QUADRUPOLE WAKE-FIELD EFFECTS IN THE KEKB LINAC

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Abstract

Quadrupole wake-field effects were observed for the first time in a single-bunch, high-intensity primary electron beam for positron production at the KEKB linac. The observed effects, which emerged as x-y coupling in transverse motion, were in good agreement with a simple calculation based on a two-particle model. The observational results and the calculation are presented with special emphasis on the parametric-excitation nature of the phenomena.

1 INTRODUCTION

Wake-field instability concerning a high-intensity electron linac is considered to be one of the most crucial issues for stably operating the existing linacs, such as the KEKB linac [1], as well as of future linear colliders. In this connection, some experimental investigations of the linac instability have so far been carried out, while focusing mainly on a transverse dipole wake-field effect and its cure. The quadrupole wake-field effects in linacs have been considered to be negligibly small, simply because they are essentially second-order phenomena. A beam of increasing intensity, such as the 8-nano Coulomb single-bunched beam in the primary high-intensity electron linac for positron production at the KEKB linac, however, may allow the quadrupole wake-filed effects to emerge as observable x-y coupling phenomena of parametric-excitation nature. Simple calculations based on the two-particle model as well as the first experimental results obtained for a high-intensity single-bunched electron beam at the KEKB linac are presented.

2 TWO-PARTICLE MODEL

2.1 Equation of Motion

Consider a round single-bunch electron beam in transverse space, which is accelerating under a constant gradient (g) with an initial energy of γ_0 in the presence of an external smooth focusing system:

- $\gamma = E / mc^2 = \gamma_0 + g\, s$,
- $\frac{d}{ds}\left(\gamma \frac{dx}{ds}\right) + \gamma k^2 x = F$,

where E is energy, m the electron mass, c the speed of light, k the betatron wave number, F the external force due to a transverse wake field, and s is the distance down the linac.

To simplify the effects of the transverse wake in a single-bunch beam, let us introduce a rigid two-particle model [2]. Half of the charge is placed into each macro-particle, while two particles are separated by a distance of about $2\sigma_z$ of the longitudinal bunch distribution, which is assumed to be rigid in the relativistic region. The wake fields at the trailing particle generated by the leading particle are thus fixed; the dipole wake, $W_1(2\sigma_z)$ and the quadrupole wake, $W_2(2\sigma_z)$ are constant in this model.

The round-shaped beam has quadrupole moments (Q_n, Q_s) as well as dipole moments (D_x, D_y) due to a transverse offset of (x_1, y_1) of the leading particle:

- $D_x = x_1$, $D_y = y_1$,
- $Q_n = x_1^2 - y_1^2$, $Q_s = 2\, x_1\, y_1$,

while the transverse wake-field effects at the trailing particle (x_2, y_2) are characterised as the external-force terms:

- $F_x = -Nr_0/2\, (W_1\, D_x + 2W_2 Q_n\, x_2 + 2W_2 Q_s\, y_2)$,
- $F_y = -Nr_0/2\, (W_1\, D_y - 2W_2 Q_n\, y_2 + 2W_2 Q_s\, x_2)$,

where N is the total number of particles in a bunch, and r_0 is the classical electron radius (r_0 = 2.818 10^{-15} m).

The equations of motion for the two particles are written as follows:

$$\frac{d}{ds}\left(\gamma \frac{dx_1}{ds}\right) + \gamma k^2 x_1 = 0,$$
$$\frac{d}{ds}\left(\gamma \frac{dy_1}{ds}\right) + \gamma k^2 y_1 = 0,$$
$$\frac{d}{ds}\left(\gamma \frac{dx_2}{ds}\right) + \gamma k^2 x_2 = F_x,$$
$$\frac{d}{ds}\left(\gamma \frac{dy_2}{ds}\right) + \gamma k^2 y_2 = F_y.$$

2.2 x-y Coupling and Parametric Excitation

The equation of motion indicates two distinct features for quadrupole wake-field effects:

- x-y coupling phenomena mainly relevant to the Q_s terms,
- parametric excitation nature explicitly related to the Q_n terms.

Although the effects are mutually dependent so that, rigorously, they can not be separated from each other, let us consider two extreme cases for numerical simulations in order to verify that the quadrupole wake-field effects at the KEKB linac are really observable:

- Case I : $x_1^2 = y_1^2$, that is, $Q_n = 0$,
- Case II: $x_1^2 >> y_1^2$, that is, $Q_s \cong 0$.

