# ELIMINATION OF DIGITIZING ERRORS IN A ROTATING COIL MAPPER

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*Abstract*

A Labview program for a Rotating Coil Mapper (RCM) has been adopted by SNS/ORNL to measure the SNS magnets. Detailed study has found that the data analysis code in the program underestimates the calculation of magnetic harmonic contents due to the finite number of points sampled per revolution. The resulting digitizing errors could be as large as 0.26% for integrated quadrupole fields, 2.4% for integrated dodecapole terms, 6.9% for integrated 20$^{th}$-pole terms, etc. This is not acceptable for many applications. With a new data analysis method, this error can be eliminated and the measurement accuracy can be improved. This paper describes an analytical solution of this problem, with an experimental demonstration of its effect.

## 1 INTRODUCTION

The Spallation Neutron Source (SNS) requires hundreds of magnets for its linac, accumulation ring, and transfer lines. We have newly established a magnet measurement lab at ORNL to handle two thirds of these magnets. A Labview [1] program for a Rotating Coil Mapper (RCM) has been adopted for our measurements. The mapper mainly consists of a radial coil, a digital integrator, and a computer, controlled by a Labview program for data acquisition and analyses. The radial coil rotates in magnetic fields and senses their azimuthal components $B_\theta$. The electrical signal induced on the coil is fed into a digital integrator PDI-5025 [2], which yields the information about the magnetic flux through the coil at different azimuthal angles θ. In the program the number of data points (P) per revolution for digital integration is set to P=50, 100, or 200. The data is further analyzed by the Labview program to produce harmonic contents of the measured magnetic fields.

In a rotating coil mapper the measurement accuracy depends on many parameters. One of them is a digital error due to the finite number of points sampled per coil revolution. Detailed study of the Labview program has found that the data analysis code in the program underestimates the calculation of magnetic harmonic contents. The errors thus produced become excessive and not acceptable for many applications when the sampling points are small. For instance, when P=50 the resulting digitizing errors is about 0.26% for integrated quadrupole fields, 2.4% for integrated dodecapole terms, 6.9% for integrated 20$^{th}$-pole terms, etc. Though these errors can be reduced to a tolerable level by using a large number of P, we have found that there is a better way to eliminate the digitizing errors completely.

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In this paper, we will first briefly review the principle of a RCM operation and mathematic formulas in the code to calculate the harmonics. Then, the errors are analyzed and improved formulas to eliminate the digitizing errors are introduced. Finally, we show some experimental results to confirm our analysis.

## 2 PRINCIPLE OF OPERATION

In a cylindrical coordinate system with the z-axis along the length of a magnet and the origin located at the center of the magnet aperture, the azimuthal component of a two dimensional field in a current free region can be written as:

$$B_\theta(r,\theta) = \sum_{n=1}^{\infty} C_n \left(\frac{r}{R_0}\right)^{n-1} Cos[n(\theta-\alpha_n)] \ , \qquad (1)$$

where $C_n$ and $\alpha_n$ are the amplitude and phase angle of the 2n-pole component of the total field and $R_0$ is an arbitrary reference radius, typically chosen to be 50-80% of the magnet aperture [3]. In a radial coil with its two-side wires parallel to the z-axis and having the radii $R_1$ and $R_2$, the magnetic flux through the coil at any angular orientation θ can be obtained by

$$\Phi(\theta) = NL\int_{R_1}^{R_2} B_\theta(r,\theta)dr = \sum_{n=1}^{\infty} C_n L \frac{1}{nR_0^{n-1}} N\left(R_2^n - R_1^n\right)Cos[n(\theta-\alpha_n)]$$
$$= \sum_{n=1}^{\infty} A_n Cos[n(\theta-\alpha_n)] \ , \qquad (2)$$

where N is the number of turns of coil winding, L is the length of the coil along the magnet axis, and $A_n$ is the total amplitude of each harmonic, given by

$$A_n = C_n L \frac{1}{nR_0^{n-1}} N\left(R_2^n - R_1^n\right) = C_n L \frac{UB_n}{nR_0^{n-1}} \ , \qquad (3)$$

