NON-LINEAR BEAM DYNAMICS STUDY IN CANDLE LIGHT SOURCE

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Abstract

We present a summary of non-linear beam dynamics study in CANDLE, a project of a third generation intermediate energy light source. The study includes the effects of non-linear fields both in the storage ring magnet system and insertion devices. The results of analytical approach and tracking simulation study are discussed.

1 RING LATTICE PARAMETERS

The present CANDLE storage ring lattice design consists of 16 identical straight sections, 13 of which are available for Insertion Devices (ID). Each straight section has a length of 4.8m with 4m available for installation of ID's. Because the CANDLE beam emittance is larger than diffraction limited emittance for expected photon wavelength range, we find the lattice optical functions at the middle of straight section (horizontal beta 7.89 m and vertical beta 4.87m) are an optimal one for both machine reliable and radiated photon beam quality point of view [1]. Table 1 lists the main characteristics of electron beam and the storage ring optics.

Table 1: Ring Optical Parameters

Parameter	Value
Hor./Vert. Emittance [nmrad]	8.4/0.084
Relative Energy spread [%]	0.1
Straight Section Length [m]	4.8
Horizontal beta at straight[m]	7.89
Vertical beta at straight [m]	4.87
Dispersion at straight [m]	0.18
Hor./Vert.tune	13.22/4.26
Momentum Compaction	0.002

2 ID CHARACTERISTICS

Insertion devices are indispensable part of third generation light sources. With the installation of insertion devices (ID's) into the ring one expects to observe three main classes of perturbations to the beam dynamics:

- closed orbit distortion;
- tune shifts and betatron beat;
- reduction of dynamic aperture
- reduction of beam lifetime.

The closed orbit distortion is induced by the field and positioning errors of the magnets. A modern technology of proper shimming and high precision alignment ensure a negligible closed orbit distortion due to installation of insertion devices. We will restrict ourselves by the study of pure magnetic field effects.

In Table 2 the basic parameters of primary insertion devices foreseen for installation into the ring straight sections at first phase of machine commissioning are summarized. There is one conventional planar wiggler and two undulators incorporated in the lattice design to cover radiated photon spectrum range of 5 to 30 keV. Such a devices with very similar parameters are widely used in many synchrotron radiation facilities with good performance and reliable operation [2],[3].

Table 1: Insertion Devices Parameters

Insertion Device	Wiggler	Und.I	Und.II
Magnetic field [T]	1.98	0.7	0.3
Period Length [m]	0.17	0.022	0.05
Number of periods	23	72	79
Gap height [mm]	18	5.6	24

2.1 Magnetic Fields

The magnetic field of quadrupoles is included in equation of motion up to the third order displacement from the ideal closed orbit. The magnetic field of combined function bending magnet is taken into account up to second order. The magnet element arrangement and its longitudinal field distribution are described by the expression

$$f(s) = \frac{1}{2}\left[\frac{(s-d)}{(a^2+(s-d)^2)^{1/2}} + \frac{(L+d-s)}{(a^2+(L+d-s)^2)^{1/2}}\right] \qquad (1)$$

where L is the magnet length; a is the magnet half aperture, d is the distance of magnet entrance location from coordinate system origin, s is the longitudinal coordinate along the ideal closed orbit.

The box-shaped magnetic field approximation is obtained when one takes the ratio $a/L \rightarrow 0$. In this case one has δ -function like edge fields at the both ends of the magnets.

The components of the insertion device magnetic field used for the derivation of equation of motion are as follows:

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$$B_x = (k_x / k_y) B_0 \sinh(k_x x) \sinh(k_y y) \cos(ks)$$
$$B_y = B_0 \cosh(k_x x) \cosh(k_y y) \cos(ks); \quad (2)$$
$$B_s = -(k / k_y) B_0 \cosh(k_x x) \sinh(k_y y) \sin(ks);$$

with

$$k_x^2 + k_y^2 = k^2 = (\frac{2\pi}{\lambda})^2; \quad (3)$$

where λ is the period length of the ID and B_0 is the peak magnetic field. For the chosen planar insertion devices k_x=0 and the effect of focusing of ID's is presented in the vertical plane with the equivalent quadrupole strength

$$K_{Q_y} = \frac{1}{2}\left(\frac{k_y}{\rho k}\right)^2 = \frac{1}{2}\frac{1}{\rho^2} \quad (4)$$

where ρ is the particle bending radius.

