# PLASMON LINAC

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#### Abstract

A linac is proposed in which a laser first excites plasmons along the inner surface of a metallic acceleration tube. Potential of the plasmon oscillation then accelerates electron beams. It features beam size in nm range and good conversion efficiency from laser intensity to acceleration gradient; a MW laser will attain a gradient exceeding GeV/m, though the current is very small.

## **1 INTRODUCTION**

Although acceleration based on lasers and plasmas has been successful in laboratories to attain an acceleration gradient over GeV/m[1], it has not yet been applied to design and constriction of real accelerators. This is partly because of difficulties to handle gaseous plasmas. This paper proposes use of solid-state plasmas in place of gaseous plasmas. A plasmon linac is a miniature linac which uses a hole in a metal with radius around the laser wavelength as an acceleration tube. A laser pulse excites plasmons along the inner wall of the hole. Test beams are then accelerated by the potential of the plasmons, as Fig.1 shows.

The plasmons are the energy quanta of the volume oscillations of solid-state plasmas, or collective excitation of the electron gases. Such electron gases are realized by the valence electrons of a metal, whose density exceeds  $10^{22}$  cm<sup>-3</sup>. Such high electron density is able to hold ultrahigh acceleration gradient. Another feature of the solidstate plasmas is that they are almost always near thermodynamic equilibrium, contrasting with gaseous plasmas which are easily driven out of equilibrium and reveal instabilities[2].

One can regard this method as a laser wakefield acceleration in a hollow channel[3], using an overdense metal plasma instead of an underdense gaseous plasma. The structure resembles that of a dielectric linac[4], which has once been studied as a rival of a linac with periodic structure. The size of this linac is much smaller than the existing ones, smaller than that of the gaseous-plasma accelerators, but larger than those of the crystal accelerators[5], and in the same order with those of photonic crystal accelerators[6].

It features beam size in nanometer range and good conversion efficiency from laser power to acceleration gradient; it attains a gradient exceeding GeV/m by a MW laser instead of a TW one, though the current is very small.

In section 2 of this paper we derive dispersion relations of the plasmons, and explicit expression of the accelerating



Figure 1: Schematic view of the plasmon linac.

field. We then derive the relation between the field and the laser power, integrating the Poynting vector in section 3. A numerical example is given in section 4. The last section contains discussion and conclusions.

## 2 DISPERSION RELATIONS OF PLASMONS

Suppose a tube with inner and outer radii *a* and  $\infty$ , made from a medium with dielectric function  $\varepsilon_1(\omega)$ . Axial symmetric components of electric and magnetic fields inside the tube ( $r \leq a$ ) are

$$E_{r} = -\frac{ik}{K_{0}}A_{0}J_{1}(K_{0}r),$$

$$E_{z} = A_{0}J_{0}(K_{0}r),$$

$$B_{\theta} = -\frac{i(K_{0}^{2}+k^{2})}{\omega K_{0}}A_{0}J_{1}(K_{0}r),$$
(1)

and those in the medium (r > a) are

$$E_{r} = -\frac{ik}{K_{1}}A_{1}H_{1}^{(1)}(K_{1}r),$$

$$E_{z} = A_{1}H_{0}^{(1)}(K_{1}r),$$

$$B_{\theta} = -\frac{i(K_{1}^{2}+k^{2})}{\omega K_{1}}A_{1}H_{1}^{(1)}(K_{1}r),$$
(2)

where

$$K_i^2 = \frac{\omega^2 \varepsilon_i(\omega)}{c^2} - k^2, \quad i = 0 \text{ or } 1, \tag{3}$$

and  $J_n(x)$  and  $H_n^{(1)}(x)$  are Bessel function and Hankel function of the first kind, respectively[7].

