HIGH CURRENT SINGLE BUNCH TRANSVERSE INSTABILITIES AT THE ESRF: A NEW APPROACH

P. Kernel, R. Nagaoka, J.-L. Revol, ESRF, Grenoble, France
G. Besnier, University of Rennes, France

Abstract

Single bunch operation at the ESRF is severely limited by vertical coherent instabilities. The user requirement (15 mA) is much above the mode-coupling instability threshold (0.7 mA), observed at a low chromaticity. The nominal current is reached by increasing the vertical chromaticity to a large positive value, and by lengthening the bunch with a reduction of the RF voltage. The transverse feedback implemented and tested at various chromaticities is not quite as efficient. Although the classical head-tail theory could be applied below the threshold, it does not explain the hard edged and higher threshold observed. The measured fast growth of the instability invalidates the notion of synchrotron motion and in consequence the notion of head-tail modes. The idea of a post-head-tail mechanism is introduced. The theory, simulation results and experimental verifications are presented. A model of the machine impedance is also discussed.

1 INTRODUCTION

Initially efforts were concentrated towards the classical head-tail theory in order to understand the ESRF vertical instability threshold. The discrepancies between the predictions of the head-tail theory and the observations realised at the ESRF led us to imagine that the ESRF does not evolve in the head-tail regime in the vicinity of the threshold at positive chromaticity. Measurements and tracking simulations confirmed that the implied instability in the threshold mechanism is much faster than the synchrotron motion. The head-tail theory, which deals with collective modes based on the synchrotron motion, is not appropriate to describe such a phenomenon. A post-head-tail theory that considers instabilities faster than the synchrotron motion is proposed in this paper. The theoretical post-head-tail intensity threshold is consistent with the measured one at the ESRF.

2 LIMIT OF THE HEAD-TAIL THEORY

At positive vertical chromaticity ($\xi_v \geq 0.2$)[1], the mode coupling no longer appears because the low-order head-tail modes ($m = 0$, $m = -1$) are damped, and the higher-order modes are too weak to present a sufficient detuning to couple.

[1] $\xi_v = (\mathrm{d}Q_\beta / Q_{\beta 0}) / (\mathrm{d}E / E)$ is the **reduced chromaticity**

As a consequence, the instability mechanism that could be envisaged at this chromaticity is a head-tail instability of decoupled head-tail modes. By increasing the chromaticity, successive head-tail modes interact with the real negative impedance in the negative frequency domain and become unstable. The most unstable head-tail mode would then define the instability threshold when its theoretical growth-time becomes shorter than the radiation damping time. It appears that the ESRF theoretical threshold would be very low, ten times lower than the measured one. In addition, the measured growth-time at threshold is much shorter than the radiation-damping time, equal to 7 ms at the ESRF.

Therefore, it must be considered that another stabilising effect exists at the ESRF in addition to the radiation damping. This effect appeared to be the large spread in incoherent synchrotron frequency induced by the distortion of the RF-potential by the longitudinal collective effect. The tracking simulation confirms that the **incoherent synchrotron frequency spread** is large enough to entirely stabilise the higher-order head-tail modes. **At the ESRF, the head-tail regime is stable.**

In this condition, the head-tail regime can reach its limit. This occurs when the theoretical growth-time of the instability becomes shorter than the synchrotron period. After this, the classical head-tail theory, based on the synchrotron motion, is no longer valid (all the head-tail modes must be considered mixed-up). A theory in which the longitudinal motion is restricted to the particle diffusion due to the energy dispersion can be used. This theory, called hereafter "**post-head-tail**" **theory** (PHT) is demonstrated below. The transition from head-tail to post-head-tail regimes is illustrated in figure 1: at large intensity, the measured synchrotron satellites (characteristic of the head-tail regime) disappear, and a single broad peak remains.

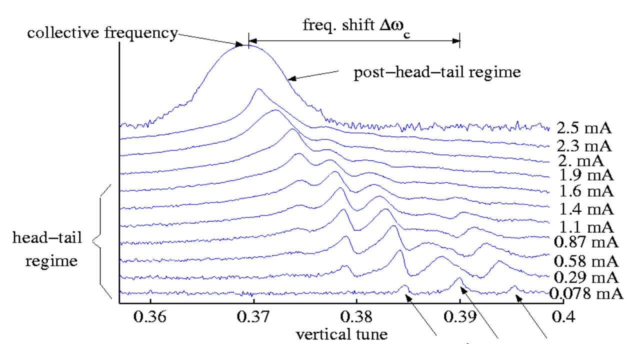

Figure 1: Transition from head-tail to post-head-tail

3 POST-HEAD-TAIL THEORY

The post-head-tail regime considers instability faster than the synchrotron motion. Therefore for this regime, the longitudinal motion is restricted to:

$$\tau = \tau_0 + \dot{\tau}\, t \quad , \quad \dot{\tau} = \alpha \frac{\delta E}{E} \quad , \quad \ddot{\tau} = 0$$