here we introduce a so-called coil unbucked geometry factor $UB_n$:

$$UB_n = N\left(R_2^n - R_1^n\right) \ . \qquad (4)$$

When the radial coil rotates in the magnetic field a voltage signal is generated, which is

$$V(t) = -\frac{\partial\Phi(\theta)}{\partial t} = \sum_{n=1}^{\infty} A_n n\omega Sin[n(\omega t-\alpha_n)] \ , \qquad (5)$$

with θ=ωt. This signal is fed into a digital integrator, which performs a seamless, definite integration of P intervals per revolution, where P is set to 50, 100, or 200 in the program. The integrated signal for each interval can be expressed by

$$\Phi_i(\theta) = -\int_{\theta_i}^{\theta_{i+1}} V(t)dt = \sum_{n=1}^{\infty} A_n Cos[n(\theta-\alpha_n)]\Big|_{\theta_i}^{\theta_{i+1}} \ , \qquad (6)$$

where i=0 to P-1. The assembly of these P individual $\Phi_i$'s constitutes a digitized magnetic flux signal, which is a superposition of infinite harmonics $\Phi_{ni}$'s. The largest value for each harmonic $\Phi_{ni}$'s takes place around $n(\theta-\alpha_n)=\pi/2$ and is given by

$$A_n' = A_n Cos\left(\frac{\pi}{2} - \frac{n2\pi}{P}\right) = A_n Sin\left(\frac{n2\pi}{P}\right) \quad . \qquad (7)$$

This could be treated as the amplitude of the output harmonic signal $\Phi_{ni}$'s from the integrator. When P is very large, i.e. P>>n2π, Eq. (7) can be approximated as

$$A_n' \approx A_n (n2\pi / P) \quad . \qquad (8)$$

The signal $\Phi_i$'s obtained in experiments is Fourier-analyzed in the Labview program, that yields the amplitude $F_n$ of each harmonic in the frequency domain. The relationship between $F_n$ and $A_n$ can be found as

$$F_n = A_n' \frac{P}{2} = A_n Sin\left(\frac{n2\pi}{P}\right)\frac{P}{2} \quad , \qquad (9a)$$

$$F_n \approx A_n (n\pi) = C_n L \frac{UB_n}{R_0^{n-1}} \pi \quad , \text{ if } P>>n2\pi \, . \qquad (9b)$$

The analysis program is to find $C_n$ from $F_n$, which is the frequency domain representation of the coil signal processed by the digital integrator. Thus, for n=1, the integrated dipole field B*L is

$$B * L = C_1 L \approx \frac{F_1}{\pi U B_1} \quad . \qquad (10)$$

For n=2, the integrated quadrupole gradient G*L is

$$G * L = (C_2 / R_0) L \approx \frac{F_2}{\pi U B_2} \quad . \qquad (11)$$

And, for n>2, the higher harmonic contents are usually expressed by the ratio of $C_n/C_1$ or $C_n/C_2$, depending on a dipole or quadrupole magnet in consideration:

$$C_n / C_1 \approx \frac{F_n UB_1}{F_1 UB_n} R_0^{n-1} = \frac{F_n}{\pi (UB_n)(B*L)} R_0^{n-1} \quad , \qquad (12a)$$

$$C_n / C_2 \approx \frac{F_n UB_2}{F_2 UB_n} R_0^{n-2} = \frac{F_n}{\pi (UB_n)(G*L)} R_0^{n-2} \quad . \qquad (12b)$$

Equations (10) to (12) form the basis in the original analysis program.

It is common practice to use a bucking coil, such as the one called the Halbach type coil [4], for measuring the higher harmonic contents in order to improve the measurement accuracy. The bucked signal is processed and analyzed in the same way as for the unbucked signal shown above, except that the unbucked coefficients $UB_n$ are replaced by the bucked coefficients $B_n$.

