A PROPOSAL TO MEASURE THE DODECAPOLE COMPONENT OF THE LHC TRIPLET MAGNETS USING A WOBBLING METHOD

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Abstract

Due to finite manufacturing tolerances the triplet quadrupoles used in the interaction regions of LHC will have non-vanishing multipole errors which need to be measured and corrected in order to guarantee sufficient dynamic aperture at collision energy. Here we discuss a method to measure the unwanted multipole components by oscillating a closed orbit bump in the interaction region and observing the orbit at pickups outside the bump. The beam's response will contain very weak signals at harmonic frequencies of the sinusoidal excitation. Even though the amplitude of the harmonic signal will be below the resolution of the position monitor system, it can be made visible by adding noise to the original pickup data and subsequent careful filtering and averaging. We use a simple computer model to simulate the oscillating bump that generates pickup signals and then analyze those in a sophisticated signal processing chain in order to retrieve the magnitude of the unwanted multipole components.

1 INTRODUCTION

In LHC the dominant sources of non-linearities at collision energy – apart from the beam-beam non-linearities – are the quadrupoles nearest to the interaction points (IP) which are very strong in order to provide small beam sizes at the IP and have a large aperture to accomodate the beams' separation due to the finite crossing angle. Since the quadrupoles are relatively far away from the IP the beta functions are very large, around 4500 m [1]. Furthermore, finite manufacturing tolerances will cause the presence of higher multipoles in the quadrupoles, especially at full excitation. These non-linearities, together with the large beta functions, will be the dominant factors that restrict the dynamic aperture at collision energy. Of the non-linearities, the dodecapole component will be one of the most pronounced [2] and, if left uncorrected, will reduce the dynamic aperture by more than 20 %. This poses the question whether there are beam based methods to determine the magnitude of the dodecapole component and whether the triplet correction magnets – they will be introduced to compensate the dodecapole's adverse effect – are actually compensating rather than enhancing the detrimental non-linearities.

In this report we will discuss a method that is based on driving an oscillating closed local bump across the triplet quadrupole magnets which is similar but simpler to the one presented in Ref. [3]. The x^5 kick from the dodecapole component will cause the local bump to be not closed and harmonics of the wobbling frequency will be visible as very weak signals in the orbit outside the bump. In ref. [4] we estimate the magnitude of the harmonic oscillations to be on the order of a few times 10^{-8} m which is well below the sensitivity and also the digitization threshold of the beam position monitor (BPM) system [5]. We will show, however, that by adding a finite amount of noise – equivalent to the magnitude of the BPM resolution – to the BPM signal, the weak signal can be elevated to observable levels. This can be understood in the sense that the small harmonic oscillations modulate the noise and appear thus above the BPM's quantization threshold. Of course the wanted harmonic signals are completely buried in the noise and we need to employ some advanced digital signal processing (DSP) [6] algorithms such as image-rejection mixers (IRM) and adaptive line enhancers (ALE) in order to extract the weak harmonic signals.

2 ANALYTICAL ESTIMATES

In order to estimate the magnitude of the mixing signals we will consider a simple model consisting of an ideal closed bump made up of two orbit correctors 180 degree apart that are excited sinusoidally at around 1 Hz. The dodecapole non-linearity will cause frequency up-mixing and a slight non-closure of the bump that will perturb the closed orbit at the exciting sine and its harmonics. In ref. [4] it is shown that the amplitude of the fifth harmonic $\mathcal{A}(5)$ which is due to the dodecapole is

$$\mathcal{A}(5) = 2 \times \frac{1}{32}\frac{K_5 L}{5!}\sqrt{\beta_1\beta_2}\,\hat{x}^5 \qquad (1)$$

where $K_5 L$ is the integrated non-linear field and $\hat{x}$ is the bump amplitude at the dodecapole, $\beta_1 = 150$ m is the beta function at the orbit corector and $\beta_2 = 4500$ m at the non-linearity inside the triplet.

If we choose $\hat{x} = 5$ mm and $K_5 L = 7200/\mathrm{m}^5$ which corresponds to 1/6 of the nominal dodecapole error of $b_6 = 10^{-4}$ we obtain $\mathcal{A}(5) \approx 10 \times 10^{-9}\mathrm{m}$. Choosing another observation point with larger beta function can increase the amplitude by a factor $\sqrt{4500/150}$ such that we achieve an amplitude of $\mathcal{A}(5) \approx 50 \times 10^{-9}\mathrm{m}$. This is clearly below the resolution and the digitization steps of the beam positioning system. Before we start discussing how to retrieve these weak signals from the beam we describe the simulation program that will help to verify the estimates and test the signal processing methods.