2.2 Analytical Estimations

This section presents the results of the IDs effects on the electron beam parameters and storage ring optics based on analytical approach developed in [4]. The equations of particle motion in real insertion devices besides above mentioned linear terms, contains also nonlinear part whose effect comes from sextupole and octupole-like terms. In horizontal plane only the oscillating over the one period of the ID term is non-vanishing (averaging to zero over the period). In the vertical plane the first non-linear wiggler effect comes from the octupole like field, which generates quadratically increasing vertical amplitude dependent tune shift:

$$\Delta Q_y^{oct} = \varepsilon_y \frac{\pi N \beta_y^2}{4\lambda\rho^2}\left[1 + \frac{2}{3}\left(\frac{N\lambda}{2\beta_y}\right)^2 + \frac{1}{5}\left(\frac{N\lambda}{2\beta_y}\right)^4\right] \quad (5)$$

The non-linear effects are proportional to the ratio $(B/E\lambda)^2$ and therefore are more severe in case of small period undulators.

Analytic calculations of the amplitude dependent tune shifts for all three ID's have been performed. The results are presented in Table 3 where the ID's fields calculated at vertical distance y= 5mm from the magnet main axis.

Table 3. Nonlinear tune shifts from ID fields

Insertion device	Wig.	Und. I	Und.II
ΔQ_y^{oct} [10^{-5}]	3.73	11	1.0

As it is seen from Table 3 the nonlinear effects arising from octupole-like field of ID's are small due to very small vertical beam emittance in storage ring.

3 TRACKING RESULTS

The tracking simulations have been performed by the direct integration method of canonical equations of particle transverse motion. The symplectic property of integration method as well as the stability criteria were checked during the numerical mapping by the solving of the linear system of equations [6]

$$B' = A * B \quad (6)$$

where B is 4x4 matrix, A is 4x4 Jacobian matrix associated with the canonical equations of motion. The tracking results are presented in the following subsections of this section.

3.1 Effects on Dynamic Aperture

The additional linear and non-linear effects applied by IDs may bring to dynamic aperture (DA) decreasing and affect beam lifetime limiting. In Fig.1 are shown the dynamic aperture simulations using the magnetic field expression (3) in natural equation of motion.

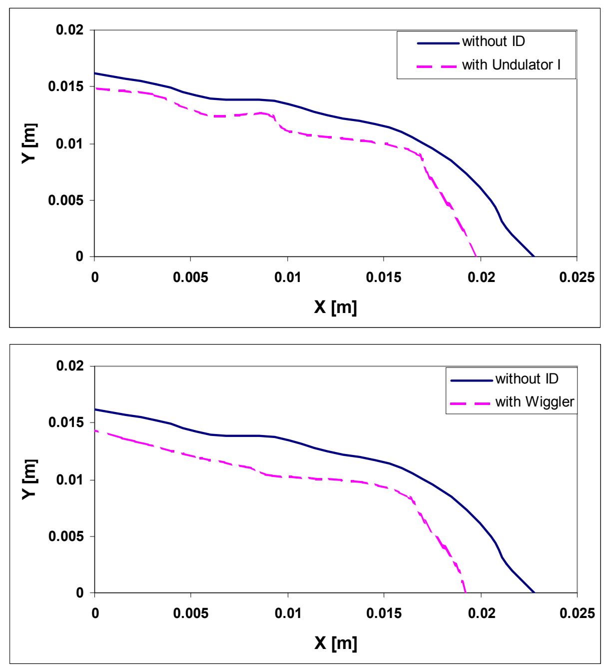

Fig.1 Dynamic aperture reduction induced by Undulator I (upper) and by Wiggler (lower).

The simulations were done for conservative scenario when ID's occupies all the straight sections. Reduction of dynamic aperture to 14.5 mm in vertical plane and 18 mm in horizontal plane is still tolerable and aperture remains well outside the ID's physical aperture.

In case of Wiggler magnets, the Touschek lifetime of the electron beam is reduced from 36 hour for bare lattice to 27 hours.

Simulations performed for the storage ring with Undulator I and Undulator II in operation show comparatively small beam lifetime reductions.

