ONE-DIMENSIONAL ORDERING IN A COLD HEAVY-ION BEAM

H. Danared, A. Källberg, K.-G. Rensfelt and A. Simonsson
Manne Siegbahn Laboratory, Frescativägen 24, S-104 05 Stockholm, Sweden

Abstract

We report on observations of beam ordering and suppression of Schottky-noise power in electron-cooled beams of highly charged Xe ions at CRYRING. We also propose a model of the ordered beam, characterized by a certain minimum distance between the ions. The Schottky-noise power of a beam with string-like order is calculated according to the model, and the good agreement with the experimental data is interpreted as evidence for a spatial ordering of the ions.

1 OBSERVATIONS

At the ESR storage ring at GSI, Darmstadt, it was seen that the momentum spread of beams of electron-cooled highly charged ions dropped abruptly to very small values when the particle number decayed to 10 000 or less. A reduction in Schottky-noise power was also noticed for particle numbers between 1 000 and 10 000 [1]. The observations were interpreted as an ordering of the beams, where the ions line up after one another, without being able to pass each other due to the strong repulsion between the highly charged ions and small energy spread due to the cooling [2].

Similar observations have been made at CRYRING with ions such as Ni^{17+}, Kr^{33+}, Xe^{36+} and Pb^{55+} [3, 4]. The results in this paper were obtained with $^{129}Xe^{36+}$ ions at an energy of 7.4 MeV/u, where the beam lifetime (without electrons in the cooler that cause recombination and a reduced lifetime) is around 190 s.

Fig. 1 shows an example of how the Schottky signal from such a beam evolves as a function of time. Guided by earlier interpretations and the model presented in this paper we refer to the drop in momentum spread as beam ordering.

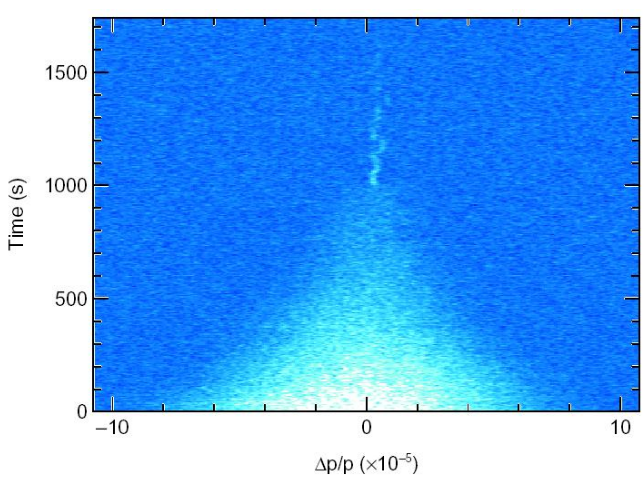

Fig. 1. Schottky signal from a beam of Xe^{36+} ions. At t=0, around 30 000 particles are injected, and it is seen how the relative momentum spread $\Delta p/p$ suddenly shrinks approximately 1 000 seconds after the injection

Sequences like the one in fig. 1 were recorded for a wide range of electron densities in the electron cooler, i.e., for different cooling rates. It was then seen how the number of stored particles at the transition to the ordered state depends on the cooling rate. Some results are shown in fig. 2. Transitions have been seen to take place when the particle number is less than 100, implying that the average distance between the particles in the 52-m-circumference ring is approaching 1 m. This distance can be compared with the diameter of the beam pipe which is 10 cm.

Another finding is that ordering does not seem to appear with more than some 5 000 stored particles. The existence of such a maximum particle number is consistent with the model for string-like order described below, although it cannot be excluded that the limit has a technical cause.

The Schottky-noise power as a function of particle number is plotted in fig. 3 for three different electron densities in the cooler. For an ordinary beam with uncorrelated particles, the Schottky power is proportional to the number of particles in the beam. This proportionality is seen to hold in fig. 3, except for the ordered beam above some 1 000 particles where the power is reduced by a factor of up to three.