2.3 Numerical Simulation

The linac parameters used for numerical simulations relevant to the KEKB linac operation configuration at the first two sectors, A and B [3], are summarized in Table I:

Table 1: Linac parameters

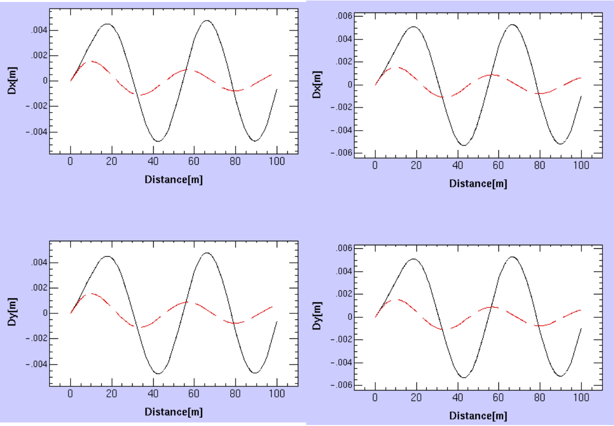
Parameter	**Symbol**	**Value**
Number of Particles	N	$5\ 10^{10}$ (8 nC)
Bunch Length	σ_z	1.5 mm
Betatron Wave Number	k	$2\pi/46$ 1/m
Initial Energy	γ_0	230/0.511
Energy Gain	g	20/0.511 1/m
Dipole Wake	$W_1(2\sigma_z)$	$4.5\ 10^5$ 1/m^3
Quadrupole Wake	$W_2(2\sigma_z)$	$5.0\ 10^9$ 1/m^5

Case I: $Q_n = 0$

To clarify the quadrupole wake-field effects, two simulation results with/without W_2 are compared for the same initial conditions:

- $x_1 = y_1 = 0$, $x_1' = y_1' = 0.3$ mrad ($s = 25.4$ m).

Figure 1 clearly shows mutual x-y coupling effects due to the quadrupole wake. The relative amount of about 12% increase in the orbit oscillation is attributed to the quadrupole wake-field effect.



(a) (b)

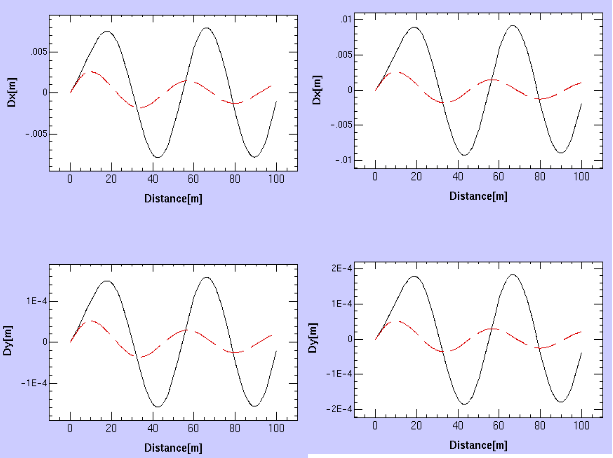
Fig. 1: Simulations for x-y coupling phenomena due to the quadrupole wake-field effects in linacs. The betatron oscillations were calculated for the KEKB linac by the simulation based on the two-particle model using the linac parameters given in Table I. The initial conditions are set to $x_1 = y_1 = 0$, and $x_1' = y_1' = 0.3$ mrad ($s = 25.4$ m), corresponding to Case I ($Q_n = 0$). (a) $W_2 = 0$, and (b) $W_2 \neq 0$. The orbit oscillation in (b) is larger than that in (a) by about 12%, which is attributed to the quadrupole wake-field effects. The dashed line shows a simulation without any wake fields.

Case II: $Q_s \cong 0$

In this case, the initial conditions are set so that the coupling from the vertical to horizontal direction is small enough to evaluate any horizontal effect of parametric-excitation nature:

- $x_1 = y_1 = 0$, $x_1' = 0.5$ mrad, $y_1' = 0.01$ mrad ($s = 25.4$ m).

It is shown in figure 2 that the horizontal oscillation increases by about 15% due to parametric excitation, while the vertical oscillation also increases by about 13% through x-y coupling from a large horizontal oscillation.



(a) (b)

Fig. 2: Simulations for the parametric-excitation nature of the quadrupole wake-field effects in linacs. The betatron oscillations were calculated for the KEKB linac by a simulation based on the two-particle model using the linac parameters given in Table I. The initial conditions are set to $x_1 = y_1 = 0$, $x_1' = 0.5$ mrad, $y_1' = 0.01$ mrad ($s = 25.4$ m), corresponding to the Case II ($Q_s \cong 0$). (a) $W_2 = 0$, and (b) $W_2 \neq 0$. The horizontal oscillation increases due to the parametric excitation by about 15% compared to (a), while the vertical oscillation also increases by about 13% through x-y coupling from a large horizontal oscillation. The dashed line shows a simulation without any wake fields.