## 3 ERROR ANNALYSIS AND IMPROVEMENT

There are two approximations in obtaining Eqs. (10) to (12). First, it is easy to see that we approximate Sin(n2π/P) as n2π/P in Eqs. (8) and (9b) under the condition P>>n2π. This underestimates $C_n$ since Sin(n2π/P)<n2π/P. The error from this approximation gets large when the number of points P per revolution is small or the harmonic number n is large. Second, it may not be so obvious that both Eqs. (7) and (9a) are also approximations. In fact, Eq. (7) is exact only in the limit of an infinitely large P, which yields an infinitely small $A_n'$. For a finite P in practice, the true maximum of the piece-wise integrated harmonic signal $\Phi_{ni}$'s could be assigned by the interpolation around $n(\theta-\alpha_n)=\pi/2$, i.e.

$$A_n' Sin\left(\frac{\pi}{2} - \frac{1}{2}\frac{n2\pi}{P}\right) = A_n Sin\left(\frac{n2\pi}{P}\right) \quad , \qquad (13a)$$

which yields

$$A_n' = 2 A_n Sin(n\pi / P) \quad . \qquad (13b)$$

Thus, we have

$$F_n = A_n' P / 2 = A_n P \cdot Sin(n\pi / P) \quad . \qquad (14)$$

This is the correct relationship between the Fourier amplitude $F_n$ in experiments and the harmonic amplitude $A_n$ in Eq. (3). The errors caused by using n2π/P in Eqs. (10) to (12), instead of 2Sin(nπ/P), are, therefore,

$$error = 1 - (n\pi / P) / Sin(n\pi / P) \quad . \qquad (15)$$

Table 1 lists the errors for different harmonic number n and points P per revolution. It shows that for P=50 the original analysis program underestimates the measured quadrupole term (n=2) by 0.26%, the dodecapole term (n=6) by 2.4%, the $20^{th}$ pole term (n=10) by 6.9%, etc.

Table 1: Errors in Calculating Harmonics from Eqs. (10) to (12)

| Errors (%) | P=50 | P=100 | P=200 |
|---|---|---|---|
| n=1 | -0.066 | -0.016 | -0.0041 |
| n=2 | -0.26 | -0.066 | -0.016 |
| n=3 | -0.59 | -0.15 | -0.037 |
| n=4 | -1.1 | -0.26 | -0.066 |
| n=5 | -1.7 | -0.41 | -0.10 |
| n=6 | -2.4 | -0.59 | -0.15 |
| n=7 | -3.3 | -0.81 | -0.20 |
| n=8 | -4.3 | -1.1 | -0.26 |
| n=9 | -5.5 | -1.3 | -0.33 |
| n=10 | -6.9 | -1.7 | -0.41 |

These digital errors can be eliminated by using Eq. (14) instead of Eq. (9b). The results are

$$B * L = C_1 L = \frac{F_1}{UB_1} \frac{1}{P \cdot Sin(\pi / P)} \quad . \qquad (16)$$

$$G * L = (C_2 / R_0) L = \frac{F_2}{UB_2} \frac{2}{P \cdot Sin(2\pi / P)} \quad . \qquad (17)$$

And,

$$C_n L = \frac{F_n}{UB_n} \frac{n}{P \cdot Sin(n\pi / P)} R_0^{n-1} \quad . \qquad (18)$$

As mentioned before, in Eq. (18) $UB_n$ should be replaced by $B_n$ to process the bucked signal for higher harmonic contents.

## 4 EXPERIMENTAL VERIFICATION

Our analysis above has been compared with the measurements of two SNS quadrupoles: one is for the linac and another for the transfer line. Table 2 lists only the results for n=2, 6, and 10 from the transfer line quadrupole 12Q45. The magnet current is 272.17 A, and the reference radius $R_0$ is 4.2 cm. For two versions of the processing codes the experiment is repeated three times for each P number. The entries in Table 2 are the averages of three mappings.

