

# TOUSCHEK LIFETIME OF A ROUND AND ELLIPTICAL ELECTRON BEAM

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*Abstract*

An approximate expression for the Touschek lifetime of an elliptical electron beam with an arbitrary ratio of height and width is presented for the convenience of practical applications. The dependence of the lifetime on the coupling constant of betatron oscillations are calculated with the formula for the SPring-8 storage ring having strong variation of lattice functions, and compared with the one of the conventional ribbon beam formula and also with experimental observations.

## 1 INTRODUCTION

In the third generation light sources, the emittance of electron beam is reduced as much as possible to produce a highly brilliant synchrotron radiation. In addition, the coupling of horizontal and vertical betatron oscillations is extremely reduced for a higher brilliance. As a consequence the bunch volume of electron beam is very small, so that the Touschek effect induced by the electron-electron collision in a beam bunch has become a dominant factor to determine the beam lifetime. This effect is especially serious in a lower energy storage ring since the effect becomes stronger drastically as the decrease of beam energy. And even in a higher energy storage ring, this effect is important for several bunched beam operation because of rather high electron density in a beam bunch. The lifetime can be increased with the coupling, but the brilliance is decreased. Therefore there is a compromise between the lifetime and brilliance in storage ring operation.

Theoretically, Touschek lifetime has mostly been discussed on a ribbon beam, where only the horizontal betatron oscillation is considered [1]. Recently, detailed discussion on the lifetime of an elliptical electron beam with an arbitrary ratio of height and width, considering horizontal and vertical oscillations, was reported [2], and an approximate formula of the lifetime was derived for practical convenience in parallel to the formalism given by Bruck [3]. It was also demonstrated that the experimental lifetime for different coupling constants in a small storage ring with slowly varying lattice functions agrees fairly well with the calculated one using the approximate formula [3]. In the present paper the lifetime of an electron beam in the SPring-8 storage ring, where the lattice functions vary very rapidly around the ring because of DBA structure, is discussed, and compared with experimental results as well as the ribbon beam formula.

## 2 THEORETICAL

Now we use the following definitions.  $r_0$  is the classical electron radius,  $c$  is the light velocity,  $N_e$  is the number of electrons per bunch,  $\gamma = E/m_0c^2$ ,  $E$  is the beam energy,  $m_0$  is the rest mass of electron,  $V_B$  is the bunch volume,  $\delta p_{x,y}$  is the horizontal and vertical momentum spread of electron beam, and  $\Delta p_{rf}$  is the RF bucket height. Momentum is given in the unit of  $m_0c$ .

In the case of a ribbon beam, Touschek lifetime is expressed as [1]

$$\frac{1}{\tau_T} = \frac{\sqrt{\pi} r_0^2 c}{\delta p_x (\Delta p_{rf})^2} \frac{N_e}{V_B} C(\epsilon) \quad (1)$$

with  $V_B = (4\pi)^{3/2} \sigma_x \sigma_y \sigma_z$  for the beam size  $\sigma_{x,y,z}$  and

$$C(\epsilon) = \epsilon \int_{\epsilon}^{\infty} \exp(-u) \frac{1}{u^2} \left[ \frac{u}{\epsilon} - 1 - \frac{1}{2} \ln \frac{u}{\epsilon} \right] du$$

$$= \begin{cases} -\ln \epsilon - 2.08 & \text{for } \epsilon < 0.01 \\ \exp(-y) & \text{for } \epsilon > 0.01 \end{cases} \quad (2)$$

where  $\epsilon = (\Delta p_{rf}/\gamma \delta p_x)^2$ , and  $y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 x^4$  with  $x = \ln \epsilon$ ,  $c_1 = 3.0897$ ,  $c_2 = 2.3783$ ,  $c_3 = 0.82021$ ,  $c_4 = 0.18332$  and  $c_5 = 0.0615$ .

