transverse phase portraits for two cases of matched and mismatched beam in longitudinal plane.

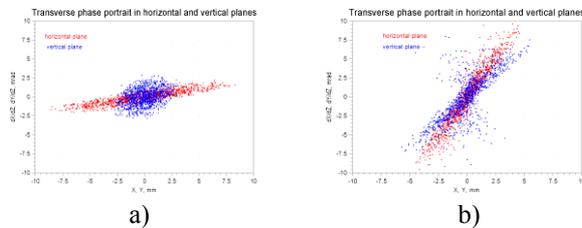


Figure 3: Transverse phase portrait on exit of linac for matched (a) and mismatched (b) beam in longitudinal plane.

You can see for the mismatched longitudinally beam emittance is much larger. Therefore the longitudinal beam dynamics is most important problem.

### 2.2 Longitudinal motion

First of all, we investigated single particle motion to make the appropriate choice of how many half-wave cavities families we actually need. In the QWC version, we had two such families for 160MHz and one family for 320MHz. Figure 4 shows the transient time versus the energy for the option with two families of 160MHz cavities (4 cavities with  $\beta = 0.072$  and 16 cavities with  $\beta = 0.116$ ) and the option with one family of 160MHz (20 cavities with  $\beta = 0.116$ ).

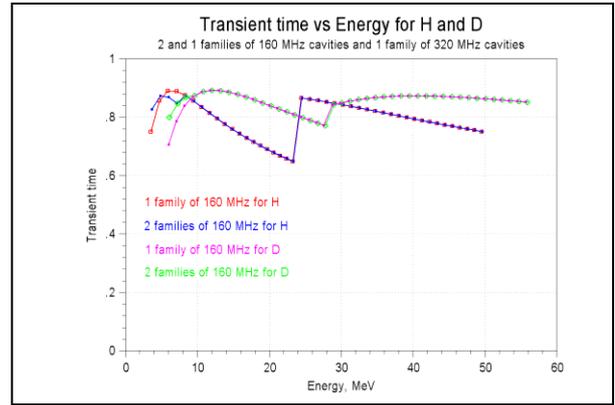


Figure 4: Transient time for H and D beams (44 cavities totally).

In both options the number of 320MHz cavities equals 24, or totally 44 cavities. One can see from fig. 4 that the first HWC family for 160MHz does not lead to significant advantages in the total ion energy. We therefore decided to use only one family for 160MHz cavities, which is also simpler and more convenient for construction. The two HWC constructions are designed for  $\beta=0.116$  and  $\beta=0.2$  at 160MHz and 320MHz, respectively. One can see that the transition between the two families takes place approximately at energy between 24MeV and 28MeV.  $H^-$  and  $D^-$  ions can be accelerated up to kinetic energies of 50MeV and 56MeV, respectively. In both options the number of 320MHz cavities equals 24, or totally 44 cavities.

In a linac with separated functions, the focusing elements are positioned in drift spaces between cryo-modules. In our case it is comparable to the length of four HWRs per cryostat. Hence the periodically repeated drifts can excite the parametric resonance in the longitudinal phase plane of the beam if the conditions are right. This effect is extremely important for our lattice and therefore we have investigated it in more detail.

In linear approximation, the equation describing the beam interaction with a drift space is given by  $d^2\Phi/d\tau^2 + \Omega^2(1 + e_1 \cos\tau)\Phi = 0$ , where  $\Omega^2 = (e E_0 T_g T_c \lambda \sin\varphi_s) / (2\pi \gamma^3 m_0 c^2 \beta) \times L_{cav} / L_\Sigma$  is the longitudinal oscillation frequency, and  $e_1 = 2\sin(\pi L_{cav} / L_\Sigma) / (\pi L_{cav} / L_\Sigma)$  is the

first harmonic amplitude of the perturbation. The parametric resonance arises when the double value of the frequency perturbation is near to the longitudinal frequency  $\nu = 2\Omega + \varepsilon$ , where the width  $\varepsilon$  of the resonance is limited by  $-e_1 \Omega/2 < \varepsilon < e_1 \Omega/2$ . Figure 5 shows several cases of the parametric resonance crossing for a different number of cavities placed in a cryomodule. One can see that in the option of “4 cavities per cryomodule” the particles appear inside the parametric resonance right from injection up to between 12MeV and 15MeV for H and D, respectively. For the “2 cavities per cryomodule” option, the H bunch experiences the resonance only from 2.5MeV up to 4MeV, but the D bunch does not cross the resonance at all. According to the oscillation theory in the central region of the separatrix, the parametric resonance acts like a linear stretcher into the opposite dipole directions. Near the boundaries, the particle motion becomes non-linear and unstable. One of the methods to minimize the parametric resonance action is to modulate the synchronous phase with  $\Omega$  frequency. The term  $\Omega^2 (1+e_1 \cos\nu t)$  will then be proportional to  $(1+e_1 \cos 2\Omega) \sin \Omega$ , and the most dangerous harmonic with frequency  $2\Omega$  could be significantly decreased.

