OPTIMISATION OF MODERN LIGHT SOURCE LATTICES

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Abstract

The 3rd generation of light sources have exceptionally low emittances (1-5 nm-rads) at energies of a few GeV. Generic design procedures for their storage ring lattices are now available and will be reviewed in the talk. These procedures involve both linear matching of arc sections (eg achromats) to straights of variable length, based on both accelerator and user needs, and nonlinear dynamics compensations providing adequate dynamic aperture in the presence of strong sextupoles and other higher order fields. Such procedures will be described and their application to real design problems illustrated.

1 INTRODUCTION

The first part of any optimisation is to choose an appropriate overall design for the lattice. The starting point should be the high level specification of user requirements and general definition of the project. Consideration is then given to which general lattice type, when optimised, best fits this specification. The choice of energy, lattice structure, symmetry etc. should be dictated by this specification and there is no optimisation tool that will dictate a particular direction. It is interesting to note that an intermediate energy double bend achromatic (DBA) lattice of 8 to 24 cells appears a very popular choice for proposed "national" sources covering hard x-rays from small gap insertion devices e.g. SOLEIL, DIAMOND, SSRF etc.[1]. This paper looks at the generally used procedures applied to optimise a chosen design and illustrates how these can be verified.

2 EVOLUTION OF OPTIMISATION STRATEGY

2nd generation light sources were generally based on relaxed, simple, highly symmetric lattices. The linear optimisation could be carried out by a simple scan of two quadrupole families and an inspection of the properties of the stable neck-tie area. Some attention was given to correction of modest chromaticities and avoiding low order (1-5) resonances. In some cases dynamic apertures were calculated after the design to confirm that apertures exceeded the physical apertures.

Early 3rd generation sources utilised low emittance structures, triple bead actromat (TBA) and DBA. Although there was strong chromaticity to correct, which introduced significant nonlinearities, the structures remained relatively simple and highly symmetric. Nonlinear optimisation could be performed by choosing a quiet region in tune space and if necessary tuning a relatively small number of harmonic sextupole families placed in the zero dispersion straights.

Modern 3rd generation "high performance" sources, pushing for reasonable lifetime from low emittance intermediate energy machines, require large dynamic acceptance. This has to be achieved often from complex, lower symmetry lattices. In these lattices the nonlinear optimisation has a strong impact on the linear design and there are no longer distinct sequential steps between linear and nonlinear lattice optimisation but iteration between the two.

3 LINEAR OPTIMISATION

The linear optimisation is a relatively straightforward matching procedure to meet the demands of the nonlinear optimisation while satisfying various constraints required to deliver a high performance source. These constraints include efficient injection (in top-up mode?), low emittance, low sensitivity to vibrations and errors, accommodation of low gap insertion devices etc. A typical set of linear matching criteria could be :-

- Reasonable maximum βr and $\beta v < 35$ m
- Reasonable beta split at the centre of the achromat
- Natural chromaticities $\zeta r < -120$, $\zeta v < -50$
- Dispersion at the centre of the achromat > 0.25 m
- Sextupoles in the high dispersion region K2<45 T/ m
- 1.5 nm-rad < Natural radial emittance < 3 nm-rad
- 0.04 m < Dispersion, centre standard straight > 0.1 m
- βr, βv at centre of standard straight < 5m, < 2.5m
- βr, βv at the injection straight >10 m, < 8 m
- αr, $\alpha v = 0$ at the centre of "achromat" & ID straight
- Integer part of radial & vertical tune < 0.5
- Working point clear of structure
- Phase optimisation to minimise nonlinear effects

Figure 1 shows the results of a linear matching exercise to these constraints, which also satisfied the nonlinear optimisation requirements for reasonable dynamic acceptance for the DIAMOND light source [2].