Through these two simulation cases, it is verified that the quadrupole wake-field effects could emerge as observable phenomena at the KEKB linac.

3 OBSERVATION

The situation in the simulation conditions, however, should not be realistic for a direct observation of the quadrupole wake-field effect at the KEKB linac, since in a real machine the quadrupole wake field (W_2) could not be eliminated as in simulations, and any comparison with/without W_2 could be totally impossible.

Measurements were therefore carried out so that a horizontal kick was applied to the beam at the initial point of an energy of 230 MeV in the middle of sector A; the resulting vertical betatron oscillation was observed down the linac in sectors A and B by a beam position monitor (BPM) system. Figure 3 shows the x-y coupling effects observed in the vertical orbit when a horizontal kick of about 0.3 mrad was set to the 8-nC beam at the initial point. The solid line indicates a simulation using the parameters listed in Table I.

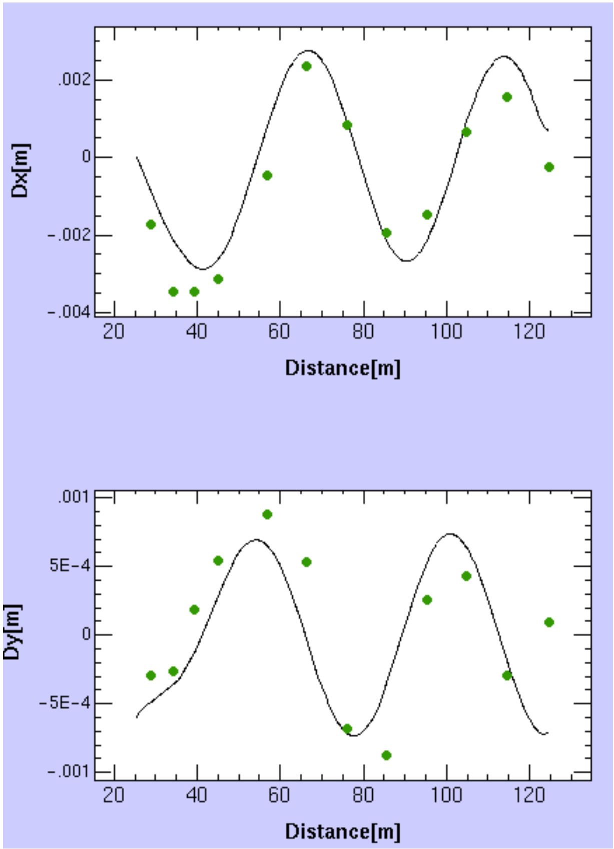


Fig. 3: The x-y coupling effects observed in the vertical orbit when a horizontal kick of about 0.3 mrad was set to the 8-nC beam at the initial point of an energy of 230 MeV (s = 25.4 m). The solid line indicates the simulation results using the parameters listed in Table I.

In order to check the results, the same horizontal kick was applied to a low-current beam. No vertical betatron oscillation due to x-y coupling was observed within the precision of the BPM system (< 0.1 mm). This signifies that the observed x-y coupling for an 8-nC beam is inherent in the high-intensity beam and could be of wake-field origin. On the other hand, the x-y coupling caused by a rotational misalignment of the steering coil used for a single kick was estimated to be smaller than 0.03%. In the case of a high-current beam, however, this coupling could be enhanced through the dipole wake-field term (W_1D_y) in the equation of motion, although the above estimation assures that the term could not be as large as the quadrupole wake-field terms. Therefore, the observed results concerning the x-y coupling might comprise both effects originating from dipole and quadrupole wake fields, regardless of the extent of their contribution.

Since the separation of these two effects in the observation is in principle difficult, we rely on the agreement between the observation and the simulation: figure 3 shows that the agreement is quite good within the experimental errors. We may conclude that quadrupole wake-field effects were surely observed for the first time at the KEKB linac.

4 CONCLUSIONS

The quadrupole wake-field effects were calculated for the KEKB linac using a rigid two-particle model. The simulation showed that the effects could be observable for an 8-nC beam at the KEKB linac if the initial conditions are appropriately chosen. Taking into account the recipe obtained by the simulation, measurements for x-y coupling effects were carried out. The obtained results are in good agreement with the simulation, suggesting that quadrupole wake-field effects were observed for the first time at the KEKB linac. Further investigation on the quadrupole wake-field effects will be continued in order to clarify the parametric-excitation nature as well as x-y coupling phenomena.

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