This consideration is similar to the longitudinal motion of a coasting-beam. Therefore it is possible to deduce the post-head-tail regime from the coasting beam results (the complete demonstration is presented in [1]). For the post-head-tail theory a bunched beam is considered.

3.1 Mono-kinetic Beam ($\dot{\tau} = 0$)

A monochromatic excitation at $\omega_p = \omega_{\beta 0} + p\omega_0$ of a mono-kinetic (without energy dispersion) **coasting beam**, gives a response at the same frequency[2]. The response can be written $\sigma_{p'} = \delta_{p,p'}\, \sigma_p$. Where $\sigma_{p'}$ represents the amplitude of the beam spectrum line at the frequency $\omega_{p'}$.

The frequency response of a **bunched beam** to the same excitation presents many harmonic lines spaced by the revolution frequency. The response can be written with a matrix: $\sigma_{p'} = \mathbf{A}_{p,p'}\, \sigma_p$. For Gaussian beam:

$$\mathbf{A}_{p,p'} = \exp - \tfrac{1}{2}[(\omega_p - \omega_{p'})^2 \sigma_\tau^2] \qquad \text{(1 PHT)}$$

For an excitation from a **narrow-band impedance**, the **coasting beam** (CB) result is recalled:

$$\Delta\omega_{cp'}^0 \sigma_{p'} = \Lambda \delta_{p,p'}\, \mathrm{j} Z_\perp(\omega_p) \sigma_p \quad , \quad \Lambda = \frac{c\, I}{4\pi\, Q_{\beta 0}\, E/e} \qquad \text{(2CB)}$$

The stability of a collective motion is given by the imaginary part of the collective frequency shift $\Delta\omega_{cp}^0$. From (2 CB), the bunched beam formula, for narrow-band impedance is deduced:

$$\Delta\omega_{cp'}^0\, \sigma_{p'} = \Lambda \mathbf{A}_{p,p'}\, \mathrm{j} Z_\perp(\omega_p) \sigma_p \qquad \text{(2 PHT)}$$

In the case of **distributed impedance**, the principle of superposition leads to a **matrix equation**:

$$\Delta\omega_c^0 \sigma_{p'} = \Lambda \sum_p \mathbf{A}_{p,p'}\, \mathrm{j} Z_\perp(\omega_p) \sigma_p \qquad \text{(3 PHT)}$$

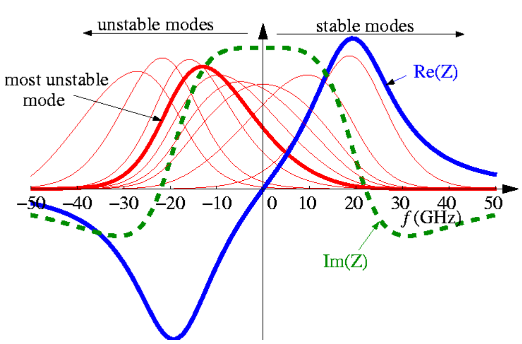

Figure 2: Computed mono-kinetic spectra

The matrix equation (3 PHT) is solved numerically, (figure 2). It appears that the modes amplitude spectra can be approximated by the Gaussian "shaker modes" $\sigma_q = \exp - \tfrac{1}{2}[(\omega_p - \omega_q)^2 \sigma_\tau^2]$.

The shaker modes are the beam response to a monochromatic excitation at ω_q. Therefore they are the eigenvectors of the matrix $\mathbf{A}_{p,p'}$, the corresponding eigenvalue is $C_q = \frac{2\sqrt{\pi/3}}{\omega_0 \sigma_\tau}$.

When the eigen-modes of the matrix equation are approximated by the shaker modes:

$$\boxed{\Delta\omega_{cq}^0 = \Lambda C_q\ \mathrm{j} Z_{\perp q}^{\mathrm{eff}}} \qquad \text{(4 PHT)}$$

with the definition of the **effective impedance**:

$$Z_{\perp q}^{\mathrm{eff}} = \frac{\sum_p Z_\perp(\omega_p) \sigma_q^2(\omega_p)}{\sum_p \sigma_q^2(\omega_p)} \qquad (5)$$

The effective impedance is computed with the **most unstable** mode peaked at $\omega_q = -11$ GHz, (figure 2). Adding the contributions of upper and lower sidebands, mono-kinetic beams are unstable with respect to any resistance.

