New Results on Electron Cooling at the TSR Heidelberg

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Abstract

At the heavy ion storage ring TSR the longitudinal electron cooling force can be measured with two methods. The first one uses stochastic noise and studies the longitudinal velocity distribution of the stored ion beam at equilibrium between stochastic heating and electron cooling, The second one uses an induction accelerator (IndAcc) and investigates the equilibrium between electron cooling and a constant auxiliary force created by the IndAcc. For the first time both methods were applied in parallel at the same storage ring and the results could be directly compared. As the ion current in such measurements is high, the influence of the ion density has to be taken into account. The results of the longitudinal electron cooling force measurements will be presented.

1 STOCHASTIC HEATING

The method [1] of measuring the longitudinal cooling force proceeds by studying the longitudinal beam distribution in the equilibrium between electron cooling and additional stochastic heating, which should dominate intrabeam scattering. To obtain absolute quantitative results it is important to ensure a reliable measurement of the diffusion constant due to the stochastic heating process. In longitudinal direction this heating is realized by a broad band stochastic noise signal applied to the gap of the resonator, with the center frequency of the noise band located close to a harmonic of the revolution frequency. The principle of the method can be understood with the aid of the Fokker-Planck equation, describing the time evolution of the longitudinal velocity distribution function $\rho(v_\parallel)$ under the influence of friction ($F_\parallel$) and diffusion (D)[1]:

$$\frac{\partial\rho(v_\parallel)}{\partial t} = \frac{\partial}{\partial v_\parallel}\left(-\rho(v_\parallel)\frac{\bar{F}_\parallel}{m_i} + \frac{\partial}{\partial v_\parallel}(\rho(v_\parallel)\cdot D)\right) \quad (1)$$

The force $\bar{F}_\parallel$ is the ring averaged cooling force: $\bar{F}_\parallel = \eta_c \cdot F_\parallel$, where η_c is the ratio between effective cooling length and circumference of the storage ring and $v_\parallel = v_{i\parallel} - \langle v_e\rangle$ is the ion velocity in the reference frame where the average electron velocity $\langle v_e\rangle$ vanishes. In the stationary case, with $\rho(v_\parallel) \rightarrow 0$ for $v_\parallel \rightarrow \infty$, the longitudinal cooling force is given by:

$$F_\parallel(v_\parallel) = \frac{1}{\eta_c} m_i D \cdot \frac{\frac{\partial}{\partial v_\parallel}\rho(v_\parallel)}{\rho(v_\parallel)}. \quad (2)$$

The diffusion constant due to the noise signal is given [2] as:

$$D = \frac{{f_0}^2 Z^2 e^2}{2{m_i}^2 {v_{i\parallel}}^2} \cdot \frac{d{U_{eff}}^2}{df} \quad (3)$$

f_0 is the revolution frequency, Z the charge of the ion of mass m_i and velocity $v_{i\parallel}$ and $d{U_{eff}}^2/df$ represents the spectral density of the noise voltage and is proportional to the noise power density experienced by the ions.

A reliable way to measure the longitudinal revolution frequency distribution $\rho(f)$ of the ions even at high beam densities is to determine the beam transfer function (BTF). In order to get the distribution and its derivative, one has to multiply equation 2 with the differential $df/dv_\parallel = \eta \cdot f/v_{i\parallel} = \eta \cdot hf_0/v_{i\parallel}$. The longitudinal cooling force is then given by:

$$F_\parallel(v_\parallel) = \frac{h\eta {f_0}^3 Z^2 e^2}{2\eta_c m_i {v_{i\parallel}}^3} \frac{dU_{eff}}{df} \frac{\frac{d}{df}\rho(f)}{\rho(f)} \quad (4)$$

where h is the harmonic at which the BTF is measured, $v_{i\parallel}$ is the ion velocity in the laboratory frame.

