PRESSURE FIELD ALONG THE AXIS OF AN ACCELERATING STRUCTURE

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Abstract

This work presents analytical and numerical results for the pressure field along the axis of a generic geometry representing an accelerating structure. Both the specific conductance and the degassing per unit length were considered in the calculation. The model is able to determine the pressure values along the symmetry axis of the structure, once the pumping speed at each extremity is given. We consider only the steady-state case, but discuss some aspects that may be relevant in the study of a transient situation, like, for instance, when the beam hits the wall, producing a gas burst.

1 INTRODUCTION

Particle accelerators use accelerating structures that are kept under vacuum to allow for high electric fields and to avoid scattering of the beam on the molecules of the gas. From the vacuum point of view, accelerating structures are usually complicated systems, due to the complex geometries they present. To operate safely, without risking sparks that could endanger the mechanical finishing, the internal pressure should be under 10^{-7} mbar. The project of a vacuum system must take into account all possible sources of gas and the conductance of each part of the accelerator. The traditional vacuum technology approach to this problem is to treat discretely each part of the system, with its respective conductance and degassing rate. This approach is limited in the sense that one cannot obtain the pressure at each point along the structure.

In this paper we present results for the pressure field along the symmetry axis of an accelerating structure, in steady state. The geometry considered for the resonating cavities is rather simplified, but keeps the basic features of cavities used in actual accelerators. We develop the concept of local specific conductance and the specific degassing rate at each point along the structure axis. In our treatment we consider two vacuum pumps, one at each extreme of the structure.

The results presented are particularly useful in the project of vacuum systems, since they allow determining the maximum distance between pumps in order to keep the pressure under specified limits [1]. The problem is treated in steady state, but we discuss briefly a situation where there is a gas burst due to the beam hitting the walls.

2 GEOMETRY AND MODELING

The model adopted is mathematically simple, but takes into account the essential details of accelerating cavities actually used. We assume the pumping at the extremes of the structure, as shown schematically in Fig. 1.

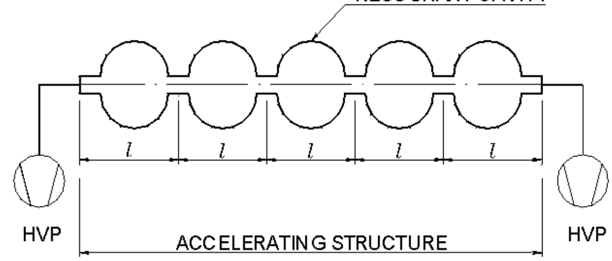

Figure 1. Schematic drawing of the accelerating structure used in the model

The system consists of 5 resonant cavities, with one vacuum pump at each extreme. The effective pumping speed will be determined by the conductance of the connections between the structure and the pumps. We assume that this conductance to be S_{eff} = 10 l.s^{-1}. The gas source is considered to be due to the natural degassing of the materials of the structure, adopted to be made of copper. In this case the degassing rate per unit area is q_{Cu} = 1.0x10^{-7} mbar.l.s^{-1}. These values are typical for the start-up of an accelerating structure [2]. A schematic drawing of the resonant cavity is presented in Fig. 2.

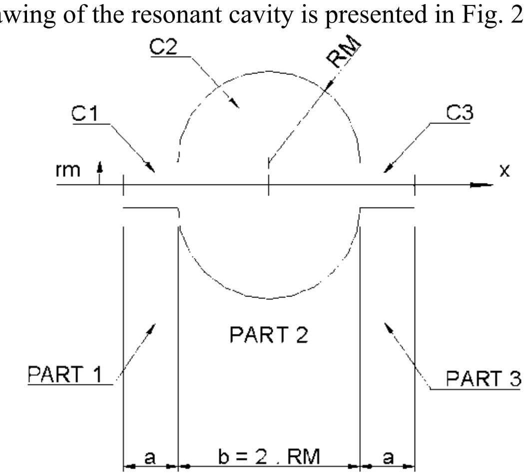

Figure 2. Schematic drawing of the resonant cavity.

The resonant cavity presents an approximately spherical geometry. Even though it does not present the complexity of those actually employed in particle accelerators, the adopted model can be generally applied in actual cases. The cavity is divided in 3 parts: a tube with internal radius *rm* = 0.4 cm and length *a* = 1 cm forms the first one. The second part is formed by a deformed sphere, with internal radius *RM* = 3 cm shifted by *rm* from the symmetry axis of the system. The third part is equal to the first one. The total length of each cavity is *l* = 8 cm. This basic configuration is repeated 5 times to form the structure. Due to the symmetry of the system and the fact that we adopt one pump at each extreme, we can study the problem from one extreme to the middle of the third structure, because the other half will reproduce the same pattern.

To treat the problem within the pressure field approach, we need to define the specific conductance and the specific degassing rate of the system. Those parameters are usually defined, in the literature, for tubular structures, with constant diameter. We must then define those parameters for a geometry with variable diameter, like the one found in part 2 of our cavity (see Fig. 2).

