MECHANICAL STABILITY OF THE RF SUPERCONDUCTING CAVITIES

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Abstract

The mechanical deformations of a radiofrequency superconducting cavity could be a source of instability, both in CW operation or in pulsed mode. The electromagnetic fields exert a radiation pressure on the cavity's walls in such a way to lower the resonant frequency. The cavity's passive stiffening allows to reduce the frequency shift. In this paper, static and dynamic detuning have been analyzed, the computation results concerning the frequency shift have been compared successfully to the HF measurements. Recently, calculations have been contributed to the design of high power protons cavities.

1 INTRODUCTION

The mechanical stability is a fundamental problem for a RF superconducting cavity: the cavity could be driven out of resonance by the submicronic mechanical deformations under Lorentz forces, since such deformations increase with the square of the accelerating gradient. Naturally the wall thickness assures the mechanical stability, but thin walls are less costly, easier to handle and effectively cooled, stiffening systems become very useful since the cavities performance reaches $40\mathsf{MV/m}$ actually. However, each system has its limits, the new idea proposed here is to combinate two stiffening methods. The mechanical instability becomes more critical in pulsed mode, adding to the environment noise, the HF pulse constitutes an internal excitation and causes the cavity to mechanically oscillate. A original direct integration method is proposed.

2 FREQUENCY SHIFT CALCULATIONS

To evaluate the frequency shift due to Lorentz forces, a mechanical computation code must be involved, since the mechanical deformations are the source of the perturbation. Then, to obtain the frequency shift, usually two methods are used: write down a computer program according to Slater's formula [1]:

$$\frac{\Delta f}{f_0} = -\frac{1}{4W}\int\int\int_{\Delta V}(\mu_0 {H_s}^2 - \epsilon_0 {E_s}^2)dV \qquad (1)$$

where: W is the total electromagnetic energy stored in the resonator, E_s and H_s are the electromagnetic surface fields and f_0 is the fundamental mode of the unperturbed cavity; or recompute the new resonance frequency from deformed shape. But in both cases, the double precision is required and the numerical precision could be lost.

The originality of this work is to develop the Slater's theory directly in a high level mechanical computation code, preserving the internal double precision and avoiding the numerical truncation. The high level mechanical computation code is "CAST3M " [2], developed at CEA France, his particularity is to be open. That means that all links to others applications are possible. Here, a special development concerning electromagnetic HF is realized.

Based on the mesh used for mechanical calculations, the elemental volume change can be written from displacements [3]. Granting that the electromagnetic surface fields E_s and H_s are unchanged after deformations, the formula 1 can be discretized to:

$$\frac{\Delta f}{f_0} = -\sum_{j=1}^{N+1}[\frac{P_j}{W}(f_1 dr_j + f_2 dz_j + f_3 dr_{j+1} + f_4 dz_{j+1} + o(dr^2))] \qquad (2)$$

where $P_j = \frac{1}{4}(\mu_0 {H_s^j}^2 - \epsilon_0 {E_s^j}^2)$ is the radiation pressure at node j, dr_j, dz_j are its displacements, f_1, f_2, f_3, f_4 are functions of the geometric parameters [3], and $o(dr^2)$ contains all negligible terms depending on order from two of the displacements. Since we remain on the linear analysis, all displacements are proportional to the pressure P, which is proportional to the square of the accelerating gradient $E_{\rm acc}$. On the other side, the stored energy W is also proportional to the square of the $E_{\rm acc}$. At last, we have:

$$\Delta f = -K{E_{\rm acc}}^2 \qquad (3)$$

where K is a ratio of proportionality. The formula 2 has been programmed in CAST3M , so the frequency shift can be obtained directly from mechanical displacements.