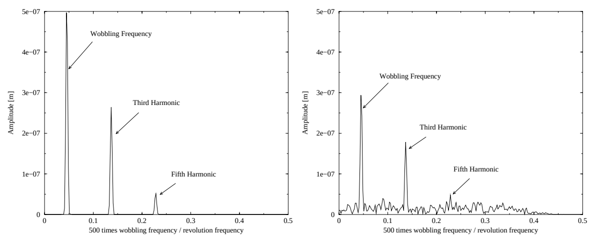

Figure 1: The low frequency spectrum of the horizontal beam position sampled at the viewpoint after decimating the raw data stream by a factor of 500 without noise on the left and with $5\,\mu$m noise added on the right.

3 SIMULATION

The simulation is based on a four-dimensional tracking program in which the beam is propagated by 4×4 transfer matrices between the dodecapoles that are modeled as thin lenses and the oscillating dipole corrector magnets. The beam propagates through a linear piece of beam line and encounters a wobbling steering magnet at a position with beta function β_1. Then it traverses a 90 degree section and meets the first triplet with beta function β_2 that contains a dodecapole error. After the triplet quadrupoles the beam traverses the IP section. Since there is a very pronounced waist at the IP the phase advance between the triplets is very close to 180^o. The beta function at the outgoing triplet is also $\beta_2 = 4500\,$m. After another 90^o section the wobbling steering magnet on the outgoing side is traversed. After the steering magnet a section is inserted that allows to adjust the tunes to the standard LHC values of $\nu_x = n.31$ and $\nu_y = n.28$. The final section is a piece of linear beam line that brings the beam to the viewpoint in another IP's triplet magnet which is both the starting and end-point of this small LHC model.

In order to test the simulation we ran about 10^6 turns and analyze the data to verify the estimates of the amplitudes given in the previous section. Since the oscillating frequency is on the order of 10^{-4} times the sampling frequency it is impossible to directly Fourier transform the entire data set and resolve the low frequencies with some accuracy. Instead we low pass filter the entire data set by a FIR-filter that cuts everything above 0.05 times the sampling frequency and only keep every tenth sample, we thus decimate the low-pass filtered data by a factor 10. The filtering makes sure that no high frequency noise is aliased into the base band which contains the desired wobbling signals. This procedure expands the frequency axis near zero frequency by a factor 10. We apply two such by-10 decimation stages and one by-5 decimation stage which expands the frequency axis near low frequencies by a factor 500. Fourier transforming the decimated signal then reveals the low frequencies present in the beam.

The left graph in Fig. 1 shows the results of the analysis of the horizontal beam position signal. Note that the horizontal axis is stretched by a factor of 500 due to the decimation process. We clearly see three peaks at one, three, and five times the wobbling frequency which roughly have peak ratios of 10 to 5 to 1 as is expected from Ref. [4]. Moreover the peak amplitude of the fifth harmonic is about 5×10^{-8} m and agrees well with the analytical prediction.

4 SIGNAL PROCESSING

In order to make the weak signals visible in the presence of digitization we add gaussian distributed random numbers with a RMS of $5\,\mu$m to the output of the tracking program and then digitize it with a granularity of $5\,\mu$m. The resulting data stream is then passed through the same three decimating stages discussed before. The resulting spectrum is shown on the right in Fig. 1 where we see that the peaks are now a little bit smaller than those of the unperturbed system and they are placed on top of a noise floor which has a magnitude of about 2 to 3 times 10^{-8} m.

This noise floor is considerably less than the $5\,\mu$m noise we added initially. This considerable reduction comes from the fact that the original noise is spread evenly over the full spectral range from 0 Hz to the Nyquist frequency at half the sampling rate which is about 5.6 kHz in our case. By decimating by a factor 500 we thus remove a considerable amount of noise. In order to quantify the reduction we realize that low pass filtering is equivalent to averaging and that averaging reduces the noise level proportional to the inverse square root of the number of averages. We can thus expect a reduction of about $1/\sqrt{500} \approx 23$ through the decimation process. A further reduction can be expected from the Fourier transformation. The noise power per bin is then reduced in much the same way as discussed above and reduces the rms noise level by a factor $2/\sqrt{N}$ where $N = 512$ is the number of samples used to calculate the Fourier transformation. We can thus expect a reduction of $2/\sqrt{512} = 1/11$. The total reduction of the noise level due to decimation and Fourier transforming is then about 1/250. With an initial noise level of $5\,\mu$m and a reduction by 250 we expect the noise level to be on the order of 2×10^{-8} m which is consistent with what we observe in Fig. 1.