In the case of an elliptical beam, when the horizontal and vertical betatron oscillations are considered, Touschek lifetime is expressed as [3]

$$\frac{1}{\tau_T} = \frac{r_0^2 c}{\gamma \Delta p_{rf}} \frac{1}{\delta p_x \delta p_y} \frac{N_e}{V_B} D(\epsilon, \zeta) \quad (3)$$

$$D(\epsilon, \zeta) = \frac{\sqrt{\epsilon}}{2} \int_0^{\infty} d\Phi \int_{\epsilon}^{\infty} \exp[-\Gamma(\Phi)u] \frac{1}{u\sqrt{u}} \left( \frac{u}{\epsilon} - 1 - \frac{1}{2} \ln \frac{u}{\epsilon} \right) du$$

where  $\Gamma(\Phi) = \cos^2 \Phi + (1/\zeta) \sin^2 \Phi$ ,  $\zeta = (\delta p_y/\delta p_x)^2 = \kappa \beta_x/\beta_y$ ,  $\kappa$  is the coupling constant and  $\beta_{x,y}$  is the beta function.  $D(\epsilon, \zeta)$  was calculated numerically for different  $\epsilon$  and  $\zeta$ .

With the transformation  $D(\epsilon, \zeta) = (\pi\zeta/\epsilon)^{1/2} H(\epsilon, \zeta)$ , we obtain the following formula,

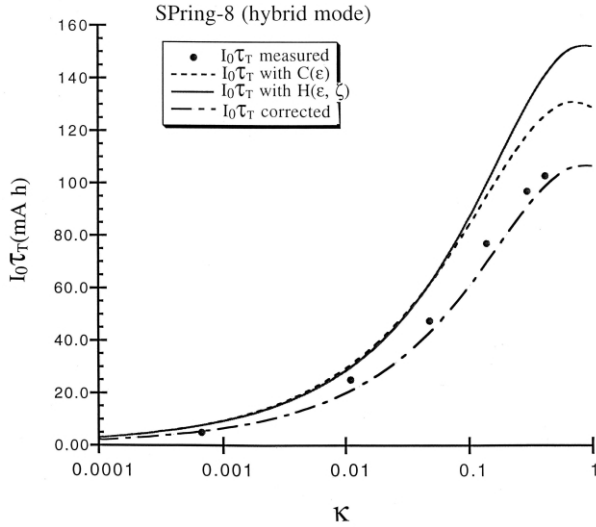


Fig.1 The Touschek lifetime in the SPring-8 storage ring calculated with  $H(\epsilon, \zeta)$  and  $C(\epsilon)$  and experimental values against coupling constant  $\kappa$ . The line-and-dot line represents the calculated lifetime with  $H(\epsilon, \zeta)$ , which is corrected considering the saturation of the lifetime, experimentally observed.

$$\frac{1}{\tau_T} = \frac{\sqrt{\pi} r_0^2 c}{\delta p_x (\Delta p_{rf})^2} \frac{N_e}{V_B} H(\epsilon, \zeta) \quad (4)$$

This is the same form as Eq.(1).  $H(\epsilon, \zeta)$  for  $\zeta/\epsilon < 0.1$  is expressed as

$$H(\epsilon, \zeta) = (1/2)[a_1 + a_2 X + (a_3 X^2 + a_4)^{1/2}], \quad (5)$$

where  $X = a_0 + \log_{10} \epsilon$ , with  $a_0 = 0.6920$ ,  $a_1 = -2.3258$ ,  $a_2 = -2.0504$ ,  $a_3 = 7.9408$  and  $a_4 = 10.867$ . It is shown that  $H(\epsilon, \zeta) = C(\epsilon)$  for  $\epsilon < 0.01$ .  $H(\epsilon, \zeta)$  for  $\zeta/\epsilon > 0.1$  is expressed as

$$H(\epsilon, \zeta) = -\ln(8.0\epsilon)[H_n(\epsilon, \zeta) / 2.30], \quad (6)$$

$$H_n(\epsilon, \zeta) = (1/2)[b_1 + b_2 Y + b_3(Y-2.5)^3 + b_4(Y-2.5)^5] \quad (7)$$