3 RESULTS AND MEASUREMENTS

For TESLA 9-cells cavities [4], the HF measurements were realized in a horizontal cryostat (CHECHIA) at DESY (Deutsches Elektronen-Synchrotron). The superconducting cavity is tested with its helium vessel which constitutes also the cavity support. In this conditions, the total frequency shift Δf_t comes from two parts: at first, a Δf_1 due to Lorentz forces of the electromagnetic field, considering the cavity is held by some ideally rigid tank; but the Lorentz forces constraint the cavity to be shorter, so the tank which is interlocked to the cavity receives an axial force F, removing the tank. The tank and the cavity change their lengths at same time and contribute a second frequency shift Δf_2, which depends on the stiffness of the helium tank. Δf_2 could be calculated by: $\Delta f_2 = F \times \frac{\partial L}{\partial F} \times \frac{\partial f}{\partial L}$. Where $\partial F/\partial L$ is the stiffness of the helium tank, it is evaluated to be $85\mathsf{kN/mm}$ for TESLA tank. $\partial f/\partial L$ and F were calculated by CAST3M : $\partial f/\partial L = 436\mathsf{kHz/mm}$ and $F = -22\mathsf{N}$ (compressive force) at $E_{acc} = 25\mathsf{MV/m}$. So, $\Delta f_2 = -159\mathsf{Hz}$ at $E_{acc} = 25\mathsf{MV/m}$. Δf_1 has been also calculated by CAST3M : $\Delta f_1 = -337\mathsf{Hz}$ at $E_{acc} =$

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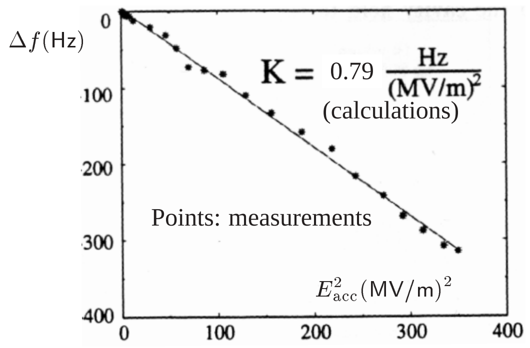

Figure 1: Measurements and calculations

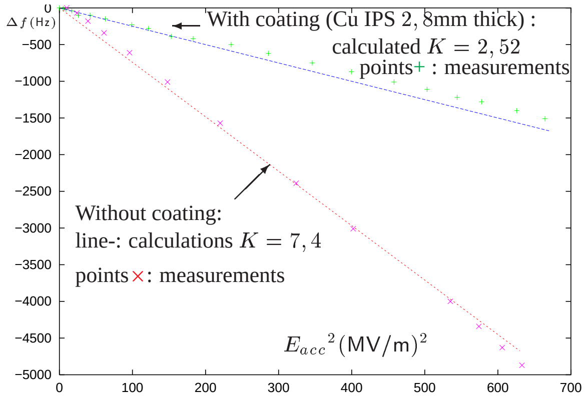

Figure 2: Improvement of the cavities stability by coating

25MV/m At least, $\Delta f_t = 496$Hz at $E_{acc} = 25$MV/m. From this result, the linear dependence (formula 3) between Δf and $E_{acc}{}^2$ can be ploted, fig.1. A good agreement with the measurements was observed.

4 CAVITY STIFFENING

The TESLA 9-cells cavity have welded rings between two cells, the bandwidth of the cavity at the specified $Q_{ext} = 3 \times 10^6$ is 434Hz [4]. From figure 1, we can see that this cavity is not rigid enough if $E_{acc} > 23$MV/m: the frequency shift becomes larger than the bandwidth. Actually, some TESLA cavities can reach an accelerating gradient up to 40MV/m and the ultimate objective is that each cavity works at 34MV/m. An another stiffening method, proposing a copper coating over the external niobium wall, has been studied. Some monocell cavities have been coated, and a real stiffening effect was observed, both from experiments and simulations (fig.2): the detuning parameter K is divided by $2,5$ after coating (2.8mm, Cu IPS). But for a 9-cells cavity, the coating should have a quite high thickness at iris (near 1cm) [5]. Adding to the technical difficulties, the force necessary for cavity tuning increases. Finally, a mixed solution using both the stiffening rings and coating seems more interesting: spraying a 2mm Cu IPS coating on a cavity with stiffening rings, fig.3, the stability is guaranteed until $E_{acc} = 34$MV/m [3], and the cavity remains tunable. The interest of this mixed solution resides on the reduction of the axial displacements by rings and the reduction of the radial displacements by the coating.