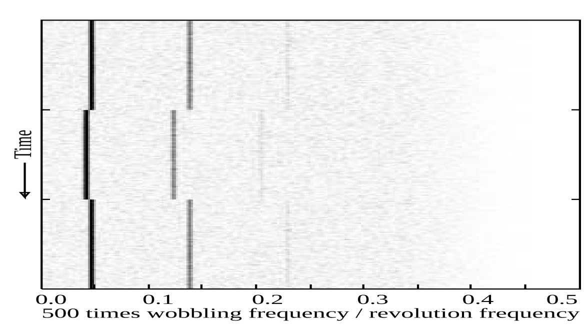

Figure 2: Spectrogram of the fifth harmonic when the wobbling frequency was temporarily lowered. The horizontal axis corresponds to that in Fig. 1 and time increases from top to bottom and the total time corresponds to 3×10^8 turns.

Another feature that is clear from looking at Fig. 1 is that the fifth harmonic signal is barely elevated above the noise floor. We can, however, display the spectra as a function of time in the form of a spectrogram as can be seen in Fig. 2 where the vertical axis from Fig. 1 is translated into a grey-scale value and higher values are displayed as darker spots. The horizontal axis covers the same frequency range as is shown in Fig. 1 and the vertical axis corresponds to time as one spectra after another is translated into grey-scale and then displayed. Time runs from top to bottom and the ticks on the vertical axis mark the time it takes to complete 10^8 turns. Also note that we show all spectrograms on a linear scale with auto-scale enabled such that the highest peak is always the maximum and the smallest value is zero. In Fig. 2 we can clearly see the fundamental, third and fifth harmonic as vertical bands running from top to bottom, albeit the fifth harmonic is barely visible. During the center third of the observation time we have reduced the wobbling frequency by a small amount and observe steps in the observed spectra of the wobbling harmonics.

In order to improve the sensitivity to the signature of the dodecapole we zoom in on the fifth harmonic and decimate the frequency range again by a factor of 5. To do this we use an IRM and mix the data stream with a frequency of 0.18, then low-pass filter and decimate by-5. We use an IRM to avoid the noise from the lower sideband to alias into the observation range. The result of frequency zooming can be seen on the left in Fig. 3 where the weak fifth harmonic and the frequency step is now clearly visible above the noise floor.

The signal to noise ratio of the fifth harmonic can be further improved by passing the signal through an ALE [7] before displaying. The ALE is an adaptive filter that very efficiently picks up coherent signals in a noisy background. We show the result of applying the ALE after the image rejection mixer and obtain the spectrogram shown on the right in Fig. 3 where the signal to noise ratio is increased significantly. We also observe equally spaced horizontal lines that are the product of re-initializing the ALE in order to ease finding the new frequency after the jump. This works because the convergence time of the adaptive filter is much shorter than the time between re-initializations.

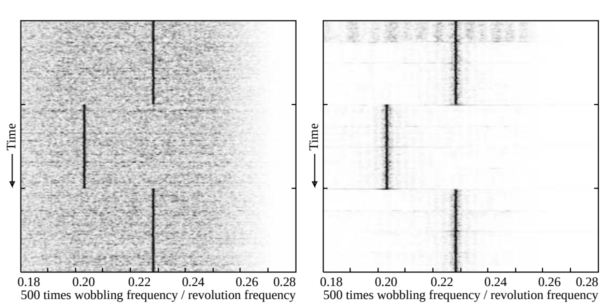

Figure 3: Spectrogram of fifth harmonic after using an IRM and a decimation factor of 5 using the same raw data that led to Fig. 2 on the left and with ALE added on the right.

5 IMPERFECTIONS

In Ref. [4] we analyze down to what dodecapole excitation the fifth harmonic is still visible and found that we can observe signals corresponding to and below $b_6 = 0.1 \times 10^{-4}$. Furthermore, redoing simulations with different noise excitations, we find that noise in the range 5 to $15\,\mu$m will permit the reconstruction of the dodecapole signature.

We investigated the effect of poor closure of the bump and found that a 10 % change in the excitation of one of the steering magnets caused the fundamental to grow dramatically, but the use of the IRM made it still possible to zoom in on the fifth harmonic. Other perturbing frequencies will have a small influence, since we can very efficiently zoom in a very small frequency range.

We have not investigated the effect of separate correction dodecapoles, but expect, if the phase advance between the triplet and the correction magnets is small, that this non-locality will have a small impact.

A rather crucial point is *how* to apply noise to the system. Directly exciting the beam will not be an option, because that will cause the emittance to grow. The noise must be applied before the analogue-to-digital converters used in the BPM system.

6 CONCLUSIONS

We show the feasibility of measuring the harmonic signals generated by the dodecapole components of LHC's triplet magnets when driving a sinuoidally excited closed bump with an amplitude of 5 mm across them.

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