A NEW EXPERIMENTAL APPROACH TO SPACE CHARGE EFFECTS

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Abstract

In recent papers[1, 2], we proposed a novel experimental approach to investigate various collective effects in space-charge-dominated beams. It was demonstrated that either a radio-frequency quadrupole trap or a solenoidal trap could reproduce nonlinear processes equivalent to those in a beam transport channel. In the present paper, we outline the essence of the idea, showing typical trap configurations for beam-physics applications. We also briefly discuss possible trap experiments that greatly deepen our current understandings of collective beam instabilities including coherent resonances and halo formation.

1 INTRODUCTION

The applications of high-power ion beams to diverse purposes, such as the production of tritium, the transmutation of radioactive waste, and heavy ion fusion, have been proposed in recent years. Since these applications require average currents much higher than those in existing accelerators, extra attention must be paid to the collective beam instabilities induced by the Coulomb self-field. It is, however, practically very difficult to observe the dynamic behavior of moving ions with a sufficient resolution in a non-destructive way. We inevitably face many limitations in accelerator-based experiments not only because the beam is travelling at great speed but also due to various noise sources that complicate the output data.

In recent papers [1, 2], Okamoto proposed a novel experimental method to explore various features of space-charge-dominated beams. The idea is based on the simple fact that a particle beam seen from the rest frame is physically almost equivalent to a single-species plasma in a trap system. Two types of trap configurations, i.e. a radio-frequency quadrupole trap (Paul trap) and a solenoidal trap, were considered. It was demonstrated that these systems enable us to replicate the collective motion of ions propagating through a periodic magnetic lattice. This fact implies that we can experimentally study the space-charge effects by using a compact tabletop device instead of expensive large accelerators. Since the plasma centroid is at rest in the laboratory frame, we can easily and directly measure the particle distribution in trap experiments. In the following sections, we discuss this new experimental scheme illustrating typical trap systems.

2 BASIC EQUATIONS

Suppose a longitudinally uniform beam propagating through a linear transport channel. The Hamiltonian for the transverse dynamics is given by

$$H_{beam} = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}K(s)(x^2 - y^2) + \frac{q}{p_0\beta_0 c\gamma_0^2}\phi, \quad (1)$$

where q is the charge state of particles, c is the speed of light, β_0 and γ_0 are the Lorentz factors, p_0 is the kinetic momentum of the beam, and the independent variable s represents the distance measured along the design beam orbit. Here, the focusing function $K(s)$ has been defined by $K(s) = -qG(s)/p_0$ with $G(s)$ being the gradient of the quadrupole fields. The scalar potential ϕ can be derived from the Poisson equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi = -\frac{q}{\varepsilon_0}\iint f dp_x dp_y, \quad (2)$$

where f is the particle distribution function in phase space. Provided that the effect of intra-particle collisions is negligible, f satisfies the Vlasov equation $df/ds = 0$. These equations clearly form a closed set self-consistently describing the collective beam motion.

Let us now consider a single-species plasma confined in a compact trap system. Assuming that the plasma motion is non-relativistic, we have the Hamiltonian

$$H_{plasma} = \frac{1}{2m}\Big[(p_x - qA_x^{ext})^2 + (p_y - qA_y^{ext}) + (p_z - qA_z^{ext})^2\Big] + q(\phi_{ext} + \phi), \quad (3)$$

where ϕ_{ext} and $\mathbf{A}_{ext} = (A_x^{ext}, A_y^{ext}, A_z^{ext})$ are the scalar and vector potentials of external plasma confinement fields, m is the rest mass of confined particles, and ϕ is the space-charge potential satisfying the Poisson equation (2). If the plasma is either hot or thin, the effect of Coulomb collisions among individual particles is negligible. The distribution function then obeys the Vlasov equation, similar to the case of particle beams.

In a Paul trap where no magnetic field is used, we can simply put $\mathbf{A}_{ext} = 0$. The transverse plasma confinement is achieved by an electric potential of the form

$$\phi_{ext} \approx \left(\frac{x^2 - y^2}{R^2}\right)V(t), \quad (4)$$

where $V(t)$ is the radio-frequency voltage applied to the electrodes, and R is the aperture radius corresponding to the minimum distance between the longitudinal axis and the electrode poles. Under these conditions, Eq. (3) can be rewritten as [1, 2]

$$H_{RFQ} = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} K_p(\tau)(x^2 - y^2) + \frac{q}{mc^2}\phi, \qquad (5)$$

where $K_p(\tau) = 2qV(\tau) / mc^2 R^2$, the scaled independent variable is $\tau = ct$, and we have ignored the longitudinal motion.

