DESIGN OF END TURNS IN CURRENT-DOMINATED DIPOLE AND QUADRUPOLE MAGNETS FOR FIELDS WITH LOW HIGHER-HARMONIC CONTENT*

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Abstract

A design approach based on a stream function with a single angular Fourier component can be used to specify the end-turn configuration in magnet windings with discrete blocks of conductors. The design approach is especially appropriate for large-bore, current-dominated magnets with many turns. The design method can be summarized as follows: First find a winding-block layout (block angles, number of turns in each block, etc.) for the central, straight-conductor part of the windings that produces two-dimensional fields with negligible higher harmonic content. Next, specify the numbers of groups of conductors into which each block of the 2-D part of the winding fans out in the end-turn region. Also specify the end-zone axial length and the shape-function profile for the end zones. Finally, generate turn contours by finding conductor-group centerline curves in the developed (flattened) cylinder surface that are contours of constant stream function. Individual turns are specified by constant parallel displacement from the group centerline curve in the developed winding surface. An interactive computer program performs the above steps has been written and has been used to design end windings for a test quadrupole example. The unwanted higher harmonics in both peak and integral fields as computed by the Biot-Savart law are remarkably low.

1 THE STREAM-FUNCTION CONCEPT

A smooth curve on a surface (which may represent the position of a conductor centerline) can be described as the locus of points for which a continuous stream function ψ of two surface coordinates is constant. If the surface is a cylinder, convenient coordinates are the axial coordinate z and the azimuthal angle ϕ. A family of nested curves representing an N-turn surface winding (without turn-to-turn connections) is defined by a function $\psi(\phi,z)$ and a set of constants C_n , n=1, 2, 3,... N. In the limit of an infinite number of turns the current-carrying conductors form a field of flow; hence the terminology "stream function". It can be shown that if the stream function for continuous surface windings is of the product form

$$\psi(\phi,z)=\sin m\phi f(z) \quad (1)$$

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and the boundaries are either open or are of infinite permeability and form a surface of rotation around the winding axis, the magnetic fields produced by the windings have pure sin $m\phi$ symmetry, even in three dimensions. That is, the three components of magnetic field around the magnet axis are the three components of the gradient of a scalar magnetic potential of the form

$$V(r,\phi,z)=\sin m\phi H_m(r,z) \quad (2)$$

For dipoles, m=1; for quadrupoles, m=2, etc. The function $f(z)$ is called the shape function, since for a given winding-cylinder radius and m value, the family of winding contours is completely specified by it.

It is usually convenient to choose a normalization factor for ψ such that $f(z)$=1 at the pole centers and $f(z)$=0 at the ends of the windings. With this normalization, the stream function in the straight section of the windings between the end-turns regions is simply sin $m\phi$. In the end regions of the windings, $f(z)$ varies smoothly between 0 and 1. The winding design approach described here is based on the idea that if discrete windings can be found that approximate the flow field of Eq.1, the magnetic field produced by them will have nearly pure sin$m\phi$ symmetry; i.e., have relatively low values of "allowed" Fourier components K=3m, 5m, … (2k+1)m, etc. (The present work concerns only the so-called "allowed" field errors, and does not address "non-allowed" error harmonics due to random errors in conductor placement, etc.).

Practical winding considerations place constraints on the detailed form of the shape function $f(z)$. The shape function $f(z)$ must have zero first derivative at the z value where the turns leave the straight section of the windings and enter the end-turn region, in order to avoid sharp bends. The maximum absolute value of the derivative of $f(z)$ in the end-zone region must at a minimum be low enough to prevent overlap of adjacent finite-width conductors. However, the end-zone region would typically be made longer in z than the minimum allowed by finite conductor-width considerations in order to reduce field peaking and to reduce unwanted Fourier components.

Many functions could be used for the shape function $f(z)$. One convenient shape function $f(z)$ for a winding of length L with end-winding zones of length w_1 and w_2 is as follows. For the left-hand end zone with $-L/2 < z < -L/2+w_1$, take

$$f(z)=1+A_1(z+L/2-w_1)^2 + B_1 (z+L/2-w_1)^4, \qquad (3)$$

where $A_1=(s_1/2-2) / w_1^2$, $B_1=(1-s_1/2) / w_1^4$. For the 2-D straight-conductor central part of the windings, with $-L/2+w_1 < z < L/2-w_2$, take $f(z)=1$. This produces the usual $\cos m\phi$ 2-D windings in this region. For the right-hand end zone, with $L/2-w_2 < z < L/2$, take

$$f(z)=1+A_2(z-L/2+w_2)^2 + B_2(z-L/2+w_2)^4, \qquad (4)$$

where $A_2=(s_2/2-2)/w_2^2$, $B_2=(1-s_2/2)/w_2^4$. Elsewhere, $f(z)=0$.