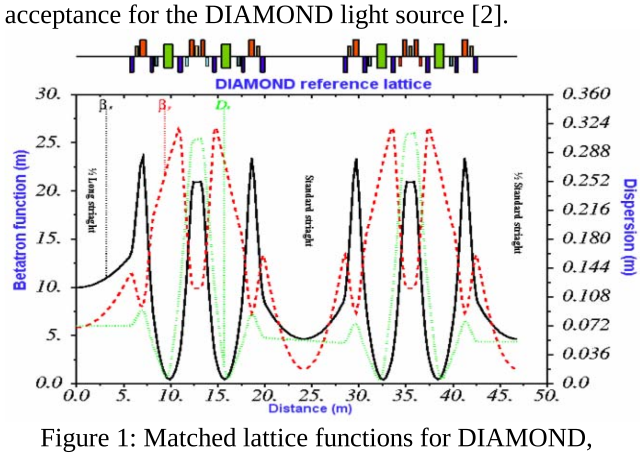

Figure 1: Matched lattice functions for DIAMOND, satisfying an extensive set of linear matching criteria.

The last three points in the criteria bullet list link the nonlinear and linear optimisations strongly together. With the possibility of iterations between the two procedures to ensure the optimal overall tune point, optimal phase between many sextupole families and to specifically cancel certain nonlinear effects using the linear phase advance over sections of the lattice.

4 NONLINEAR OPTIMISATION

4.1 Light source issues

The major concerns to be addressed by the optimisation of the nonlinear dynamics are ensuring reasonable lifetime and efficient injection, possibly in top-up mode. The nonlinearities occur principally from the chromaticity correcting sextupoles and to a lesser degree from magnetic errors and insertion devices.

Touschek lifetime is a major concern. The lifetime is proportional to the square of the momentum acceptance, which itself is the minimum of that derived from RF, physical aperture and dynamic aperture. Several sources (ALS, ESRF and APS) measure dynamic momentum acceptances ~2%, less than the design study prediction. Medium energy, small emittance light sources (SLS, SOLEIL, DIAMOND etc.) require 4-6 % momentum acceptance and would suffer a significant drop in lifetime if these values failed to be achieved in practice.

4.2 Overview

The challenge is that small target emittance entails a large natural chromaticity, the correction of which by the chromatic sextupoles induces strong nonlinearities, (“kicks”) and a consequent limitation of the dynamic aperture.

The solution is to minimise these nonlinearities or “to cancel the kicks” by choosing carefully the phase between sextupoles or adding additional sextupoles and so maximise the dynamic aperture.

The analysis of the problem starts by defining the nonlinear Hamiltonian for single particle motion, $H(x,p_x,y,p_y,\delta;s)$ as there is no analytic solution the problem is tackle by two complimentary approaches[3].

- Using numerical tools to provide an approximate solution to the equations of motion (tracking codes). This method has the advantage of coping with all regimes including strong nonlinearities but is less likely to give significant insight into the understanding of the underlying behaviour. Tracking should be based on symplectic methods so as not to introduce spurious damping or instability. This can be achieved through “kick” approximations or by sympletic maps (derivable by Lie formulisation).
- A perturbation analysis which then derives analytical dynamic quantities such a distortions, resonances etc. Fails when the nonlinearities are strong (large amplitude) but provides a very useful insight into the nonlinearities and their effects within the bounds of its applicability. These perturbation methods can be based on a canonical approach or as is now popular the application of one turn maps, normal forms and differential algebra techniques.

4.3 Single resonance approach

The single resonance approach [4], used extensively in nonlinear optimisation of many light sources (for instance, the ESRF [5] and SLS [4]) illustrates the useful parameterisation possible through the application of perturbation theory. In this approach the Hamiltonian can be written as a series of driving terms of different orders in field gradients (sextupole and quadrupole etc). Following the derivations in [4].
There are 9 first order terms

$$h_{jklmp} \propto \sum_n^{N_{sext}} (b_3L)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} \eta_n^p e^{i[(j-k)\varphi_{xn}+(l-m)\varphi_{yn}]} - \left[\sum_n^{N_{quad}} (b_2L)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} \eta_n^p e^{i[(j-k)\varphi_{xn}+(l-m)\varphi_{yn}]} \right]$$

The b_2L and b_3L are the integrated strengths of the individual quadrupoles and sextupoles, β, η, φ the betafunctions, dispersion and betatron phases. The second set of terms only contributes to the chromatic terms ($p \neq 0$).

- There are 4 Chromatic terms

h_{11001} and h_{00111} drives chromaticities

h_{20001} and h_{00201} drives off-momentum, 2Qx, 2Qy resonances and cause beta-function beats and 2nd order chromaticity

- There are 5 Geometric terms

h_{21000} and h_{10110} drives integer resonances of type Q_x .

h_{30000} drives 3rd-integer resonances of type $3Q_x$.

h_{10200} and h_{10020} drives coupling resonances $Q_x \pm 2Q_y$.

