MEASURING BETA FUNCTIONS AND DISPERSION IN THE EARLY LHC

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Abstract

Achieving nominal beam conditions in the LHC requires a small beta beating and a small dispersion mismatch. We compare the linear optics errors expected from linear and nonlinear magnetic field imperfections and magnet misalignments with the required values. The measurement procedures are simulated in a machine with collimators, realistic aperture restrictions, and different bunch intensities, in order to infer the expected measurement accuracies for beta beating and dispersion. Consequences for possibly required beam-based correction tools are discussed.

1 OUTLINE

In Section 2 we simulate the expected optics errors due to field errors in the superconducting magnets, and due to misalignments of sextupoles and beam-position monitors (BPMs). In Section 3 we describe simulated measurements of beta function and dispersion, including their dependence on the coupling correction and on the BPM noise.

2 EXPECTED MISMATCH

To assess the expected optics imperfections, we have performed a simulation using LHC optics version 6.2. As specified in [1], we here considered the random and systematic multipole components in all s.c. arc dipoles (MBs) and quadrupoles (MQs) ranging from b_1, a_1 to b_{11}, a_{11}, a correction of measured average b_3, b_4, and b_5 components per arc, and absolute misalignments of the b_3 spool-piece correctors and beam-position monitors (BPMs) by 500 μm rms, truncated at 2.5 σ, and the lattice sextupoles by 250 μm. The quadrupoles and dipoles were not misaligned, so that the 250 μm and 500 μm also represent the relative misalignments of lattice sextupoles and BPMs with respect to the adjacent quadrupoles, respectively, and 500 μm the misalignment of the sextupole spool pieces with respect to the dipoles. The initial rms orbit is of order 3–5 mm horizontally and 2 mm vertically. This is not representative of the real closed orbit distortion expected, as we have not misaligned the quadrupoles. (The purpose of our study is to look at optics errors which arise from the relative misalignments. We here profit from the fact that the dispersion mismatch caused by normal and skew quadrupole field errors is much larger than that expected from the quadrupole misalignments.)

Using MAD we next compute the beta functions and dispersion (1) for the uncorrected machine, (2) after correcting the orbit to 1 mm rms (according to BPM readings), and (3) after also correcting the tunes varying the arc quadrupoles. We repeated this exercise for 10 different random seeds, and thereby obtained average values and rms spread of the optical functions, which we can compare with the values obtained without magnet errors and misalignments.

Figures 1 shows the simulated horizontal beta mismatch across octant 7, where the (betatron) cleaning insertion is located. The maximum beta beating (solid line) is of the order 17–20% initially, and decreases to 15% after orbit correction and tune adjustment (dashed lines).

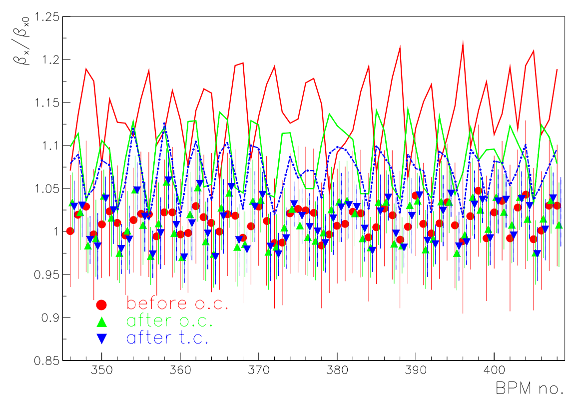

Figure 1: Simulated horizontal beta function β_x/β_{x0} vs. BPM number before and after orbit correction, and after tune adjustment, for LHC octant 7; error bars reflect the rms spread over 10 random seeds; lines indicate the maximum mismatch.