2 MODEL OF SPATIAL ORDER

The signal from the Schottky detector can be approximated by a delta pulse each time a particle passes

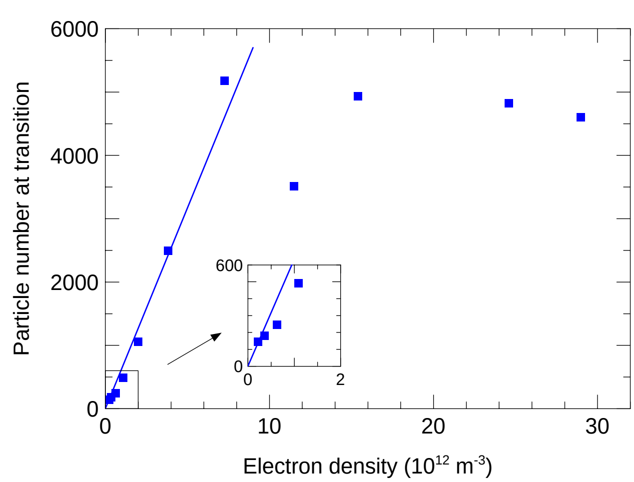

Fig 2. Number of particles in the beam when the transition to the ordered state takes place as a function of the electron density in the cooler.

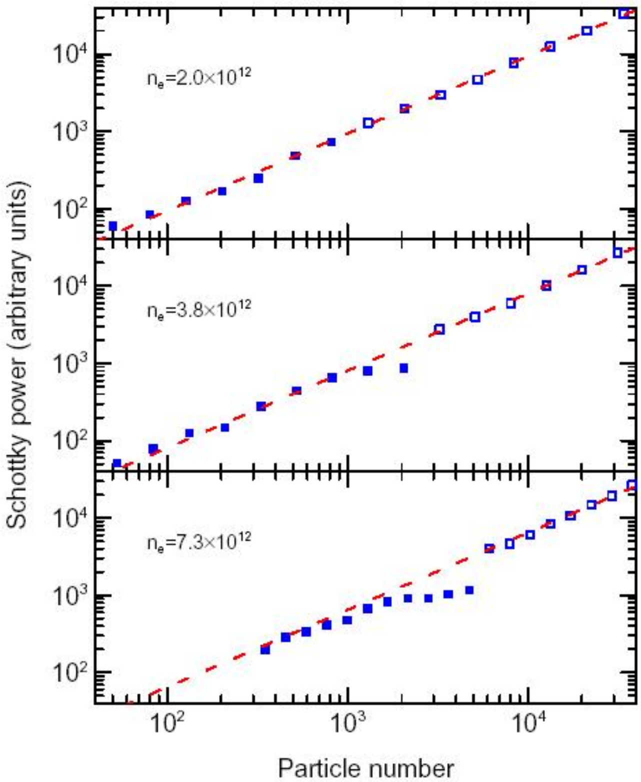

Fig. 3. Schottky power as a function of particle number when electron densities n_e as written in the figure. Open symbols how the power before the transition and filled symbols represent the ordered beam. Uncorrelated particles should have a Schottky power proportional to the particle number, as indicated by the dashed lines which have unit slope.

through it. The total signal from N particles can thus be written as [5]

$$I(t) = Zq\sum_{a=1}^{N}\sum_{n=-\infty}^{\infty}\delta(t-\frac{2\pi n+\theta_a}{\omega_a}), \qquad (1)$$

where Zq is the charge of the ions, θ_a the initial phase of ion a in its motion around the ring and ω_a its revolution frequency. For uncorrelated particles the spectral density of this signal is

$$S(\omega) = \frac{(Zq)^2}{2\pi}\sum_{a=1}^{N}\sum_{n=-\infty}^{\infty}\omega_a^2\,\delta(\omega-n\omega_a) \qquad (2)$$

and is concentrated in bands around the revolution frequency and its harmonics. With ω_0 being the nominal revolution frequency, the integrated noise power in each (non-overlapping) band is then

$$P_n = \frac{(Zq)^2}{2\pi}\sum\omega_a \approx \frac{N(Zq)^2\omega_0^2}{2\pi}, \qquad (3)$$

and it is thus proportional to the particle number.

Ordering introduces some correlation between the particles, however, and this modifies the Schottky signal.