In general each sextupole and quadrupole will contribute to a driving term as a complex number in the overall summation. It is helpful to view these as vectors on an Argand diagram. A number of optimisation lattice codes such as OPA[6] and BETA[7] display these such diagrams to assist in visualisation of the optimisation process.

There are also 13 second order terns from which are derived the linear tune shift with amplitude, octupole-like resonances and the 2nd order chromaticies.

4.4 Codes

The most commonly used lattice design codes provide access to the outputs of both approaches. Often the faster analytic output is used for a general insight to the problem and to perform a quick iterative optimisation. The tracking results are often used to test these outputs in a more rigorous manner, beyond the limits of validity of the analytic approach and also to input to a slower iterative optimisation. Commonly used codes include MAD[8], BETA[7], OPA[6] and RACETRACK[9]. Many of these codes, which originate from the early period of light source design, have been enhanced in an evolutionary way to reflect the requirement for additional features. For example, the more rigorous inclusion of nonlinear lattice function effects in lifetime calculations (BETA), the more sophisticated inclusion of ID effects (RACETRACK and BETA) or the inclusion of the output from modern 1 turn map analysis (MAD). There are also Lie algebra based codes designed specifically to produce the coefficients of the one turn map, one of the earliest and most widely used being MARYLIE[10], which provides output of the nonlinear terms in the generator.

4.5 Nonlinear lattice optimisation

The first step is to design the linear lattice to assist in cancelling as much of the underlying nonlinear behaviour as possible. Without a strong consideration of the suppression of nonlinear effects at the start of the optimisation process, subsequent attempts at suppression by additional sextupoles etc. will tend to fail. It should also be recognised that this initial linear solution could need revisiting as the detailed optimisation progresses to ensure the best overall optimisation. The second stage is the detailed optimisation of sextupole families, strengths, position and smaller tune or phase changes.

4.6 Phase optimisation over a section of lattice

There are many examples of the very effective use of this first stage to mitigate strong nonlinearities by cancellation of geometric terms by setting the phase advance over sections of the lattice.

For instance, good dynamic behaviour as shown in Figure 2 was achieved in the low symmetry SPEAR 3 lattice by setting the phase advances in the DBA arc cells close to ¾ x 2π horizontally and ¼ x 2 π vertically[11]. A similar approach was taken in the initial design of the DIAMOND, 6 fold symmetric design [12], where partial cancellation was achieved within a four cell super-period by setting **–I** transformer conditions over 2 cells horizontally and 1 cell vertically.

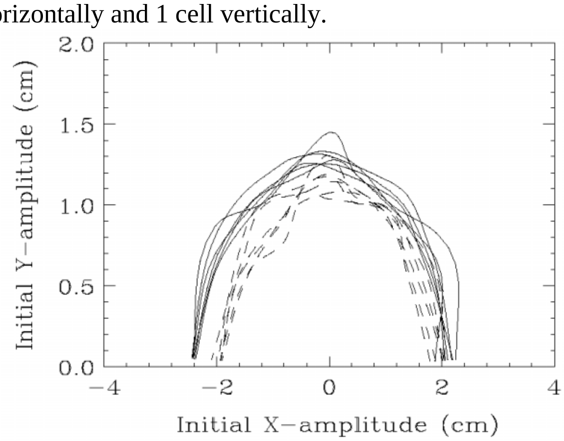

Figure 2: SPEAR 3 Dynamic aperture including error effects, no harmonic sextupoles required.

The ASTRID II design, which has two arcs of minimum emittance cells and two long straights, achieves nonlinear control by ensuring **–I** cancellation between every N and N+4 cell horizontally and every N and N+2 vertically. Setting the overall tune close to an interger + 1/3 by control of the phase across the long straights, creates a strong nonlinear tuneshift which can actually stabilise the motion. There are no uncompensated sextupoles in the long striaghts, as all chromatic correction is done in the arcs. This design has produced good dynamic behaviour, in a low period machine [13].