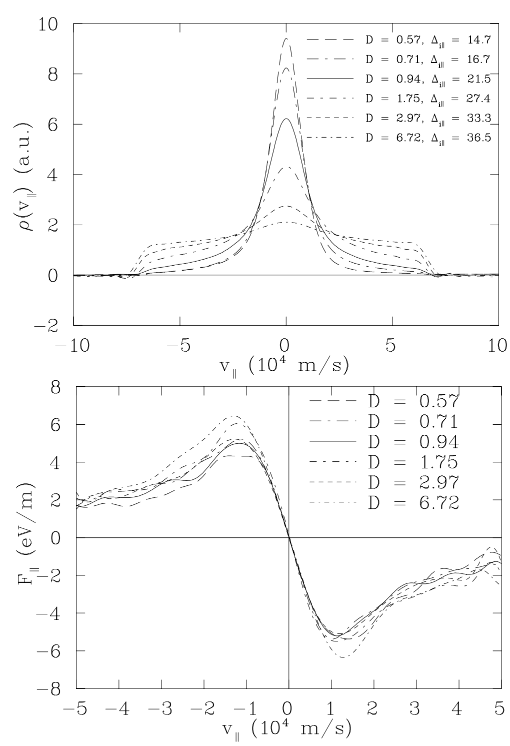

Figure 1: Cooling force measurements with stochastic heating for different noise levels ($^{12}C^{6+}$, 73.3 MeV, $n_e = 8.0 \cdot 10^6$ cm^{-3}, $I_{ion} = 20\mu$A, B_{cool} = 418 Gauss, expansion 9.6). The upper part of the figure shows the beam velocity distributions, the resulting cooling forces are shown in the lower part (D in units of 10^{10} m^2/s^3, $\Delta_{i\parallel}$ in units 10^3 m/s , where $\Delta_{i\parallel}$ is the variance of the longitudinal velocity distribution)

Examples for the beam distribution and the resulting cooling force for different noise levels are shown in figure 1. D is varied within about one order of magnitude, keeping the noise level always high enough to ensure that the width of the longitudinal beam distribution was determined by the width of the noise signal. The ion current for these measurement was $I_{ion} \approx 20$ μA. With increasing noise level the width of the longitudinal distribution $\Delta_{i\|} = \langle v_\|{}^2\rangle^{1/2}$ increases ($\Delta_{i\|} \approx 2500$ m/s for the cooled beam). Within $v_\| \leq 5000$ m/s the cooling forces deduced from the measurements performed at different values of D agree quite well. For velocities close to the maximum of the force and higher, the uncertainty increases.

The friction coefficients $\alpha_\|$ were determined by fitting a straight line to the cooling forces in figure 1 for velocities between -1000 m/s and 1000 m/s. The results scatter with a standard deviation of $0.56 \cdot 10^{-4}$ eVs/m^2 around the mean value of $\alpha_\| = 7.53 \cdot 10^{-4}$ eVs/m^2. This standard deviation is about 7.4 % and is of the order of the uncertainty of the diffusion term. The average of the absolute values of the extrema of the forces is (5.43 ± 0.60) eV/m, the average of the position of the extrema is $(12.2 \pm 1.2) \cdot 10^3$ m/s, with a somewhat higher uncertainty. Thus the cooling force, in particular the friction coefficient and the position of the extreme values of the force, can be determined with an experimental accuracy of about 10 %.

2 INDUCTION ACCELERATOR

The longitudinal cooling force within its linear regime can also be measured very conveniently with the aid of the induction accelerator. The IndAcc applies a longitudinal accelerating or decelerating force which is constant for all ions. The change in energy experienced by an ion is typically of the order of 1 eV per turn. Detailed description of the IndAcc and the methods for obtaining the longitudinal cooling force can be found in [3]. In the linear regime of the longitudinal cooling force the ion beam is accelerated or decelerated by the IndAcc until the ion beam has reached a relative velocity $v_\|$, where the force by the IndAcc is compensated by the electron cooling force, leading to a shift Δf in the revolution frequency. The relative velocity $v_\|$ can be extracted by Schottky noise analysis, and by measuring the strength of the applied voltage U_{ind} the cooling force of that relative velocity can be evaluated:

$$F_\|(v_\| = \frac{1}{\eta}\frac{v_{i\|}}{f_h}\Delta f) = -\frac{Ze\,U_{ind}}{L_C} \qquad (5)$$

$v_{i\|}$ is the velocity of the ion beam in the laboratory frame and f_h is a harmonic of the average revolution frequency without the additional IndAcc force, η is a machine parameter (η = 0.89 for the standard mode of the TSR), Z is the charge of the ion and L_C = 1.2 m is the effective length of the interaction region.

An example of such a measurement with $^{12}C^{6+}$ (73.3 MeV, I_{ion} = 20 μA, $n_e = 8 \cdot 10^6$ cm^{-3}) is shown in fig. 2. The full curve shows the spectrum of the cooled beam without the additional force by the IndAcc. The dashed curve is the spectrum taken while inducing U_{ind} = 0.4 V. The first momenta of the spectra are indicated by straight lines. These two spectra yield one point of the cooling force.