3.2 Beam with energy dispersion

The spread in energy induces an incoherent dispersion of the betatron frequencies $\omega_\beta(\dot{\tau}) = \omega_{\beta 0} + (\omega_\xi - \omega_{\beta 0})\dot{\tau}$, which one hopes that it stabilises the post-head-tail instability, where $\omega_\xi = \xi_v\, \omega_{\beta 0} / \alpha$ is the chromatic modulation. With Gaussian energy dispersion $\gamma_0(\dot{\tau})$, every line $\omega_{p'}$ of the mono-kinetic bunch is **widened** and becomes **a band** of r.m.s. width $|\omega_{p'} - \omega_\xi|\sigma_{\dot{\tau}}$, (figure 3). For a bunched beam with energy spread, the previous mono-kinetic matrix equation (3 PHT) becomes:

$$J_{p'}^{-1}(\Delta\omega_c)\sigma_{p'} = \Lambda \sum_p \mathbf{A}_{p,p'}\, \mathrm{j} Z_\perp(\omega_p) \sigma_p \qquad \text{(6 PHT)}$$

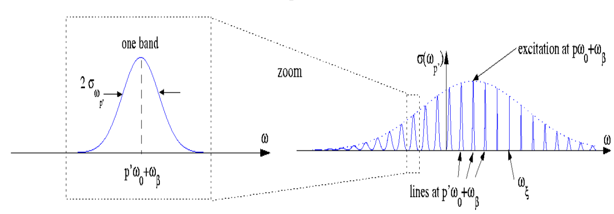

Figure 3: Widened bands with energy dispersion

The complex frequency shift is now hidden in the **dispersion integral**:

$$J_{p'}(\Delta\omega_c) = \int \frac{\gamma_0(\dot{\tau})}{\Delta\omega_c - (\omega_{p'} - \omega_\xi)\dot{\tau}} \mathrm{d}\dot{\tau} \qquad \text{(7 PHT)}$$

By inserting the shaker modes σ_q in the matrix equation, **a dispersion integral relation** is obtained:

$$\boxed{\Delta\omega_{cq}^0\, J_q(\Delta\omega_{cq}) = 1} \qquad \text{(8 PHT)}$$

The collective frequency shift for a beam with dispersion is $\Delta\omega_{cq}$, and for a mono-kinetic beam it is $\Delta\omega_{cq}^0$.

[2] J.L. Laclare's formalism is considered [2].

The dispersion integral $J_q(\Delta\omega_{cq})$ is computed at the limit of stability $\mathrm{Im}(\Delta\omega_{cq}) = 0$, with $\mathrm{Re}(\Delta\omega_{cq})$ as a variable parameter. The result is plotted in a reduced impedance plane:

$$W = U + jV = \frac{\Lambda C_q Z_{\perp q}^{\mathrm{eff}}}{\sqrt{2}|\omega_\xi - \omega_q|\sigma_{\dot{\tau}}} = \frac{-j\Delta\omega_{cq}^0}{\sqrt{2}|\omega_\xi - \omega_q|\sigma_{\dot{\tau}}}$$

with $\Delta\omega_{cq}^0 = J_q^{-1}(\Delta\omega_{cq})$. The stability domain of the mono-kinetic beam is reduced to the imaginary axis $U = 0$ (any resistance is unstable). With energy spread, the stability area is widened between the two curves with a maximum excursion $1/\sqrt{\pi}$, (figure 4).

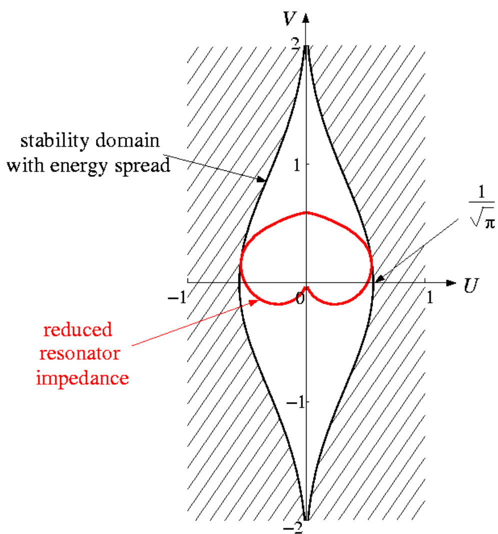

Figure 4: Stability region of the reduced impedance plane

The reduced impedance curve is the adapted image of a resonator impedance and is close to a circle. Therefore the stability condition is approximated by **the circle criterion**:

$$|W| \le 1/\sqrt{\pi} \quad , \quad |\Delta\omega_{cq}^0| \le \sqrt{\frac{2}{\pi}}\,|\omega_\xi - \omega_q|\sigma_{\dot{\tau}} \qquad \text{(9 PHT)}$$