To make a model of the ordered beam we first neglect the velocity spread of the ions, i.e., we take $\omega_a = \omega_0$ for all a. The Schottky power then becomes

$$P_n = \frac{(Zq)^2\,\omega_0^2}{2\pi}\left[\left(\sum_{a=1}^{N}\cos n\theta_a\right)^2 + \left(\sum_{a=1}^{N}\sin n\theta_a\right)^2\right]. \qquad (4)$$

The model is then characterized by a certain minimum distance between the particles. This distance is defined by setting kinetic and potential energies equal, giving

$$d_{\min} = \frac{(Zq)^2}{4\pi\varepsilon_0 kT} \qquad (5)$$

if kT is the ion temperature, Apart from the minimum distance, the particles have random positions along the ring circumference C.

The algorithm for choosing θ_a can thus be described by: *i*) choose θ_1 randomly between 0 and 2π, *ii*) choose θ_2 randomly between 0 and 2π, too, but exclude the region $\theta_1 \pm d_{\min}/C$, and *iii*) repeat step *ii* the desired number of times, excluding the regions around all previously inserted particles. It is found numerically that the maximum number of particles $N_{\max}$ that can be inserted into the ring in this way is approximately $0.747C/d_{\min}$ when N is large. With $N_{\max}$ particles in the ring all inter-particle distances are between $d_{\min}$ and $2d_{\min}$. Note that the only free parameter in the model is kT which determines both $d_{\min}$ and $N_{\max}$.

From direct measurements on the beam, only upper limits to the temperature of the ordered beam can be obtained. Since the ions are continuously electron-cooled, however, we can assume that they are in thermal equilibrium with the cooler electrons in the absence of intrabeam scattering. The electron temperature has been calculated from peak shapes in dielectronic-recombination spectra on many occasions, and typical values are 0.05–0.15 meV longitudinally and 1–3 meV transversally.

If we choose the temperature of the Xe^{36+} ions equal to 0.5 meV, we get $d_{\min}$=4 mm and an $N_{\max}$ of 9 500 with the 52 m circumference of CRYRING. Using these parameters, the Schottky power was calculated for particle numbers up to $N_{\max}$ and shown as the curve in fig. 4. The power, with the factor $(Zq)^2\omega_0^2/2\pi$ omitted, is calculated at the first harmonic of the revolution frequency and is averaged over 100 000 simulated particle distributions. Also plotted in the figure are the same experimental data points as in the lower part of fig. 3, the vertical scale having been shifted to obtain the best agreement with the curve. The dotted line has unit slope and thus represents the power expected from uncorrelated particles. Choosing a different kT essentially has the effect of sliding the curve along the dashed line.

While the frequency spread of the Schottky signal gives information about the momentum spread of the beam, the Schottky power, according to our model, gives information about the spatial correlation between the ions. The good agreement between the model and the experimental

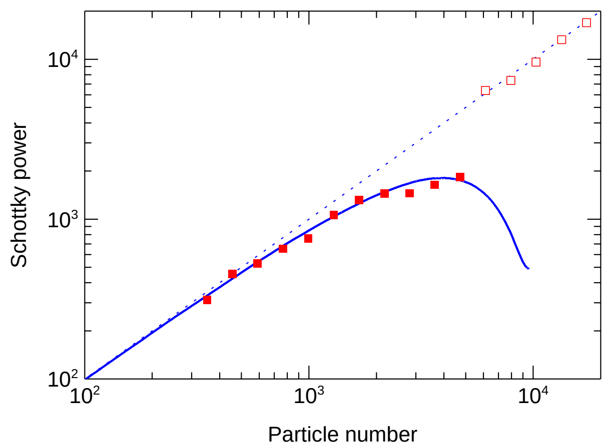

Fig. 4. Schottky power as a function of particle number, calculated according to eq. (4) and the model described in the text (curve), and experimental data points (squares).