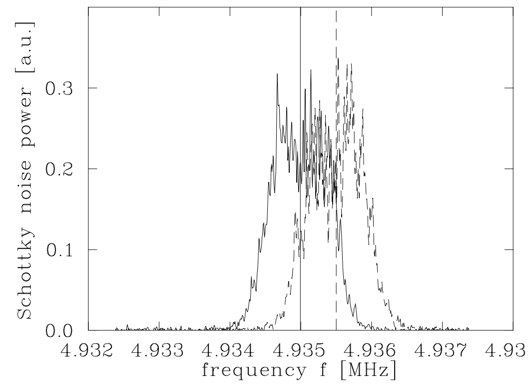

Figure 2: Longitudinal cooling force measurement with the IndAcc. Full curve is the spectrum of the cooled beam. Applying U_{ind} = 0.4 V results in a spectrum shown by the dashed curve. The first momenta are drawn as straight lines.

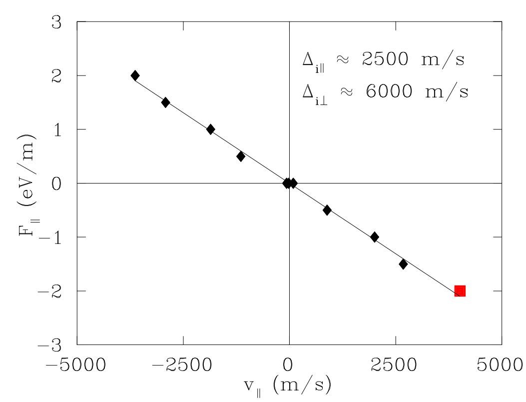

Figure 3: Longitudinal cooling force as function of the longitudinal relative velocity ($^{12}C^{6+}$, 73.3 MeV, I_{ion} = 20 μA, $n_e = 8 \cdot 10^6$ cm^{-3}). $\Delta_{i\|}$ is the longitudinal and $\Delta_{i\perp}$ the transverse velocity spread of the ion beam.

Applying this procedure for different strengths of the IndAcc force yields the longitudinal cooling force shown in fig. 3; the data point presented by a square results from the measurement shown in fig. 2. A straight line fit to the data yields a longitudinal friction coefficient of $\alpha_\| \approx 5 \cdot 10^{-4}$ eVs/m^2, which is about 40 % less than the average value from stochastic heating measurements. Furthermore the coefficients from the IndAcc measurement show a clear decrease with increasing ion density, whereas the results from stochastic heating seem to be independent of the ion density as shown in fig. 4, where the the inverse friction coefficient $1/\alpha_\|$ as function of the average ion density n_{ion} is plotted. As the coasting beam can be described by Gaussian functions in the transverse plane, n_{ion} is defined as:

$$n_{ion} = \frac{I_{ion}}{Zev_{i,\parallel}} \cdot \frac{1}{\pi\sigma_x\sigma_y} \quad (6)$$

where $\sigma_{x,y}$ is the horizontal and vertical Gaussian widths of the transverse profile. The measurements shown in fig.4 were carried out for $^{12}C^{6+}$ ions of E=73.3 MeV and an electron density of $n_e = 8 \cdot 10^6$ cm^{-3}). The inverse friction force coefficients $1/\alpha_\parallel$ measured with the induction accelerator are shown by squares, those with stochastic heating by circles.

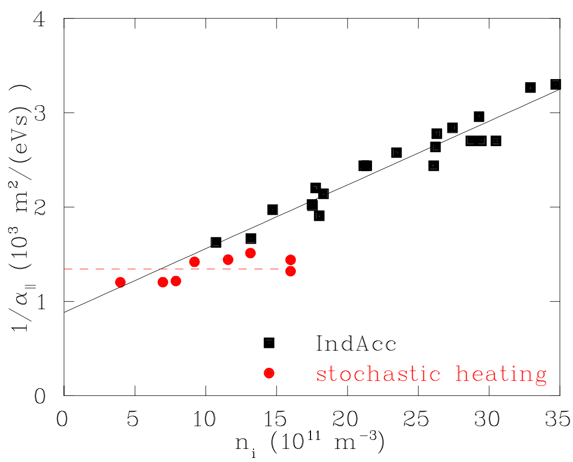

Figure 4: Inverse friction coefficient $1/\alpha_\parallel$ as function of the average ion density for $^{12}C^{6+}$ ions (E = 73.3 MeV, $n_e = 8 \cdot 10^6$ cm^{-3}. Measurements with the induction accelerator (squares) and with stochastic heating (circles) are shown. To the IndAcc data a linear function is fitted (full line), while the data obtained with stochastic heating is well represented by a constant function (dashed line).