As a final example of the application of this method, the relatively complex SLS TBA lattice is tune in such a manner as to allow the cancellation of first-order driving terms, even in the case where the ~180 degree phase advance/cell required for low emittance would otherwise couple the h_{2001} mode to the correction of chromaticity. In the zero dispersion mode the phase advances are set to $\Delta Qx \sim 7/4$ and $\Delta Qy \sim 3/4$ in each TBA. The h_{2001} and h_{00201} modes cancel between 2 TBA, and the 5 geometric modes between 2 TBA-pairs [14]. This is similar but somewhat simpler to tune than the decoupling achieved through the introduction of a "phase trombone" between the TBA arcs as used in earlier SLS designs [15].

Another way to use phase advance to assist in the optimisation of lattices, where the small periodicity comes from the inclusion of a few very long straight sections, is to arrange for the symmetry breaking section to become "transparent" by setting a 2π phase change and avoiding strong sextupoles in that region. An example of this approach within light source design is the new Spring 8 lattice, shown in Figure 3, which allows the exploitation of four extremely long straights [16]. The 2π matching section is introduced in place of the non-symmetry breaking "missing dipole cells". This section is "transparent" for on-momentum particles and in this case the off momentum acceptance is improved by the addition of relatively weak sextupoles in the matching sections.

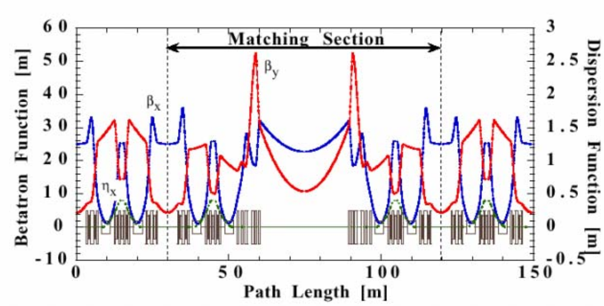

Figure 3: New spring 8 lattice with 2π matching section

4.7 Optimisation of sextupoles

The next step to producing large dynamic acceptance is the introduction of additional sextupoles, the so-called "harmonic" families, to help cancel nonlinear effects. As mentioned earlier, in highly symmetric achromatic lattices these consist of one or two families of sextupoles placed in the zero dispersion regions, which effectively reduce the strongest 1st order terms. In complex lower symmetry lattices there can be many families, and now with the drive towards minimum emittance solutions these are often in a region with significant dispersion (this blurs the distinction between chromatic and harmonic families). In this case the solution of nine 1st order equations reduces to a 9 x M_{sext} linear system, where M_{sext} is the number of sextupole families. As mentioned earlier, these equations can have significant problems with degeneracy. Also it is found that 2nd order terms, amplitude dependent tune shifts, higher order chromaticity etc., which arise from the cross talk of these sextupoles also need to be considered. The handling of this more complex analytic system, as part of the optimisation to produce good dynamic behaviour, requires a delicate balancing of various weights to cancel and minimise the terms, which show most relevance to the nonlinear motion under consideration. The SLS handbook [14] states: "eventually some skill in setting weight factors for the many terms, developed by systematic phenomenological studies".

4.8 Optimisation procedure

Optimisation can be looked upon as finding the setting of a number of "variables" to improve a number of "quality factors" which are linked ultimately to improved light source performance. The list below outlines typical variables:-

- Phase over section of lattice (see Section 4.6)
- Machine tune
- Number of sextupole families
- Sextupole positions, sensitive to <10 cm
- Sextupole strengths
- Optimisation method
- Weights given to quality factors
- Target values of quality factors

The quality factors fall into two categories:-

1. Analytic Factors : Which are relatively quick and easy to calculate, are easy to use to scan and optimise variable with but often give necessary but not sufficient conditions for good dynamic performance. E.g.

- 1st and 2nd order perturbation terms
- MAP coefficients, tune shifts with amplitude, tune shifts with momentum, higher order chromaticity, resonance, driving terms
- Off momentum lattice functions
- Maximum sextupole strengths

2. Numerical Factors: Are slow to calculate, difficult to use directly in optimisation but are often more closely related to machine performance.