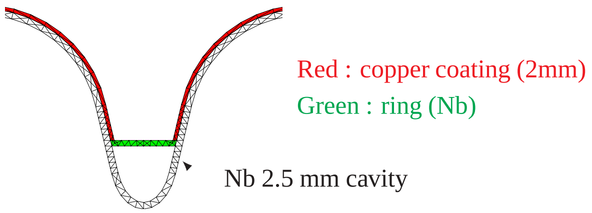

Figure 3: Use both rings and coating

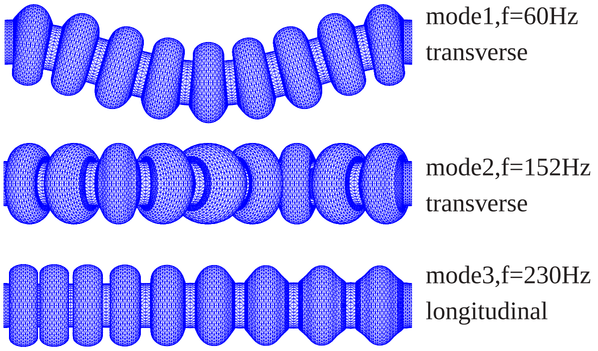

Figure 4: Three first eigen-modes

5 DYNAMIC ANALYSIS

The TESLA cavities will be operated in pulsed mode with RF pulses of 1.3ms. The mechanical vibrations mode analysis of the TESLA 9-cells cavity has been done with CAST3M , as examples, calculated first three eigen-modes are ploted at figure 4.

Disassociating the microphonic sources of noise, the RF pulse make a internal noise since it generates cycling Lorentz forces. The direct integration method has been used to solve the second degree differential equation 4:

$$M\{\ddot{q}\} + \lambda\{\dot{q}\} + K\{q\} = \{P(t)\} \tag{4}$$

where $\{q\}$ is the generalized displacement, M the mass matrix, λ the damping matrix, K the rigidity matrix, and $\{P(t)\}$ is the time depending radiation pressure distribution. The time depending $\Delta f(t)$ was calculated step by step using a adapted damping model [3]. A good agreement with measurements was observed, figure 5.

6 NEW PROTON CAVITIES DESIGN

The 700 MHz superconducting proton cavities have been proposed for high power LINACs [7], with pulse option. Even the cavities will work at a lower accelerating field, near 11MV/m, the ratios $\frac{E_{peak}}{E_{acc}} = 2.6$ and $\frac{H_{peak}}{E_{acc}} = 4.88$mT/(MV/m) [8] are higher than the ratios for TESLA electron cavities ($\frac{E_{peak}}{E_{acc}} = 2$ and $\frac{H_{peak}}{E_{acc}} = 4.26$mT/(MV/m)). The mechanical stability analysis is important for the design. The optimized wall thickness is 4mm, obtained by mean of the stress distribution analysis under vacuum (2bars) [3]. The Lorentz force detuning has been calculated for $B_{peak} = 50$mT (table 1). Concerning the monocell, a good agreement with the measurements is observed (example shown in figure 6).