3.2 Phase space dynamics

The non-linearities induced by the ID's leads to enhancement of the amplitude-dependent tune shifts and distortion of phase space. The tracking simulation study of 1000 particles beam with gaussian distribution is given in Fig.2. The model is equivalent for tracking one particle over 1000 turns in the ring. The distribution has the following standard deviations:

$$\sigma_x = 1.8mm, \quad \sigma_{x'} = 0.8mrad; \qquad (7)$$
$$\sigma_y = 0.7mm, \quad \sigma_{y'} = 0.4mrad.$$

These initial values are more than two times larger than those of electron beam delivered from booster.

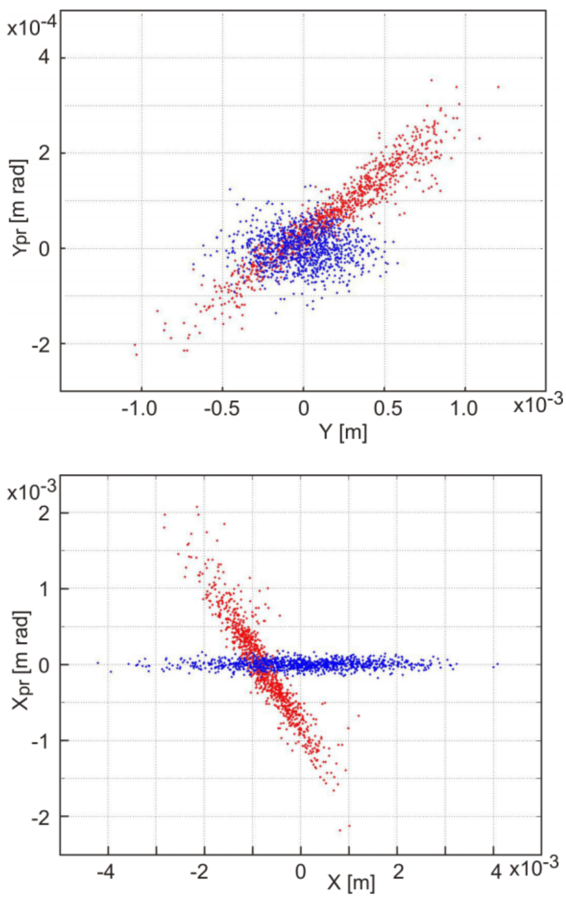

Fig. 2 Vertical (upper) and horizontal (lower) phase space evolution after one revolution in the ring. Blue colour indicates the initial beam.

The tracking has been performed at the middle of straight section when it occupied by Undulator II. Tracking results show significant phase space distortion and appearance of non-linear resonance islands due to ID's non-linearities at the distances large than one third of ID's aperture.

3.3 Chromaticity Calculations

Amplitude dependent non-linear tune shifts induce also the non-linear chromaticity. In Fig. 3 the chromaticity dependence of momentum spread is plotted in assumption that all straight sections are occupied by the strong wiggler magnets. The simulations are based on the method developed in [7].

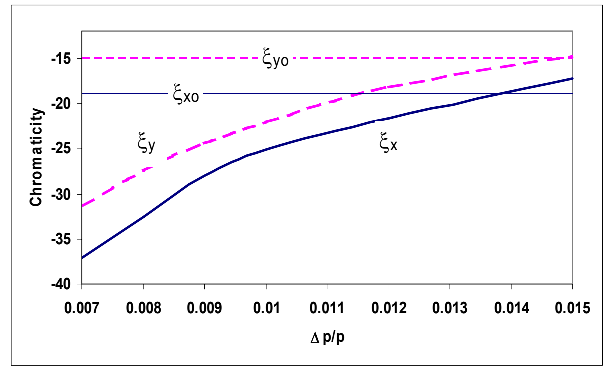

Fig.3 Non-linear chromaticity vs momentum spread

The horizontal lines in the figure correspond to the natural linear horizontal and vertical chromaticity values $\xi_{x0} = -18.91$ and $\xi_{y0} = -14.87$ respectively. As it is seen from Fig.3 at the momentum spread of about 1.4 %, the nonlinear chromaticities are approach to natural chromaticity values.

4 SUMMARY

It can be expected that non-linear fields can produce significant effects at large betatron amplitude. These betatron phase distortions breaks the periodicity of the beta functions and phase advance between sextupoles, which could lead to strong sextupole resonances and reduction of dynamic aperture. The wiggler focusing effect could be compensated using local matching well-established technique.

5 REFERENCES

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