Using the boundary conditions at r = a, we delete  $A_0$  and  $A_1$  from these to obtain the transcendental equation

$$\frac{\varepsilon_0(\omega)}{K_0} \frac{J_1(K_0a)}{J_0(K_0a)} - \frac{\varepsilon_1(\omega)}{K_1} \frac{H_1^{(1)}(K_1a)}{H_0^{(1)}(K_0a)} = 0.$$

Inserting  $\varepsilon_0(\omega) = 1$  and the dielectric function of the medium

$$\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)},$$
 (4)

with  $\omega_p$  and  $\gamma$  being plasma frequency and relaxation constant respectively, we obtain the dispersion relation

$$K_{1}H_{0}^{(1)}(K_{1}a)J_{1}(K_{0}a) - \left[1 - \frac{\omega_{p}^{2}}{\omega(\omega + i\gamma)}\right] \times K_{0}H_{1}^{(1)}(K_{1}a)J_{0}(K_{0}a) = 0.$$
(5)



Figure 2: Real part of the dispersions of the plasmons inside the vacuum tube. Two straight lines give those of particle beams with  $v_p = c$  and c/2.

Figure 2 shows real part of the dispersion relations for various  $k_p a$  values. Frequencies in time and space are normalized by  $\omega_p$  and  $k_p = \omega_p/c$ . All  $\omega$  values approaches  $\omega_p/\sqrt{2}$ , the surface plasmon-polariton frequency, with increasing *k*. Two straight lines show particle beams with velocities  $v_p = c$  (relativistic electrons) and c/2 (for comparison). Lasers with frequencies at the crossing points are able to excite the plasmons mating with the speeds of the beams.

Equations (1) include the longitudinal and transverse electric fields. In case that the phase velocity equals the light velocity, Eq. (3) gives  $K_0 = 0$  and consequently,  $J_0(0) = 1$  and  $E_z = A_0$ . The acceleration field is therefore independent of the radial position in the tube. The transverse field contains  $J_1(K_0r)$ . Because  $K_0r \ll 1$ , we approximate it as

$$J_1(K_0r) \sim \left(\frac{K_0r}{2}\right) \cdot \left(\frac{1}{\Gamma(2)}\right) = \frac{K_0r}{2},$$

to obtain

$$E_r = -\frac{ikA_0r}{2}$$

The focusing strength is thus proportional to the distance from the axis. The phase difference between the accelerating and focusing phases is  $\pi/2$  as expected.

## 3 ACCELERATING FIELD AND LASER POWER

Integration of the Poynting vector

$$S = \frac{1}{2} \operatorname{Re}\left[E_r H_{\theta}^*\right],$$

gives the laser power P which transmits along the tube;

$$P = \frac{1}{2} \int_{0}^{\infty} \int_{0}^{2\pi} \operatorname{Re} \left[ E_{r} H_{\theta}^{*} \right] r dr d\theta$$
  
$$= \frac{1}{2} \left[ \int_{0}^{a} \int_{0}^{2\pi} E_{r} H_{\theta}^{*} r dr d\theta + \int_{a}^{\infty} \int_{0}^{2\pi} E_{r} H_{\theta}^{*} r dr d\theta \right]$$
  
$$= \frac{\pi \omega k k_{p}}{c \mu \omega_{p}} \left[ \frac{A_{0}^{2}}{K_{0}^{2}} \int_{0}^{a} J_{1} (K_{0}r) \left( J_{1} (K_{0}r) \right)^{*} r dr \right]$$
  
$$+ \frac{A_{1}^{2}}{K_{1}^{2}} \left( 1 - \frac{\omega_{p}^{2}}{\omega^{2}} \right) \int_{a}^{\infty} H_{1}^{(1)} (K_{1}r) \left( H_{1}^{(1)} (K_{1}r) \right)^{*} r dr \right],$$

where  $\mu$  is the magnetic permeability in the vacuum. Using the boundary conditions of the field components

$$A_0 J_0(K_0 a) = A_1 H_0^{(1)}(K_1 a),$$

and inserting  $E_z$  in Eq. (1), we find that the accelerating field  $E_z$  inside the tube is proportional to the square root of the laser power P;

$$E_z(0) = \alpha k_p P^{1/2},\tag{7}$$

where

$$\alpha = \left[\frac{\pi\omega k}{c\mu\omega_p k_p} \left(\frac{1}{K_0^2} \int_0^a J_1(K_0r) \left(J_1(K_0r)\right)^* dr + \frac{1}{K_1^2} \left(\frac{J_0(K_0r)}{H_0^{(1)}(K_1r)}\right)^2 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \right]^{-\frac{1}{2}} \times \int_a^\infty H_1^{(1)}(K_1r) \left(H_1^{(1)}(K_1r)\right)^* dr \right]^{-\frac{1}{2}}$$
(8)

The  $k_p a$  dependency of the coefficient  $\alpha$  of the above equation is numerically calculated and given in Fig.3 for two phase velocities,  $v_p = c$  and c/2.