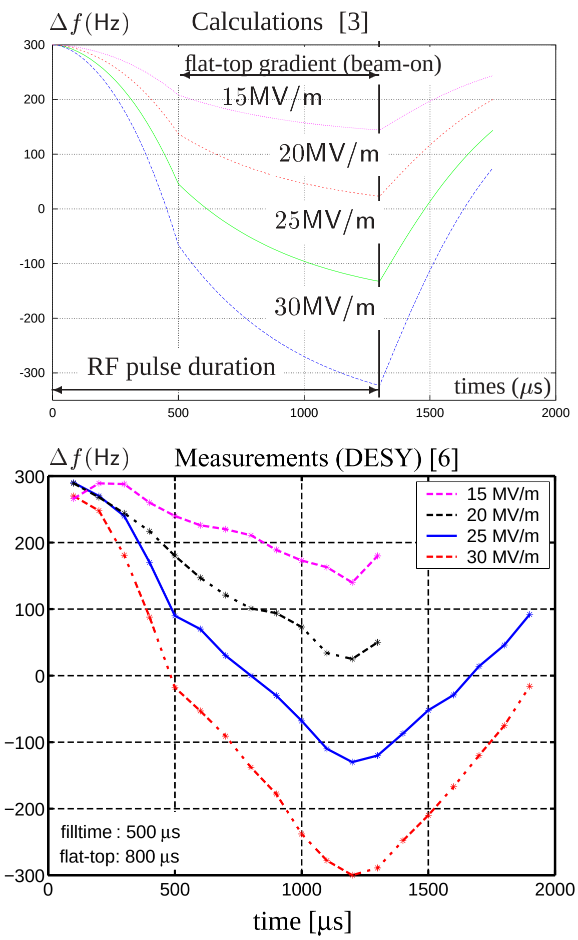

Figure 5: Frequency shift during a pulse

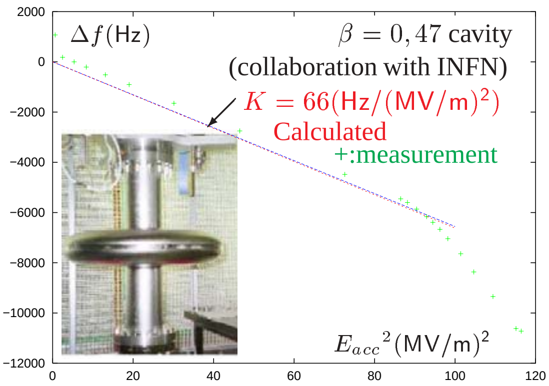

Figure 6: Calculated and measured frequency shift

The dynamic analysis has been stated. The eigen-modes of the 5-cell and the 1-cell β=0.65 cavities (4mm) have been calculated. Table 2 lists the first fifth eigen-frequencies. Less rigid, the eigen-modes of the 5-cell cavity are more closer and lower. Some eigen-modes shapes are shown in fig.7. For the monocell, the major effect of mode 1 is the displacement, others deform the cavity; for the 5-cell cavity, the first fifth modes concern cells relative displacements, the cell deformation appears from mode 6.

Table 1: Frequency shift of the proton cavities

5-cell β0,65 cavity	$K = -\Delta f/{E_{acc}}^2$	$\delta f/\delta l$
thickness 4mm	$4[\mathrm{Hz}/(\mathrm{MV/m})^2]$	260kHz/mm
1-cell $\beta 0, 65$ cavity	$K = -\Delta f/{E_{acc}}^2$	$\delta f/\delta P$
thickness 5mm	$17[\mathrm{Hz}/(\mathrm{MV/m})^2]$	300Hz/mBar

Table 2: Longitudinal eigen-frequencies (unit: Hz)

β065	f1	f2	f3	f4	f5	f6
5-cell	88	172	247	305	332	662
1-cell	357	1185	1404	2344	3885	3653

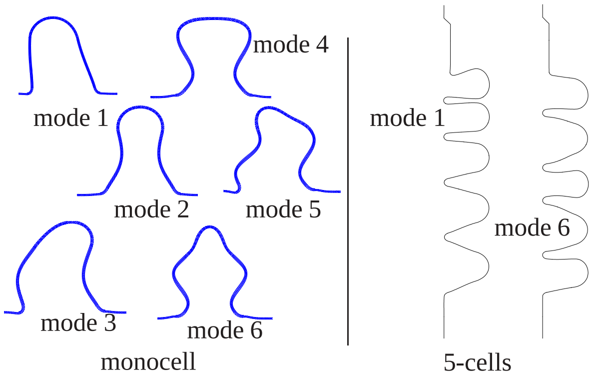

Figure 7: Eigen-modes shapes of a monocell

7 CONCLUSION AND PERSPECTIVES

Mechanical stability studies for SRF cavities are presented. The cavity stiffening has been discussed. The simulation results on TESLA or on protons prototypes have been confirmed by measurements. A dynamic analysis method has been elaborated, the calculated times depending frequency shifts are very close to the measurements. Vibration measurements are in perspective, in order to continue the dynamic analysis of the proton cavities.

8 REFERENCES

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