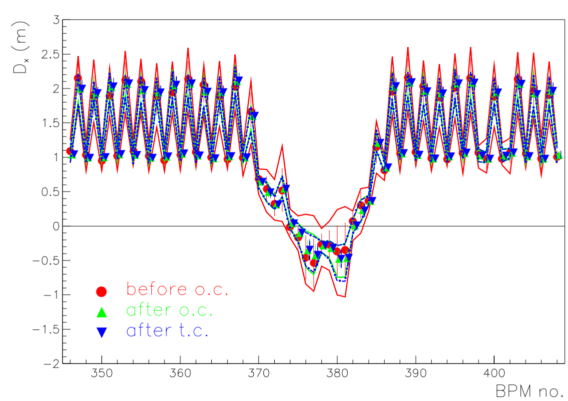

Figure 2: Simulated horizontal dispersion D_x for octant 7. Lines indicate the maximum and minimum value.

The simulated values for the horizontal dispersion are illustrated in Fig. 2, again for octant 7. We observe that the maximum residual horizontal dispersion in the straight section of IR 7 can be as large as $(-)1$ m, whereas it is of the order of 20 cm in the vertical plane. The orbit correction reduces the residual dispersion by about 30%. In the vertical plane the peak residual dispersion is about 15 cm; it is hardly affected by the corrections.

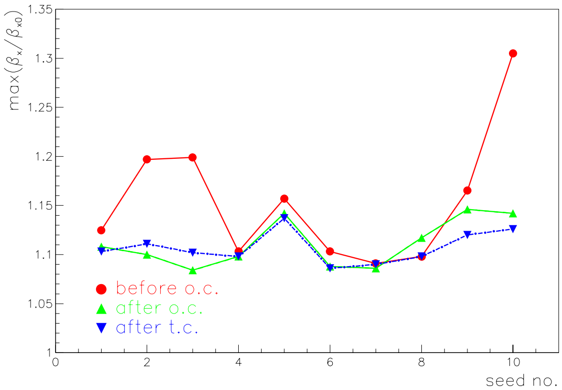

Figure 3: Simulated maximum horizontal beta mismatch $\max(\beta_x/\beta_{x0})$ versus random seed.

Figures 3, 4 present the maximum horizontal beta and dispersion mismatch (over all BPMs in the ring) as a function of random seed, again for the three cases: before and after orbit correction, and after adjustung the tunes. In case of the dispersion, we plot the normalized quantities $\max(|D_x - D_x^{(0)}|/\sqrt{\beta_x^{(0)}}) \times \sqrt{\beta^{(0)}_{\mathrm{arc-max}}}/D^{(0)}_{x,\mathrm{arc-max}}$, where the superscript (0) refers to the nominal optics, and the subindex 'arc-max' to the maximum value in the arc. Numerically, $\sqrt{\beta_{\mathrm{arc-max}}}^{(0)}/D^{(0)}_{x,\mathrm{arc-max}} \approx 6.6\ \mathrm{m}^{-1/2}$.

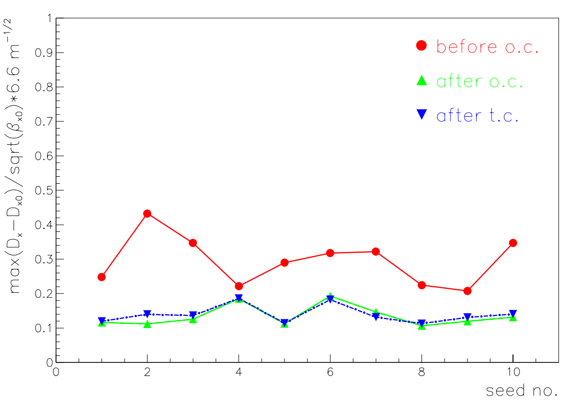

Figure 4: Simulated maximum horizontal dispersion mismatch $\max(|D_x - D_x^{(0)}|/\sqrt{\beta_x^{(0)}}) \times 6.6\mathrm{m}^{-1/2}$ versus random seed.

With regard to the ring mechanical aperture, the relative beta-beating is specified to be less than 21% everywhere around the machine and the relative (horizontal or vertical) dispersion mismatch to be less than 30% [2]. The results in Figs. 3 and 4 meet these requirements.

3 MEASUREMENT

We here assume that during the commissioning the beta functions and dispersion are measured with a single pilot bunch of $N_b \approx 5 \times 10^9$ protons. The BPM resolution for the pilot is specified to be $200\ \mu$m, both in orbit and in trajectory mode [3]. We further assume that before measuring the optics, the orbit and betatron tunes have been corrected as described in the previous section.