An alternative way to trap a large number of charged particles is the use of uniform longitudinal magnetic field. The vector potential for this type of field is $\mathbf{A}_{ext} = (-By/2, Bx/2, 0)$ where B is the field strength. Putting $\phi_{ext} = 0$, we obtain, from Eq. (3),

$$H_{SOL} = \frac{1}{2m}\left[\left(p_x + \frac{qBy}{2}\right)^2 + \left(p_y - \frac{qBx}{2}\right)^2\right] + q\phi(x, y; t), \qquad (6)$$

where the longitudinal motion has been omitted again. By employing the variables in a rotating frame, the Hamiltonian (6) is transformed to [1, 2]

$$\tilde{H}_{SOL} = \frac{\tilde{p}_x^2 + \tilde{p}_y^2}{2} + \frac{1}{2}\kappa^2(\tilde{x}^2 + \tilde{y}^2) + \frac{q}{mc^2}\tilde{\phi}(\tilde{x}, \tilde{y}; \tau), \qquad (7)$$

where $\kappa = qB/2mc$ and $\tilde{\phi}(\tilde{x}, \tilde{y}; \tau) = \phi(\tilde{x}\cos\psi + \tilde{y}\sin\psi, -\tilde{x}\sin\psi + \tilde{y}\cos\psi; \tau)$ and $\kappa = qB/2mc$. Equation (7) is similar to the previous Hamiltonians in Eqs. (1) and (5) while the stationary state is now axisymmetric.

3 TRAP CONFIGURATIONS

3.1 Radio-Frequency Quadrupole Trap

A schematic view of a typical linear Paul trap is illustrated in Fig. 1. A single-species plasma is confined near the trap axis with an radio-frequency electric field of quadrupole symmetry. For the axial confinement, static voltages are applied to two end plates or end pipes to form a longitudinal potential well. The signs of the periodic voltage functions $V_x(t)$ and $V_y(t)$ must be opposite, so that a strong quadrupole focusing field is generated. When $V_x(t) = -V_y(t) = V(t)$ and $U_x = U_y = 0$, the focusing potential is approximately expressed by Eq. (4). Detailed design considerations can be found in Ref. [3].

Since H_{RFQ} has the form identical to H_{beam} except for the coefficients, the collective motion of a single-species plasma in the Paul trap is physically equivalent to that of a particle beam in a linear transport channel. Therefore, as first pointed out in Ref. [1], we can experimentally simulate the nonlinear behavior of charged-particle beams by adjusting the form of $V(\tau)$ to the variation pattern of $G(s)$ along an arbitrary accelerator lattice. For instance, the voltage pattern in Fig. 2 simulates a FODO beam transport channel. It is, of course, possible to replicate much more complex lattice structures.

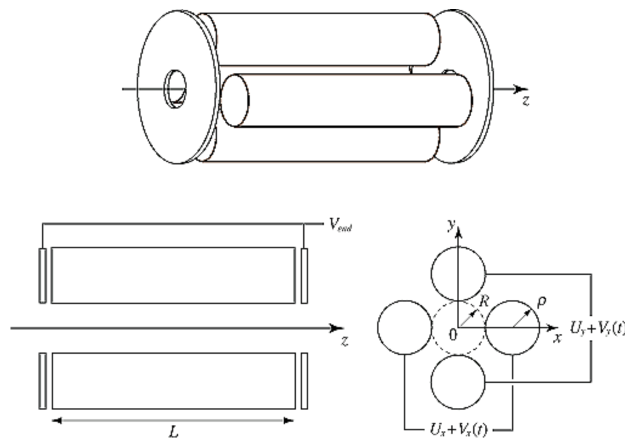

Figure 1: Schematic view of a typical Paul trap.

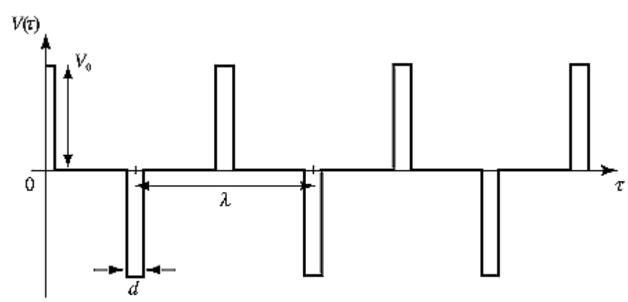

Figure 2: Example pattern of the electrode voltage $V(\tau)$.

3.2 Solenoidal Trap

Figure 3 shows the layout of a trap system which is basically composed of a solenoid coil, many ring-shaped electrodes aligned along the axis of the coil, a vacuum vessel, a source of charged particles, and multi-channel Faraday cup or phosphor plate. This system is called *multi-ring-electrode* (MRE) *trap* [4].

In order to prevent longitudinal particle loss, we apply different static voltages to the ring electrodes, creating a potential well. The well must, of course, be deeper than the initial energy of the particles extracted from the source. By changing the pattern of the applied voltages, we can form a plasma with various aspect ratios. It is thus possible not only to study the space-charge effects in a short bunch but also to approximately realize the two-dimensional situation formulated in the last section. Provided that the longitudinal potential well is deep enough, the achievable plasma density is calculated from the simple formula

$$n_{\text{limit}} = \frac{\varepsilon_0 B^2}{2m}. \quad (8)$$

This is referred to as either the *Brillouin density limit* or *space-charge limit*.