#### **4 NUMERICAL EXAMPLES**

If we use silver with parameters  $\omega_p = 13.2 \times 10^{15} \text{s}^{-1}$ and  $\gamma = 68.9 \times 10^{12} \text{s}^{-1}$ , the design conditions  $k_p a = 10$  and  $v_p = c$  give a = 227nm and the laser wavelength as 344nm. According to Eq. (7), a laser with power of 1MW attains the acceleration gradient of 45.0GeV·m<sup>-1</sup>. The acceleration length  $\sim c/\gamma$  is, however, only 4.32 $\mu$ m, giving the energy gain of 194keV.



Figure 3: Conversion coefficient  $\alpha$  between  $k_p P^{1/2}$  and the acceleration field  $E_z(0)$ , as a function of  $k_p a$  for two phase velocities,  $v_p = c$  and c/2.

This short length is due to the ohmic loss, which raises the temperature and destroys the accelerator structure. The solution is to keep the structure in a low temperature. The resistivity  $\rho$  is related to  $\gamma$  by the relation

$$\rho = \frac{m^* \gamma}{ne^2},$$

where  $m^*$  is effective mass of an electron and *n* is the electron density. The resistivity of silver at 300 K, 16.29  $\times 10^{-9} \ \Omega \cdot m$ , is reduced to  $0.0115 \times 10^{-9} \ \Omega \cdot m$  at 10 K[8]. The acceleration length and the energy gain at 10 K instead increase to 6.11 mm and 273 MeV, respectively, with the 1MW laser. The laser intensity at the inner wall is  $1 MW/(2 \times \pi \times 227 nm \times 6.11 mm) = 11.48 \times 10^9 W cm^{-2}$ . This value can be below the damage threshold of the silver, if the laser repetition rate is moderate.

#### 5 DISCUSSION AND CONCLUSIONS

Equations (1-2) give only the fundamental TM mode. Also higher TM modes and hybrid modes can, however, exist, though TE modes cannot[9]. Expanding the input laser field by the possible orthogonal modes, we could obtain its coupling to the required TM mode[11]. It should be noted that, under certain conditions, a linear-polarized laser field prefers a particular mode (corresponding to the HE<sub>11</sub> mode in optical fibers) to the TM mode. Further studies are required in order to solve this problem.

The hitherto analyses suggest some interesting physical problems in the case  $k_pa < 2$ . In spite that a usual rf accelerator tube has the cut-off frequency, the dispersion relation has solution where the tube size is much smaller than the laser wavelength. It has been recently shown that in a material with negative dielectric constant such as a metal, light can transmit along a space whose dimension is below its wavelength[9]. An interesting observation has been also reported that the optical transmission through subwavelength holes in metal films can be enhanced by several orders of

magnitude[10]. This phenomenon is understood as a result of interaction of the incident light with independent surface plasmon modes on either side of the film.

Another feature of the dispersion shown in Fig.2 is in its negative group velocity,  $\partial \omega / \partial k < 0$ , at  $k_p a < 2$ , in spite of its positive phase velocity  $\omega / k$ . We have a preceding example, a backward-wave tube, in which a forwardflowing electron beam converts its energy into a backward rf wave[12].

This linac with a fine acceleration tube is capable of producing so-called nano beams. It will make a contribution to study, manufacturing and measurement in nanometer range. A carbon-nanotube electron source will provide source beams for this linac[13].

In conclusion, we have presented an analysis of acceleration using potential of plasmons excited in inner wall of a metal tube by a laser. It is able to attain a gradient exceeding GeV/m by, not a TW, but a MW laser to produce nano beams. In addition to the technical advantages, it will provide us some interesting problems; i.e., optical transmission through the space with the subwavelength size and the negative group velocity. More work needs to be made on the interface between the accelerator structure and, both laser light and source beams.

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