To measure the beta functions, we kick the beam to an amplitude of 1σ either horizontally or vertically, and record the turn-by-turn beam position over 200 turns at all $N = 509$ BPMs. We apply two methods to infer the beta functions. In the first approach, the beta function at BPM j is estimated as

$$\beta_{x,j} = \frac{\langle (x_j - \langle x_j \rangle)^2 \rangle}{\frac{1}{N}\sum_i \langle (x_i - \langle x_i \rangle)^2 / \beta^{(0)}_{x,i} \rangle}\ , \tag{1}$$

where x_i denotes the reading of BPM no. i and the angular brackets signify an average over all turns.

We simulated the measurement procedure by deflecting a single particle in MAD (thus ignoring any possible decoherence over the first 200 turns). The simulation shows that even without any BPM noise the beta functions inferred from (1) can be wrong by as much as 30%, *i.e.*, by an amount equal to or larger than the expected mismatch. The reason for the discrepancy is a large betatron coupling arising from the a_2 field errors in the dipoles. Indeed the closest tune approach without further correction varies between $\kappa \approx 0.025$ and $\kappa \approx 0.30$. Even with perfect BPM readings, the 200-turn beta measurement exhibits a deviation between measured and actual β_x of about 7.4% rms and 19.5% maximum, due to the coupled optics.

We thus corrected the systematic skew coupling using 8 skew-quadrupole families (1 per octant), whose strengths are computed based on magnet measurements of the a_2 component in all magnets (see [4]). Subsequently, we applied an empirical fine tuning based on minimizing the closest-tune approach κ, using the same 8 families of skew quadrupoles. After the coupling correction, κ is of the order of a few 10^{-3}. We note that the correction based on magnet measurements alone is not always sufficient. In more than half of the cases the additional empirical coupling correction is required, before the maximum measurement error drops below 10%. Figure 5 demonstrates the improvement in the β measurement that can be achieved by correcting the coupling. Now the rms deviation around the ring is only 2.3% and the maximum error 5.5%.

We have performed simulations of beta functions measurements all around the ring for 10 different random seeds and different levels of BPM noise. The result for the horizontal plane is summarized in Fig. 6 (the numbers for the vertical plane are almost identical). Even with a BPM noise of only $100\ \mu$m and after coupling correction, the maximum error over all BPMs and random seeds can still be as large as 40%. At the specified noise level of $200\ \mu$m the rms error is 16%, which is slightly smaller than the allowance made for the beta beating.

In order to improve the resolution of the measurement, we apply a harmonic analysis to the BPM data [5]. That is,

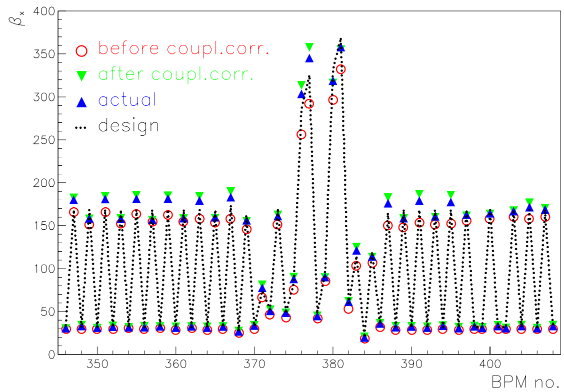

Figure 5: Simulated 200-turn measurement of horizontal beta function before and after coupling correction, without BPM noise, in octant 7. The actual and the design optics are also indicated.