The physical mechanism of a transverse Landau-damping is recovered: stability requires that the collective frequency (shifted proportionally to intensity) is kept inside the incoherent betatron frequency band (of width proportional to the chromaticity).

The **post-head-tail intensity threshold** is then:

$$\boxed{I_{th} = \frac{4(E/e)\alpha\frac{\sigma_E}{E}(I_{th})|\omega_\xi - \omega_q|\sigma_\tau(I_{th})}{\sqrt{2/3}\,|\beta Z_{\perp q}^{\mathrm{eff}}|}} \qquad \text{(10 PHT)}$$

We found that this equation was already proposed by R.D. Ruth and J.M. Wang with a different approach [3].

4 APPLICATION AT THE ESRF

From equation (10 PHT), the ESRF intensity threshold is then computed. The effective impedance used $\beta Z_{\perp q}^{\mathrm{eff}} = 8\ \mathrm{M\Omega}$, is in agreement with the one fitted from the mode-coupling measurement [4]. The results plotted in figure 5 (left) are in agreement with the measured intensity threshold. The corresponding spontaneous tune at threshold $Q_{th} = \frac{\omega_{\beta 0} + \Delta\omega_{cq}^0}{\omega_0}$, can be computed with the equation (4 PHT). It is also in agreement with the measurement, as in figure 5 (right).

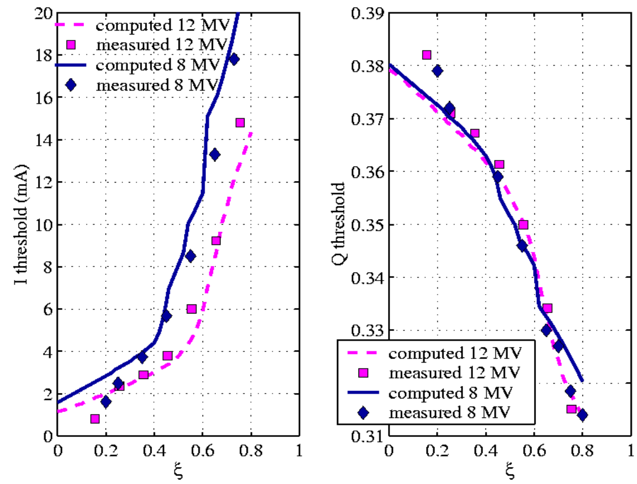

Fig.5: Theoretical and measured I_{th} (left)
Theoretical and measured Q_{th} (right)

5 CONCLUSION

At the ESRF, the coherence of the head-tail modes is broken by the large spread of incoherent synchrotron frequency, induced by the longitudinal collective effect. The longitudinal impedance helps to damp the effects of the transverse one.

Moreover, a limit of the head-tail regime was identified to appear when the theoretical instability growth-time is shorter than the synchrotron period. At any positive vertical chromaticity ($\xi_v \ge 0.2$), above the mode-coupling regime, a post-head-tail instability limits the maximum current at the ESRF. The foreseen threshold complies with the measured one at the ESRF.

The post-head-tail regime might not be a characteristic of the ESRF only. Other light sources, with large impedance and working at high chromaticity, might also be limited in single bunch by the post-head-tail instability.

REFERENCES

[1] P.Kernel, “A Contribution to a New Consideration of Single Bunch Transverse Instabilities in Storage Ring Light Sources”, PhD Thesis, J.Fourier University, Grenoble, France.

[2] J.L.Laclare, “Coasting Beam Coherent Transverse Instabilities”, in C.A.S. CERN/94-01, Switzerland.

[3] R.D.Ruth and J.M.Wang, “Vertical Fast Blow-Up in Single Bunch”. In IEEE Transactions on Nuclear science, 1981.

[4] R. Nagaoka et al., “Transverse Instabilities in the ESRF Storage Ring: Simulation, Experimental Results and Impedance Modelling”, in PAC’99 proc