DYNAMIC BEHAVIOR OF CHARGED PARTICLE BEAMS IN CURVILINEAR MAGNETIC FIELD

Z. Parsa*, Brookhaven National Laboratory, Physics Dept., 510 A, Upton, NY 11973, USA.
V. Zadorozhny†, Institute of Cybernetic, National Academy of Sciences of Ukraine.

Abstract

We have studied the steady - state solution of Vlasov - Maxwell system and have obtained the following results, by using universal Maxwell equation we construct electric and magnetic fields such that the given steady - state (for example Brilluen beams) becomes locally asymptotically stable. The proper way in synthesis of control electrical and magnetic fields are discussed.

1 INTRODUCTION

The problem of stability, acceleration and focusing bunched beam has received a great deal of attention in last two decades. After early publications concerning a methodical study of linear dynamics of a bunched beam (see reviews [1] and references given therein) a lot of works have been devoted to the investigation of the nonlinear dynamics of the one arising in the high - energy beam and adjacent question (see, e.g. [4]). The interest in the investigation of behavior, based on Vlasov - Maxwell equations (VME) [2] has increased appreciably at present. This interest is due to, on the one hand, the universality VME as a general model in the theory of behavior many particle system and, on the other hand, to remarkable contradictions in results obtained by different authors. A lot of analytical investigations and computer experiments (see, for example [4]) are devoted to the study of this questions. It is not surprising then that recent work on high- energy beams has caused a new surge in the study of VME. In this paper, an approach to finding a solution to VME is presented using direct Lyapunov method and properties of vector field with a singular point on a compact manifold. The VME is transformed to Fredholm's equations in an attraction region. These properties are linked to the self- focused and acceleration of a bunched beam. The most intensive investigations of behavior bunched beams were carried out in the model VME with power nonlinearity of electric and magnetics fields.

Solving this problem analytically we used, as a rule, that for an arbitrary motion of charged particle there exists an electric and magnetic fields yield the given motion, in other words, the Maxwell equations are universal.

Under this postulate the Vlasov equation

$$\frac{\partial f}{\partial t} + v\partial_x f + (E + [vH])\partial_v f = 0 \tag{1}$$

transform in linear equation for f , so that a field (E, H) is arbitrary parameter. Looking for a nontrivial solution of this difference equation in the form

$$f(t, x, v) = f_0(x, v)e^{i\omega t}$$

with a constant value ω, shows that f_0 satisfies the equation

$$Lf_0 \equiv v\partial_x f_0 + (E + [vH])d_v f_0 \tag{2}$$

Research in his direction is inspired by the fact that a spectrum of the operator L represents a completely characteristic of the asymptotic behavior of a solution of differential equation for given parameter (E, H). It is important to note that may be an area such that $\omega_0 = 0$ on the boundary. In this case ω_0 is a limit spectrum and eigenfunction $f_0(\omega_0)$belong to Sobolev space. Otherwise, the topological structure of the family of solutions in any neighborhood of steady- state will be constructed sufficiently simple. The question concerning transform of asymptotic motion into chaos is more subtle.

2 MAIN RESULT

Let us first look at some simple example and will find spectrum of the operator L. We choose a dynamical system as follows

$$\dot{x} = -\sin x, \qquad x \in [-\pi, \pi] \equiv \Delta \tag{3}$$

It is obvious that $x = 0$ is asymptotically stable steady -state. Consider the following operator L:

$$Lf_0 = \sin x\partial_x f_0$$

Let $\{e^n_{(x)}\}$ be a complete orthonormal set in Δ. Then $f_0 = \sum_{|n|\leq\infty} C_n e^n(x)$. Consider the following uniform problem

$$Lf_0 = \lambda f_0, \qquad f_0(0) = 0 \tag{4}$$

Then

$$Lf_0 = \int_{-\pi}^{\pi} \sum_{|n|\leq\infty} \mu_n e^n(x)e^{-n}(y)f_0(y)dy \tag{5}$$

where $\{\mu_n\}$ is a set such that $\mu_n^{-1} \in \Lambda(L)$, Λ is the spectrum of L.The consideration of (5) in more detail shows that $\Lambda$is a spectrum of Toeplitz matrix $T_n = [t_{rq}]_0^N$, $\qquad t_{rq} = \int_{-\pi}^{\pi} \sin x\partial_x e^{-n}e^n dx.$

* Supported by US Department of Energy contract Number DE-AC02-98CH10886. E-mail: parsa@bnl.gov
† zvf@umex.istrada.net.ua

Define $t_{00} = 1$, then we have: T_n which is a Toeplitz matrix. This reasoning yields a simple conclusion. If we choose a positive integer N and look for $\lambda_\nu^N \in \Lambda, \quad \nu = 1, ...N+1$ then

$$\lambda_\nu^N = -1 - \cos\frac{\nu\pi}{N+2} \tag{6}$$

$$\nu = 1, 2, ..., N+1.$$

Study of this questions more preciesly see in [3].

3 ALGORITHM OF CHOOSING STABILIZING ELECTRICAL AND MAGNETIC FIELD

Assuming the input into Brilluen flow is set by plane $z = 0$, we determine electric and magnetic fields such that motion of electrons at volume of the Plasma Lens is satisfied by Brilluen initial conditions on interval $\Delta L\epsilon[-\alpha, 0], \ \alpha \ << 1$. That means the condition of equipotentials of magnetic field lines are held and electric filed $E_\perp = 0$. It should be noted that general fields in space $R^2\Delta L$ will act as stabilizer, so after a pass through this space beam must come out at Brilluen orbit. Thus, the electron must reach plane $L = 0$ independent of the initial radial velocity. Such motion is called tautochronic.

Point of tautochrone will be the state of asymptotically stable equilibrium. We will use this general approach for finding force of electric field $E_{||}$.

Using Poisson equation and dependence of the beam on coordinates (in interval ΔL), we obtain such a charge distribution that gives possibility to electron beam to transporting on equilibrium Brilluen orbits.

Thus, we have investigated possibility of entrance on to stationary equilibrium orbits of compensated charged particle beams in curvilinear magnetic field configuration. We applied asymmetrical approach for hydrodynamic model using principle of equipotentialization of magnetic field lines for creation of stationary equilibrium orbits of charged particles typical for many plasma devices.

4 SUMMARY

We let the motion of bunched beam be described by simple equation (for example by hydrodynamical type) that don't consider the temperature disturbance of velocity of particle, and let this equation have a steady - state. The one have physical sens if and only if it is asymptotically stable with respect to the solution of (1). Moreover, if we deal from condition $|v| \leq 1$, then we have acceleration.

we have investigated possibility of entrance on to stationary equilibrium orbits of compensated charged particle beams in curvilinear magnetic field configuration. We applied asymmetrical approach for hydrodynamic model using principle of equipotentialization of magnetic field lines for creation of stationary equilibrium orbits of charged particles typical for many plasma devices.

5 REFERENCES

[1] Z. Parsa, V. Zadorozhny. Focusing and Acceleration of Bunched Beams // in AIP conf. Proc. 1999, 530, p. 249-259.

[2] A. A. Vlasov. Many-Particle Theory and its Application to Plasma Physics. New York, 1961,355p.

[3] Z. Parsa, V. Zadorozhny. Nonlinear Dynamics on Compact and Beam Stability // Nonlinear Analysis. 2001, v. 47, p. 4897-4904.

[4] Status of Muon Collider Research and Development and Future Plans Proceedings of HEMC'99, 27 Sept. -1 Oct., 1999, Montauc, NY, USA.