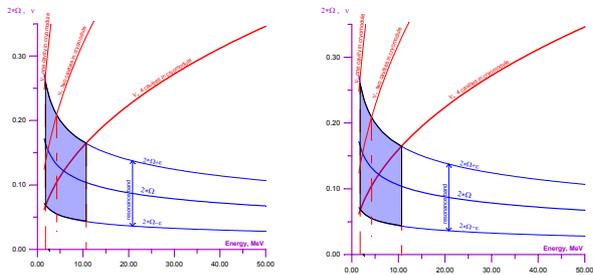


Figure 5: The parametric resonance for H and D beams due to the proposed drift space in the HWR linac.

Figure 6 shows the optimized phase portrait of the H and D beams at the exit of the linac for different numbers of cavities per cryomodule. The total drift for the whole accelerator is the same for all options. This implies that the average acceleration field over one unit of periodicity remains constant. From fig. 6 one can see that the parametric resonance stretches the bunch and also changes the effective phase area in the longitudinal plane. For the bunch at the linac entrance an energy spread of  $\Delta W/W = \pm 1.6\%$  and a phase length of  $\Delta\Phi = \pm 15^\circ$  was assumed. For the option “4 cavities per cryomodule”, the bunch has the largest longitudinal emittance, and the “zero drift” option leads to the most compact bunch. The option with 2 and 4 cavities per cryo-module for the 160MHz and 320MHz parts, respectively, has a similar area like the “zero drift” option, because in the first part the bunch does not cross the resonance due to the shorter period of perturbation, and in the second part with 4 cavi-

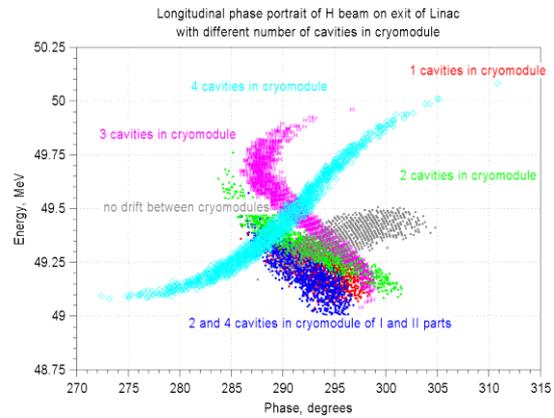


Figure 6: Phase portraits of H beam at the exit of the HWR linac, assuming different arrangement options.

ties per cryomodule the resonance is left behind already. Hence this lattice also avoids crossing the parametric resonance and would be preferable from a beam-dynamical point of view. However, technically it is difficult to fit all required equipments after each cryomodules with two cavities, and we need longer drift space also, which one compensates partly this advantage.

At energies higher than 20MeV we will not cross a parametric resonance at all. Therefore, if the transition from the 160MHz to the 320MHz HWCs takes place above 20MeV, only the 160MHz family can restrict the longitudinal acceptance of the linac. However, the bunch has to be compressed by a factor of 2 in phase during the transition because the frequency doubles. From this point of view the transition has to take place after the 20-th cavity. The length of the drift space is currently 78cm between cryomodules and determined by the required space for the focusing quadrupoles and additional equipment like diagnostic, vacuum valves and so on.

### 3 CONCLUSION

We have analysed H and D beams dynamics in the linear accelerator based on the quarter- and half-wave superconducting cavities. Unlike heavy ions the H and D beams are transversely unstable in conventional QWCs. We have investigated the drift space influence to the longitudinal beam dynamics. For H, D and particles with ratio  $m/e \leq 5$  the parametric resonance due the drift space affects significantly to the longitudinal beam dynamics.

### 4 REFERENCES

[1]W. Brütigam, O. Felden, M. Glende, H. Jungwirth, A. Lehrach, R. Maier, S. Martin, A. Schnase, Yu. Senichev, R. Stassen, R. Tölle, E. Zaplatin, “SC accelerator components for Light Ion Linacs”, Super-Conductive Workshop, Tsukuba, 2001.