The linear dependence of $1/\alpha_\parallel$ obtained with the IndAcc can be explained by the following model: By the induction accelerator the longitudinal momentum of each particle is changed by $\Delta p_{\parallel ion}$. In a stationary condition this momentum is compensated in the electron cooler $(\Delta p^{EC}_{\parallel ion})$:

$$\Delta p_{\parallel ion} + \Delta p^{EC}_{\parallel ion} = 0 \quad (7)$$

Due to momentum conservation during electron cooling:

$$n_{ion}\Delta p^{EC}_{\parallel ion} + n_e \Delta p_{e\parallel} = 0 \quad (8)$$

one gets:

$$n_{ion}\Delta p_{\parallel ion} = n_e \Delta p_{e\parallel} \quad (9)$$

The electron density n_e is considered to be constant over the cross section of the beam and is given by:

$$n_e = \frac{I_e}{\pi e \alpha_{exp} {R_i}^2 v_e} \quad (10)$$

I_e is the electron current, α_{exp} = 9.6 is the expansion factor of the beam , R_i = 4.76 mm is the cathode radius and v_e is the electron velocity.

The change in ion momentum due to the induction accelerator can be approximated by $\Delta p_{\parallel ion} = \Delta E_{ion}/(2E_{ion}) \cdot p_{\parallel ion}$, where $E_{ion} = ZeU_{ind}$, with U_{ind} being the induced voltage. Therefore the change of the electron velocity can be described by

$$\Delta v_e = n_{ion}/n_e \cdot ZeU_{ind}/(m_e v_{i,\parallel}) \quad (11)$$

where m_e is the electron mass. In the linear regime of the cooling force the inverse of the friction coefficient is thus given by $1/\alpha_\parallel = -(v_{i\parallel} - \langle v'_e \rangle)/F_\parallel$, with $\langle v'_e \rangle = \langle v_e \rangle + \Delta v_e$. In the stationary situation the electron cooling force amounts to $F_\parallel = -F_{ind} = -ZeU_{ind}/L_C$, therefore the inverse friction coefficient is given by $1/\alpha_\parallel = 1/\alpha^{meas}_\parallel - \Delta v_e/F_{ind}$. $\alpha^{meas}_\parallel$ is the friction coefficient derived from the measurement without taking the additional change of the electron velocity into account:

$$\frac{1}{\alpha^{meas}_\parallel} = \frac{1}{\alpha_\parallel} + \frac{L_C}{m_e v_{i,\parallel}} \cdot \frac{n_i}{n_e} \quad (12)$$

A linear fit $1/\alpha^{meas}_\parallel = 1/\alpha_\parallel + \kappa \cdot n_i/n_e$ to the IndAcc data in fig. 4 gives $\alpha_\parallel = 11 \cdot 10^{-4}$ eVs/m^2, which is in reasonable agreement with the average value of the friction coefficient from stochastic heating $\bar{\alpha}_\parallel = (7.5 \pm 0.7) \cdot 10^{-4}$ eVs/m^2. For κ we obtain $\kappa = 5400$ m^2/(eVs). This is in good agreement with our simple model, which gives $\kappa = L_C/(m_e v_{i,\parallel}) = 6180$ m^2/(eVs). The value of κ should be independent of the ion species. In one beam time the longitudinal cooling force was therefore measured with the IndAcc as function of the ion current for D^+, $^6Li^{3+}$, $^{12}C^{6+}$ and $^{16}O^{8+}$ (6.1 MeV/u), which were stored at the same setting of the TSR and the electron cooler (I_e = 30 mA, $n_e = 8 \cdot 10^6$ cm^{-3}). The average value amounts to $\bar{\kappa} = (6400 \pm 2000)$ m^2/(eVs), again in good agreement with our simple model.

3 CONCLUSION

At the TSR several methods to measure the longitudinal cooling force are available. One method makes use of stochastic heating. The measured force is independent of noise level and ion current and can be measured with an accuracy of about 10 %. Furthermore the position of the extreme values v_{max} can be measured. Measuring the longitudinal friction force with the induction accelerator is more accurate, but the influence of the ion density has to be taken into account. Looking at $\alpha_\parallel$ (IndAcc) and $\bar{\alpha}_\parallel$ (stochastic heating) for $n_i \rightarrow 0$ there seems to be some discrepancy, marking a point for further studies.

4 REFERENCES

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[2] S. van der Merr, CERN/PS/AA 78-6 (1978)

[3] C. Ellert et. al., NIM A 314,399 (1992)