The above function has the constraints $f = 1$ in the straight section, $f' = 0$ at the ends of the straight section, and $f = 0$ at the winding ends built in. In the present example, the end zones are taken to be equal in length with the same winding contours. Consequently $w_1=w_2=w$ and $s_1=s_2=s$; the shape function therefore has three free parameters: total winding length L, end-zone length w, and end-slope parameter s. Fig. 1 is a plot of the shape function.

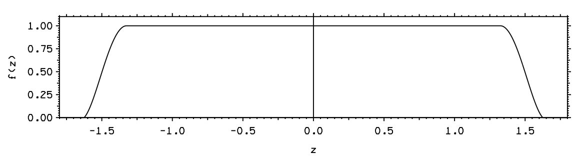

Fig. 1. Plot of the shape function of Eqs. 3-4 for the present example with L=3.25 m, $w_1=w_2$=0.3 m, and $s_1=s_2$=0.5.

2 TWO-D WINDING DESIGN

The purpose of this design stage is to find a winding layout for the straight-winding section (2-D part of the windings) that has negligible higher harmonics. Turns are separated by the fixed turn-to-turn gap width within a block; the angular spacing between blocks is adjustable. The angular position of the first block is fixed. Block angles are adjusted to null out a specified number of higher harmonics. The problem is nonlinear because the harmonics are nonlinear functions (trigonometric) of angular positions of the blocks. In principle, the 2-D optimization could be done with a nonlinear optimizer from a standard mathematical analysis software package. In the present numerical work, a "homemade" Fortran routine is used. The approach taken to reduce the number of free parameters, which still allowing all five free blocks to move, was to represent the block shifts by a Fourier series in initial block angle. With this routine, and with the proper choice of the numbers of turns in each block, it was possible to null out the harmonics m=6, m=10, and m=14. Parameters for the 2-D layout are summarized in Table I.

Table I: Parameters for the 2-D Part of the Winding

Parameter	Value
Conductor width (as radially projected on the winding cylinder)	1.17 mm
Spacing between conductors	0.085 mm
Winding radius	188 mm
Total turns/pole	69
No. of winding blocks	6
No. of turns in blocks 1, 2, 3, 4, 5, and 6, respectively	20, 19,13 ,7 ,6 , and 4
Optimized block center angles for blocks 1, 2, 3, 4, 5, and 6, respectively	3.81,12.2, 20.57, 26.75, 32.22, and 38.65 deg.

3 END-TURN DESIGN

An additional set of variable parameters is introduced in the end-turn design stage. They are the numbers of groups into which each block of the 2-D winding fans out in the end windings and the numbers of turns in each group of each block. If a block has a relatively small number of turns (as is typical for the blocks near the pole center), the number of groups in the block can be 1; this means that the conductors in the block stay together in the end-winding zone.

In general, finer division of the blocks into many groups means lower higher harmonics. In a practical design, finer division must be traded off against costs of special spacing pieces between turns, bending radii, etc. The present test example turned out to have finer end-turn division than is needed to meet most field-quality specifications and should be iterated for fewer divisions with input from magnet designers. For example, Block 3 could have only one group, etc. The end-turn group numbers are summarized in Table II below.

Table II: End-Winding Block Fanout Specification

Block No.	No. of turns in block	No. of groups in block fanout	No. of turns in Group 1	No. of turns in Group 2	No. of turns in Group 3
1	20	3	3	5	12
2	19	2	9	10	-
3	13	2	6	7	-
4	7	1	7	-	-
5	6	1	6	-	-
6	4	1	4	-	-

Fig. 4 is the winding layout produced by the present approach. Group centerline curves were determined by first setting $c_g = \sin 2\phi_g$, where ϕ_g is the center angle of the group in the 2-D part of the winding. Then the group centerline contour in the end windings is just the curve determined by $\sin 2\phi\, f(z) = c_g$, with $f(z)$ given by Eqs. 3-4, and with L = 3.25, w = 0.3, and s = 0.5. Individual turn contours for each group were then generated by laying off constant distances (i.e., appropriate multiples of the conductor width and turn-to-turn gaps) from the group centerline curve along the normal to the group centerline curve in the developed surface of the cylinder.