where  $Y = \log_{10}(\zeta/\epsilon/10)$ ,  $b_1 = 5.5700$ ,  $b_2 = -1.1947$ ,  $b_3 = 0.06489$  and  $b_4 = -0.0013898$ . Note that  $C(\epsilon) \equiv -\ln(8.0\epsilon)$  for  $\epsilon < 0.01$ , so that  $H(\epsilon, \zeta) = C(\epsilon)$  for  $H_n(\epsilon, \zeta) = 2.30$ . It is noted that  $H_n(\epsilon, \zeta) \approx 2.3$  for  $\zeta/\epsilon < 10$  and  $H_n(\epsilon, \zeta)$  decreases gradually with  $\zeta/\epsilon$ .

Since  $\delta p_x$ ,  $V_B$  and  $H(\epsilon, \zeta)$  depend on the position along the circumference of storage rings, it is necessary to take the average of Eq.(4) around the ring.

### 3 NUMERICAL EXAMPLE

The SPring-8 storage ring is one of the third generation light sources with a small emittance of 7 nm-rad operated at 8 GeV. The lattice is of the double bend achromatic type, so that the lattice functions  $\beta_{x,y}$  and  $\eta_x$  vary rapidly around the ring. The ring is operated in the hybrid mode; high and low beta functions in the horizontal direction appear alternately around the ring. In addition, there are four long straight sections in the storage ring, where bending magnets of the double bends are removed. The beta functions in the straight sections are almost the same as those of normal cells, but there are no dispersions in the long straight sections. All these complexities of the lattice functions are taken into account to calculate the Touschek lifetime.

The beam lifetime was measured by varying the coupling constant in the full range by skew quadrupole magnets. Experimental data, picked up from Ref.[4], are shown with solid marks in Fig.1, which was obtained under the experimental condition with a beam current of 1 mA per bunch in the 21 bunch operation, the chromaticity  $\xi_x=3$  and  $\xi_y=4$ , and an RF voltage of about 12 MV. Note that the lifetime is almost the same as that at 1 mA in a single bunched beam operation, so that the effect of the residual gas scattering is negligible. The solid and broken lines in the figure represent the Touschek lifetime for  $V_{rf}=12$  MV calculated with  $H(\epsilon, \zeta)$  and  $C(\epsilon)$ , respectively. We observe that the lifetimes calculated with  $H(\epsilon, \zeta)$  and  $C(\epsilon)$  are longer than the experimental one about 30 and 20 % at  $\kappa \approx 1$ , respectively.

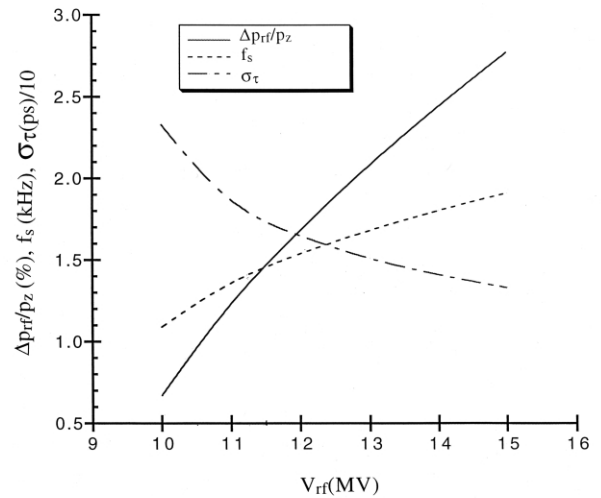


Fig.2 RF voltage dependence of the RF bucket height  $\Delta p_{rf}/p_z$ , synchrotron oscillation frequency  $f_s$ , and bunch length  $\sigma_\tau$ .

It is rather surprising that the lifetime calculated with  $C(\epsilon)$  is close to that with  $H(\epsilon, \zeta)$ . The difference appears for  $\kappa > 0.1$ , but only about 10 % even at the full coupling in spite of the fact that only the horizontal betatron oscillation is considered for  $C(\epsilon)$ .