Numerical factors, found from tracking:-

- Phase space plots
- Tune shifts with amplitude and momentum
- Frequency maps analysis

Dynamic aperture itself (short term, longer term, 1 to 6 dimensions):-

- On and off momentum
- With errors (field and closed orbit)
- With physical apertures
- With typical realistic chromaticity values
- With IDs including vertical coupling

4.7 Examples of the use of quality factors

The optimisation of SOLEIL [17] was carried out by controlling values for the desired analytically calculated tune shift terms to tailor the large amplitude tune shifts determined from tracking, and so avoid the crossing of destructive high order resonances at large amplitude.

In recent years the use of frequency maps analysis has proved a valuable tool to characterise and optimise light source lattices. These are obtained by plotting the numerically determined "tunes" of tracked particles launched over a fine X-Y grid and highlighting, through shading or colour, the level of nonlinear behaviour (derived from quantities such as the diffusion rate of these tunes). It provides a powerful picture of the important resonant features of large amplitude motion. It has now been used extensively for design, operational optimisation and characterisation of lattices (see for example [18] or [19]). Figure 4 shows an example plot for the ideal ALS lattice.

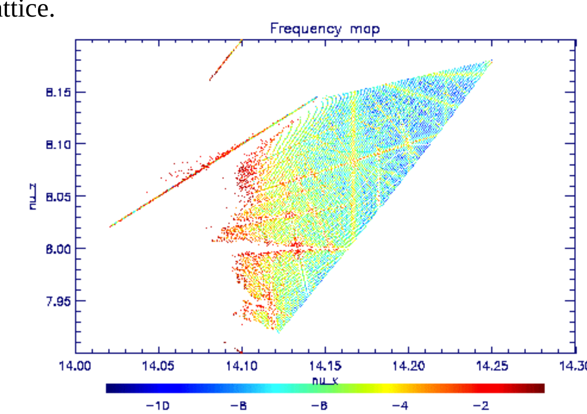

Figure 4: Frequency map of the ALS for an ideal lattice

The dynamic aperture has been used extensively as a quality factor and the off momentum aperture is an important indicator of the eventual momentum acceptance and lifetime of high performance sources. It is important to keep two things in mind when using such a quality factor to optimise derived quantities such as lifetime.

Firstly the modelling should include effects that may significantly modify the measured quantity, such as dynamic aperture, e.g. the effects of realistic element position errors, magnet field errors, insertion devices and even the physical apertures.

Secondly, it should be recognised that other factors not yet included in the model may inhibit the apparent benefits of enlarging a single quality factor. For instance there will be little benefit derived from an increase in transverse nonlinear dynamic momentum acceptance if unaccounted for nonlinear distortions reduce the real momentum acceptance due to the presence of physical apertures or the reduction of longitudinal acceptance [20].

5 VERIFICATION OF MODELING

A very important role is played by the experimental verification, using existing light sources, of the models. Beam tracking using fast turn by turn data from electron beam position monitors has provided real measurements and estimates of the quantities estimated previously from numerical tracking, such as phase space plots [21], dynamic aperture [22], [23] and even frequency map analysis [18]. Careful measurements of lifetime and injection rates can estimate the actual dynamic aperture or momentum acceptance. These "real" measurements are useful at highlighting important factors for inclusion in the modelling, often found as efforts are made to align measured and modelled results. For instance the inclusion of beam based machine errors in the determination of frequency map characteristics in ALS [18] or the effect of the assumption of thin sextupoles on the tune shift with momentum [24]. Importantly the impressive performance of existing machines such as ESRF, SLS, BESSYII etc. give some confidence in the usefulness of the optimisation methods used in their design and subsequently in the designs of proposed light sources such as DIAMOND, SOLEIL, SSRF etc.

6 CONCLUSIONS

High performance flexible light source lattices demand measures to counteract destructive nonlinear effects either through the careful choice of phase over parts of the lattice and/or the inclusion of many sextupole families to cancel nonlinear effects.

Perturbation theory gives powerful quality factors that can be used in nonlinear optimisations and it application has been enhanced and extended by the utilisation of modern techniques (Lie transforms, normal forms and differential algebra).

Dynamic aperture must be tested, in general, directly by tracking both on and off momentum. Also its accurate determination requires realistic models with errors and other features included.

Characterisation of tracking, through frequency map analysis for instance, has provided additional information for the optimisation of both operational machines and new designs.

Finally the ongoing verification and improvement of models, based on experimental measurements in existing sources, has proved extremely valuable to machine designers and developers alike.

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