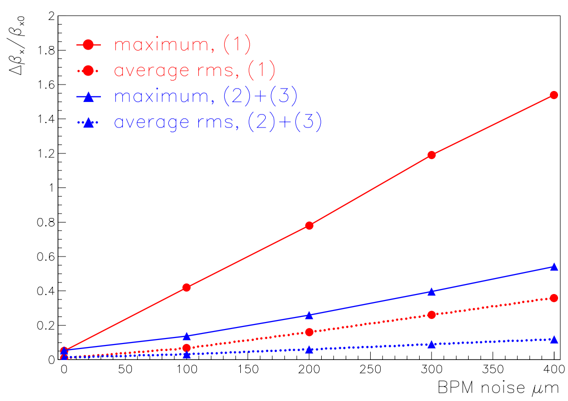

Figure 6: Simulated maximum relative error and average rms relative error over 10 random seeds of the simulated measured horizontal beta function vs. rms BPM noise after coupling correction, using either Eq. (1) or Eqs. (2) and (3).

for each BPM, denoted by k, we compute the two sums:

$$\begin{pmatrix} C_k \\ S_k \end{pmatrix} = \frac{1}{n} \sum_{m=1}^{n} x_k(m) \begin{pmatrix} \cos(2\pi m Q_x) \\ \sin(2\pi m Q_x) \end{pmatrix} \quad (2)$$

where n is the number of turns, $x_k(m)$ is the reading of the kth BPM on turn m, and $Q_{x,y}$ are the known tunes. The oscillation amplitude is $A_k = 2\sqrt{C_k^2 + S_k^2}$. Similar to before, we consider

$$\beta_{x,j} = \frac{A_j^2}{\frac{1}{N}\sum_i A_i^2/\beta_{x,i}^{(0)}} . \quad (3)$$

The advantage of this method compared with Eq. (1) is that we filter out the signal at the betatron frequency, which suppresses the noise contribution by a factor $1/\sqrt{n}$. A substantial improvement is visible in Fig. 6.

We next simulate the dispersion measurement by taking two off-energy orbits at $\Delta p/p = \pm 5 \times 10^{-4}$, which corresponds to $\pm 1\sigma_\delta$. Dividing the shift in BPM reading by the change in relative momentum yields the dispersion.

Without BPM noise we find that the maximum measurement error in $\max(|D_{x,y} - D_{x,y;0}|/\sqrt{\beta_{x,0}}) \times \sqrt{\beta_{\mathrm{arc-max},0}}/D_{x,\mathrm{arc-max},0}$ is about 2.0% and 5% for the horizontal and vertical plane. The rms deviation is of the order of 0.6%. The BPM reading error in orbit mode is also specified as 200 μm [3] for a pilot bunch. If we add this error, the maximum and rms deviations increase to 90% and 26%, respectively. This is consistent with the ratio of the expected orbit shift and the rms BPM error, but it is about 10 times larger than required. Therefore, the resolution of the dispersion measurement must be improved, either by enlarging the momentum window to $\pm 2 \times 10^{-3}$, by increasing the beam current (the BPM error in orbit mode is only 5 μm for the nominal LHC beam), or by averaging over several orbits.

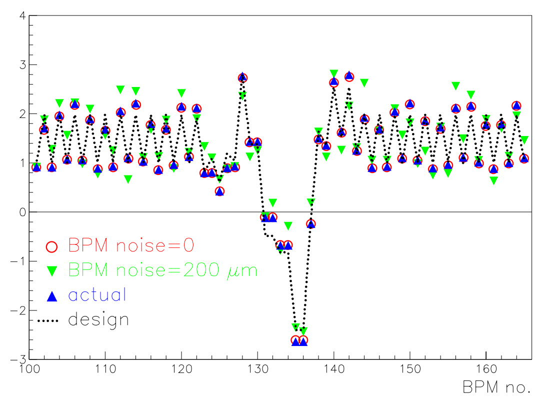

Figure 7: Simulated measurement of horizontal dispersion using off-energy orbits at $\Delta\delta = \pm 5 \times 10^{-4}$, with zero and 200 μm rms BPM noise, in octant 3. The actual and the design optics are also indicated.

4 CONCLUSIONS

The simulated optics errors are consistent with the specifications. The simulation of beta function and dispersion measurements suggests that the prior correction of linear coupling is essential for obtaining relevant results. The anticipated BPM resolution of 200 μm for the pilot bunch yields beta-function measurement errors comparable to the specified tolerances. Measuring the dispersion may require a modified scheme.

5 REFERENCES

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[2] J.B. Jeanneret and R. Ostojic, LHC Project Note 111 (1997).

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[5] J. Borer et al., Proc. EPAC92, p. 1082 (1992).