Table 2. Experimental Results

| | | P=50 | P=100 | P=200 |
|---|---|---|---|---|
| G*L (T) | Improved Formulas | 1.76727 | 1.76723 | 1.76722 |
| | Approximate Formula | 1.76253 | 1.76597 | 1.76692 |
| | Error (Experiment) | -0.27% | -0.072% | -0.017% |
| | Error (Theory) | -0.26% | -0.066% | -0.016% |
| $C_6/C_2$ | Improved Formulas | 1.85E-3 | 1.85E-3 | 1.85E-3 |
| | Approximate Formula | 1.81E-3 | 1.84E-3 | 1.85E-3 |
| | Error (Experiment) | -2.4% | -0.56% | -0.13% |
| | Error (Theory) | -2.4% | -0.59% | -0.15% |
| $C_{10}/C_2$ | Improved Formulas | 2.86E-5 | 2.86E-5 | 2.87E-5 |
| | Approximate Formula | 2.68E-5 | 2.81E-5 | 2.85E-5 |
| | Error (Experiment) | -6.8% | -2.0% | -0.74% |
| | Error (Theory) | -6.9% | -1.7% | -0.41% |

It is easy to see that with the improved formulas (16) to (18) the integrated gradient G*L, the dodecapole term (n=6) and the 20$^{th}$ pole term (n=10) remain essentially unchanged for different data points P per revolution within the accuracy limit of our system. With the approximate formulas, G*L and other harmonic terms are underestimated, especially for small P. Note that for the entries $C_6/C_2$ and $C_{10}/C_2$, both $C_2$ and $C_6$ ($C_{10}$) come from the same formulas. This should be taken into account when the experimental errors are calculated. The theoretical errors from Table 1 are also listed for comparison. The agreement in general is very good. Some minor discrepancies still remain, largely due to signal fluctuations from mapping to mapping or numerical errors rather than essential physical mechanism. In fact, we would produce exactly the same errors in experiment as predicted in theory if we process off-line the same raw experimental data with two different formulas. The data in Table 2 are plotted in Figs. 1 – 3.

## 5 ACKNOWLEDGEMENTS

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## 6 REFERENCES

[1] Labview is a graphical programming language developed by National Instruments Corp.
[2] PDI-5025 is a high precision digital integrator manufactured by METROLAB Instruments SA.
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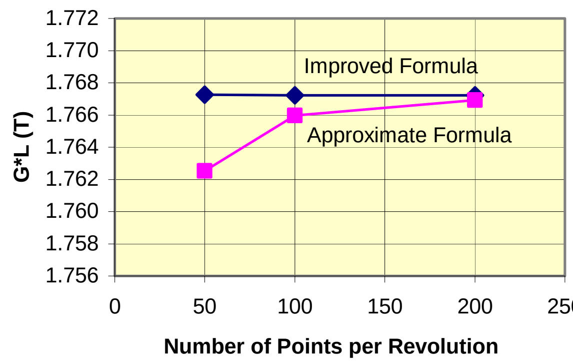


Fig. 1. Integrated gradient vs. sampling points from two formulas.

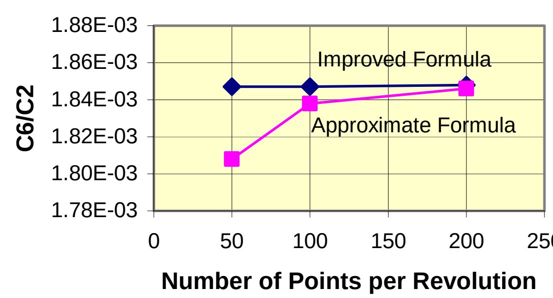


Fig. 2 Integrated dodecapole vs. sampling points from two formulas.

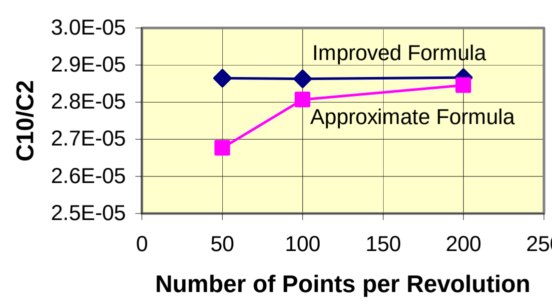


Fig. 3 Integrated 20$^{th}$ pole vs. sampling points from two formulas.