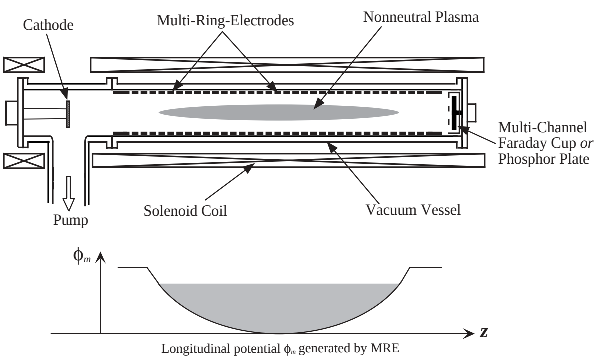

Figure 3: A multi-ring-electrode trap

4 POSSIBLE EXPERIMENTS

The typical subjects that can be studied with a single-species plasma trap are *coherent resonances* and *beam halo formation*. For a systematic exploration of these phenomena, the density of a confined plasma must be controllable. In other words, it is desirable to provide some mechanism that enables us to alter the *tune depression* η, i.e. the ratio of the space-charge-depressed tune to the bare tune. For an electron plasma, we can control the value of η to some degree by modifying the cathode condition, for example. On the other hand, a handy way to reduce the tune depression of an ion plasma is the use of a cold neutral gas; the plasma can be cooled through direct collisions with the gas atoms. An alternative, much better way is to apply the laser cooling method [5]. Since the Doppler limit of laser cooling is in a milli-Kelvin range or even below, we can cover the full range of tune depression from $\eta = 1$ (high-temperature limit) to $\eta = 0$ (low-temperature limit). Moreover, it is possible to directly and non-destructively measure the plasma motion at an extremely high resolution by detecting photons spontaneously emitted from excited ions. Amplifying the fluorescence by a photo-multiplier and then catching it with a CCD camera, we can even image the real-time evolution of a tiny portion of the plasma.

The instability due to coherent resonances can be examined easily and systematically with a Paul trap system. All we have to do is to adjust the design tune of the system after generating a plasma of a certain temperature. Needless to say, the bare tune can readily be controlled through the pulse height and width of the electrode voltages. Resonance experiments are feasible with a solenoid trap as well, while a periodic perturbation must be applied to the plasma core in order to excite a resonance of particular order.

Since we can give a specific amount of mismatch to the plasma simply by disturbing the electrode potential, it is possible to investigate diverse aspects of halo dynamics including the η-dependence of halo extent, the role of mismatch, the amount of halo particles, etc. Another major advantage of a single-species plasma trap is its long confinement time. If the vacuum pressure is sufficiently high, we can stably maintain a plasma over a few minutes or, in some cases, even longer than hours. Considering this fact as well as the high resolution of measurement data, long-term effects like *intrabeam scattering* is also regarded as an object of trap experiments. Following the time evolution of a plasma core, we can experimentally evaluate the growth rate due to incoherent Coulomb collisions and its dependence on the plasma density and lattice characteristics.

Finally, it is informative to point out that the Paul trap configuration allows the study of bunched beams with various aspect ratios. As mentioned above, such experiments can be done with a MRE trap; it is only necessary to change the pattern of the applied voltages to the ring electrodes. Similarly, in a Paul trap, we can form an ellipsoidal plasma with a particular aspect ratio by using *segmented* electrode rods [3].

5 CONCLUDING REMARKS

On the basis of the idea first proposed in Refs. [1] and [2], we studied two types of plasma trap systems as a tool for experimental beam physics. As briefly reviewed in Section 2, the present trap configurations reproduce the collective processes equivalent to those in beam transport channels. This fact indicates a unique possibility that we use a compact tabletop devise for the systematic study of space-charge-dominated beams. Considering the technical difficulties and many noise sources accompanied by accelerator-based experiments, a plasma trap simulator offers much better information about the fundamental mechanisms of collective beam instabilities.

6 REFERENCES

[1] H. Okamoto, “On dynamical analogy between linear beam transport channels and plasma trap systems” Hiroshima University Preprint HUBP-01/98 (1998).

[2] H. Okamoto and H. Tanaka, Nucl. Instr. Meth. A437 (1999) 178.

[3] H. Okamoto, Y. Wada, and R. Takai, Nucl. Intr. Meth. A485 (2002) 238.

[4] A. Mohri, H. Higaki, H. Tanaka, Y. Yamazawa, M. Aoyagi, T. Yuyama, and T. Michishita, Jpn. J. Appl. Phys. **37** (1998) 664.

[5] D. J. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. **20** (1975) 637; T. Hänsch and A. Schawlow, Opt. Commun. **13** (1975) 68.