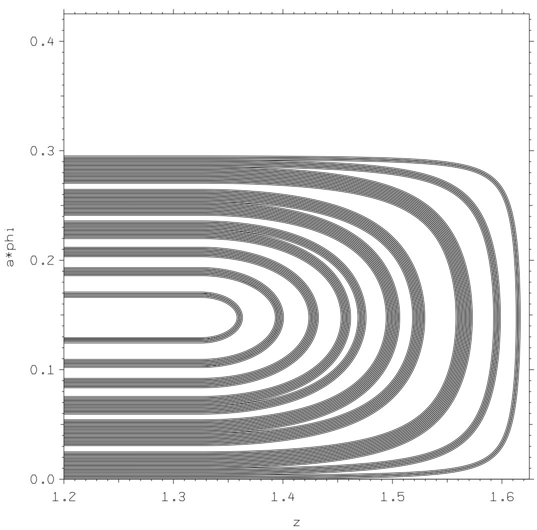

Fig. 4. End-turn layout on the developed winding cylinder with the shape function of Fig. 1, the 2-D design of Table I, and the block group definition of Table II, showing individual turns.

4 CALCULATION OF 3-D FIELD HARMONICS

Magnetic fields were computed by use of a Biot-Savart law algorithm in which individual turns were modeled by a collection of short (typically 2-mm) line segments in the ends. Single straight segments connecting the end turns were used to model the straight sections. With this model, effects of iron yokes are ignored, but it is easily shown that in the absence of nonlinear effects such as saturation effects, departures of the iron geometry from axisymmetry, etc., the ratios of higher harmonics to the fundamental m=2 harmonic are actually overestimated by the Biot-Savart field calculation if an iron yoke is present. This is a consequence of the fact that iron boosts the fundamental (m=2) harmonic component of the field in the bore more than it does higher harmonics.

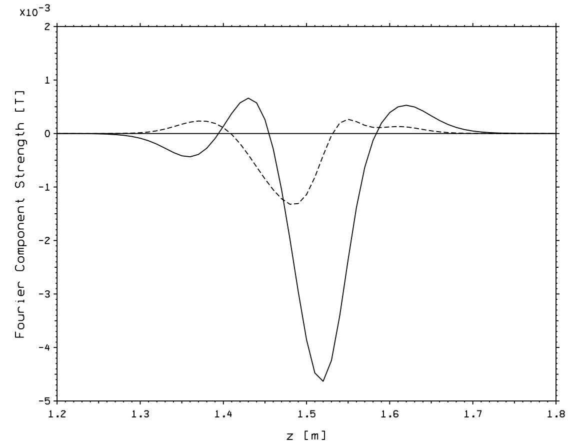

Fig. 5. Plot of the m=6 (solid curve) and m=10 (dashed curve) Fourier components of B_r in the end region of magnet. The straight part of the windings ends at z = 1.325 and the end of the windings is at z = 1.625.

Fig. 5 is a plots of the m=6 and m=10 Fourier components of the radial component of the field at the reference radius of 11.43 cm as a function of z for the end windings of Fig. 4. Curves for the azimuthal component are similar, as expected. Harmonic strengths should be compared to the fundamental m=2 value of 2.2 T at the reference radius in the center of the magnet. Higher harmonics are essentially zero in the 2–D part of the magnet. This serves as a check on the 2-D design, since it was based on Fourier analysis of the current distribution, not computed fields.

Values for the harmonics m=6 and m=10 integrated in z over a single end zone, divided by the fundamental m=2 field component integrated over the whole magnet, are 4.2×10^{-5} and 9.3×10^{-6}, respectively.

5 DISCUSSION

Probably, the most serious limitation of the field model of this paper is that a winding layer is treated as a current sheet of zero radial thickness with a radius equal to the mean radius of the finite-thickness layer. For magnets of the size of the AHF quadrupoles, this should not cause large errors. Also, for magnets with warm iron yokes, saturation effects should be small and should not significantly affect the results. Nevertheless, the results should be checked with a more realistic model with conductor turns with the actual radial depth.