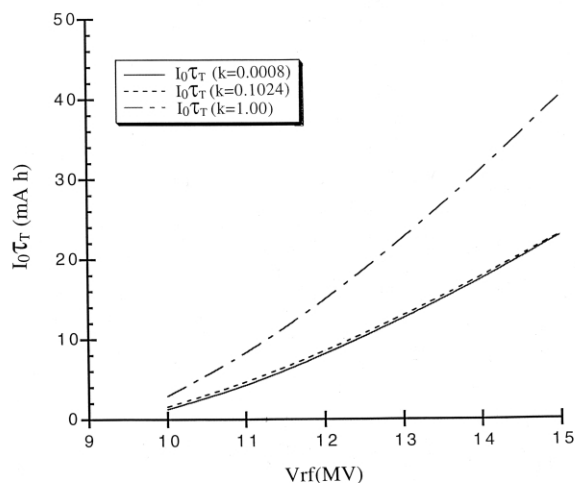


Fig.3 RF voltage dependence of the Touschek lifetime calculated with  $H(\epsilon, \zeta)$  for different coupling constants.

Since  $\epsilon < 0.012$  in the present case, this difference comes from the difference between the value of  $H_n(\epsilon, \zeta)$  in the flat region and in the decreasing region against  $\zeta/\epsilon$ . By the definition of  $\epsilon$  and  $\zeta$ , we have

$$\zeta/\epsilon = (\gamma \delta p_y / \Delta p_{rf}) = [\gamma / (\Delta p_{rf} / p_z)]^2 (\epsilon_y / \beta_y) \quad (8)$$

where  $p_z$  is the longitudinal momentum. The vertical emittance is  $\epsilon_y \approx 3.5$  nm-rad at  $\kappa=1$ , and  $\beta_y=10\sim 25$  with  $\langle \beta_y \rangle \approx 18$  m. Then we have  $\zeta/\epsilon \approx 170$ , at which  $H_n(\epsilon, \zeta) \approx 1.9$ . Accordingly the ratio of the lifetime of the present result and the ribbon beam formula at  $\kappa=1$  is understood to be about 1.2 or less.

The ring is usually operated with a coupling constant of about 0.001 to increase the radiation brightness. Accordingly, the lifetime is as short as about 7 h at 1 mA per bunch. It is desired to increase the lifetime in place of increasing the vertical emittance. Because of the weak and strong dependence of Touschek lifetime on the bunch length and RF bucket height, respectively (see Fig.2), the lifetime calculated with  $H(\epsilon, \zeta)$  increases with the RF voltage, as shown in Fig.3.

According to the experiment, however, the lifetime decreases gradually above an RF voltage of 11~12 MV, which is possibly caused by the dynamic aperture determined by momentum deviation. In addition, it is noted that the measured synchrotron oscillation frequency at the turning point of the lifetime against the RF voltage is about 1.4 kHz [5].

This implies that the critical RF voltage for the saturation is about 11.2 MV (see Fig.2). The lifetime

calculated with  $H(\epsilon, \zeta)$  for the RF voltage 11.2 MV is about 30 % lower than that for 12 MV (see Fig.3). In consequence, the calculated lifetime shown in Fig.1 should be lowered as a whole by about 30 %, which agrees fairly well with the experimental one as represented by a line-and-dot line.

## 4 CONCLUSION

Touschek lifetime in the SPring-8 storage ring was calculated with a new formula as a function of coupling constants. The rapid variation of lattice functions in the storage ring was taken into account. The calculated lifetime is about 30 % longer than the experimental one. It was experimentally observed that the lifetime saturates by some mechanism at a higher RF voltage. When this saturation is taken into account, the calculated lifetime agrees fairly well with the experimental one. Thus, in the present paper it was demonstrated that the Touschek lifetime of an elliptical electron beam can be calculated fairly well with the approximate formula even in the case of a storage ring with a rapid variation of lattice functions.

## ACKNOWLEDGMENT

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