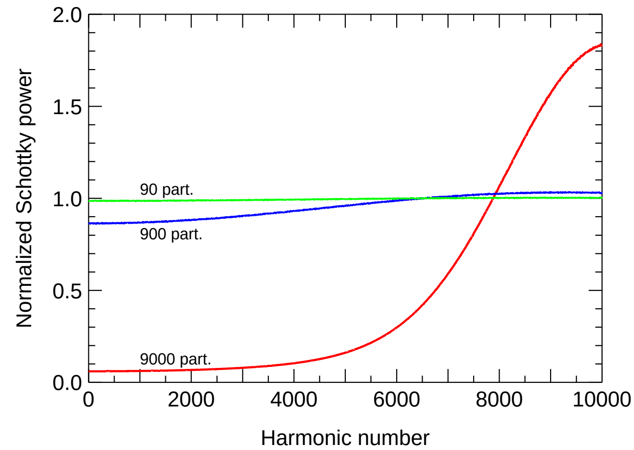

Fig. 5. Schottky power, calculated according to eq. (4), as a function of harmonic number n for 90, 900 and 9 000 particles in the beam. The power is normalized such that it is equal to one for uncorrelated particles.

data supports the idea that such correlations exists or that the beam is spatially ordered. This order becomes noticeable at particle numbers above approximately 1 000.

One can also want to compare the predicted particle number $N_{max} = 9\,500$, above which ordering should not appear, with the observed limit of 5 000 particles. However, it must be understood that the model tries to describe the beam after the transition to the ordered state, but gives no information about how the actual transition takes place. It therefore does not predict at which particle number the transition really occurs, nor how this number depends on, e.g., the cooling rate as depicted in fig. 2.

3 MODEL PREDICTIONS OF SCHOTTKY AT HIGH HARMONICS

It is often argued that if a beam with a certain number of particles n exhibits spatial ordering, the Schottky signal at or in the vicinity of the harmonic n should be very strong. At least this would be the case for a beam displaying long-range order. In our beam model, however, the particle correlations are only local, unless the particle number approaches N_{max}, where, as already mentioned, all inter-particle distances are between d_{min} and $2d_{min}$. It is then not surprising that the Schottky power is almost independent of harmonic number for low particle numbers. Only at particle numbers approaching N_{max} is there an enhancement of the Schottky signal, and then not just to harmonics close to the particle number but to a broad range. This is illustrated in fig. 5. At high particle numbers but low harmonics one instead sees the suppression of the Schottky power already discussed.

4 BUNCHED BEAMS

Indications of ordering have also been seen in a bunched beam when the rf voltage is extremely low. In this case we look with an oscilloscope at the signal from a longitudinal electrostatic pickup. Using an rf amplitude of 6 mV, we can then observe sharp transitions from a broad emittance-dominated bunch to a much narrower, approximately 3 m long space-charge-dominated bunch. The bunch contains some 400 particles at the transition, estimated from the area of the bunch, implying that the linear particle density in the bunch is quite similar to that observed in the ordered coasting beam.

The usual expression for the length of a space-charge-dominated bunch (see, e.g., ref. [6]) does not hold when the particle distance is large compared to the beam radius. Instead, we have performed numerical calculations of the theoretical bunch length for a linear particle configuration at zero temperature. We find, for instance, that if the bunch contains 400 Xe^{36+} ions in a sinusoidal rf potential that has an amplitude of 6 mV, and if the particles are located at the centre of a 10-cm-diameter conducting beam pipe, the bunch is approximately parabolic with a total length of 2.4 m. This is quite close to the observed length, and the difference might be attributed to a non-vanishing ion temperature. Such a calculation thus confirms that the rf voltage, although very low, is of the magnitude where ordering can be expected.

If these investigations can be pursued in spite of the experimental difficulty in keeping the revolution frequency equal to the rf frequency with sufficiently high accuracy, one could for instance have an interesting possibility to increase the particle density once the ordering has appeared by increasing the rf voltage.

5 REFERENCES

[1] M. Steck *et al.*, Phys. Rev. Lett. **77**, 3803 (1996).
[2] R. W. Hasse, Phys. Rev. Lett. **83**, 3430 (1999).
[3] H. Danared *et al.*, Proc. PAC2001, Chicago 2001, p. 2038.
[4] H. Danared *et al.*, Phys. Rev. Lett. **88**, 174801 (2002).
[5] S. Chattopadhyay, CERN 84-11.
[6] T. J. P. Ellison *et al.*, Phys. Rev. Lett. **70**, 790 (1993).