The specific conductance at each point along the *x*-axis of the cavity may be defined as:

$$c(x) \equiv 96.0 f^3(x) \qquad (1)$$

where the function $f(x)$ defines a surface with cylindrical symmetry by revolution around the *x*-axis. The multiplicative constant depends on the gas and the temperature, and this value is valid for N_2 at 293 K. The unit of $c(x)$ is l.cm.s^{-1}. We will call it the local specific conductance. Analogously we can define the local specific degassing rate, which is also a function of *x*, and can be defined as:

$$q(x) \equiv q_0 2\pi f(x)\left[1+\left(\frac{df(x)}{dx}\right)^2\right]^{\frac{1}{2}} \qquad (2)$$

where q_0 is the degassing rate per unit area. The unit of $q(x)$ is $\text{mbar.l.s}^{-1}.\text{cm}^{-1}$ [3].

The modeling will be done assuming the regime of molecular flow, and the diffusion equation will be:

$$c(x)\frac{\partial^2 p(x,t)}{\partial x^2}+\frac{dc(x)}{dx}\cdot\frac{\partial p(x,t)}{\partial x}=-q(x,t)+\nu(x)\frac{\partial p(x,t)}{\partial t} \quad (3)$$

Since we will deal only with the steady state, the diffusion equation reduces to:

$$c(x)\frac{d^2 p(x)}{dx^2}+\frac{dc(x)}{dx}\cdot\frac{dp(x)}{dx}=-q(x)\cdot \qquad (4)$$

The physically acceptable solutions are found imposing the boundary conditions. We will treat initially the first cavity on the left, in Fig. 1. Once the local specific degassing rate is determined, we can find the total degassing rate for the cavity, q_{RC}. The pressure at the entrance to the vacuum pump, assumed as x = 0 cm, can then be determined as:

$$p(0)=\frac{5\cdot q_{RC}}{2\cdot S_{eff}}$$

Another boundary condition is obtained from the imposition of the conservation of throughput at each point along the axis of the accelerating structure. At x = 0 cm we have that:

$$q(0)=-c(0)\left.\frac{dp(x)}{dx}\right|_{x=0}$$

but, since the total quantity of gas in the system is given by $5q_{RC}$, then:

$$q(0)=-\frac{5q_{RC}}{2}\cdot$$

The minus sign shows that the throughput at x = 0 cm is directed towards the left in Fig. 1.

So we can solve eqn. (4) for the first part of the resonant cavity. In this part, the local specific conductance is given by $c_1(x)$ = 6.14 cm.l.s^{-1}. The local degassing rate is also constant and given by $q_1(x)$ = $2.51\text{x}10^{-7}$ $\text{mbar.l.s}^{-1}.\text{cm}^{-1}$. The solution gives the pressure at x = 1 cm, and the throughput at this point is given by the throughput at x = 0 cm minus the throughput of part 1 of the first cavity.

Part 2 has a local specific conductance given by:

$$c_2(x)=96.0\cdot\left[\sqrt{9-(x-4)^2}+0.4\right]^3$$

and a local specific degassing rate given by:

$$q_2(x)=\left\{2\pi\cdot 10^{-7}\left[\sqrt{9-(x-4)^2}+0.4\right]\sqrt{1+\left[9-(x-4)^2\right]^{-1}(x-4)^2}\right\}$$

The same reasoning can be applied to the other significant points along the structure, which are: x = 7, 8, 9, 15, 16, and 17 cm.

3 RESULTS AND DISCUSSION

The solution was obtained using both analytical and numerical procedures. Figure 3 shows the pressure field profile along the *x*-axis, from 0 cm ≤ x ≤ 20 cm. The solution for the right-hand side presents the same mathematical structure, only reflected at x = 20 cm due to the symmetry of the problem.

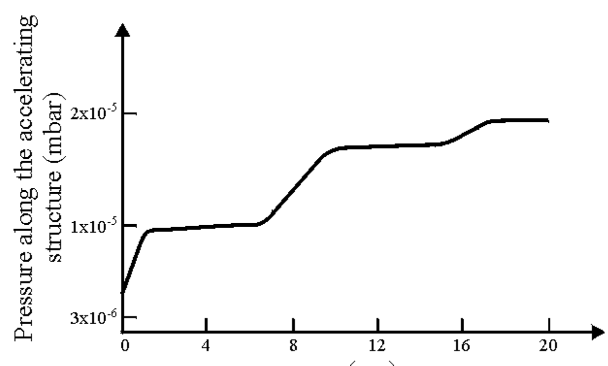

Figure 3. Pressure field along the axis of the accelerating structure.

One can see that close to the vacuum pump the pressure changes very fast, but after x = 7 cm, the pressure gradient is small. This is a consequence of the fact that what determines the effective pumping speed is the conductance along the system. Increasing the pump capacity would not change the situation significantly [3].

It is also noticeable in Fig. 3 that the pressure gradients are higher at the tubular parts of the structure than at the "spherical" parts. This is due to the local specific conductance being much higher at the "spherical" than at the tubular parts. As we get closer to x = 20 cm, the pressure gradient decreases, going to zero at x = 20 cm, showing the decrease in the throughput in this region.

It is interesting to compare this result with a situation where we have pumping at each cavity, with an effective pumping speed of 2 $l.s^{-1}$. We assume the same values for the local specific conductance and degassing rate. The pumping speed was chosen in order to obtain the same pressure at the end of the structure as in the previous analysis. With this choice, the pressure in the middle of the structure is approximately 4 times lower than in the previous case.

We have treated the steady state case, but we can use eqn. (3) to study a transient situation, like, for instance, when the beam hits the walls and produces a gas burst. We can consider the gas sources as independent of each other, and so the problem becomes linear and we can obtain the transient and stationary solutions independently. The general solution is given by their superposition.

4 REFERENCES

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