APPLICATION OF WIGGLERS TO QUASI-ISOCHRONOUS TRANSPORT SYSTEMS

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Abstract

The transition energy of a circular machine based on standard FODO lattices depends on the horizontal optics of the cell and, therefore, on the average radius of the machine. This property imposes tight constraints on the design of isochronous machines that are frequently considered for applications such as electron and proton accumulators or muon colliders. The price to be paid to satisfy the isochronous condition is to have a dispersion function with very large peak values along the machine circumference. In this paper, a number of optical modules, which we call *wigglers*, in analogy with the devices used in electron machines, are presented that overcome such difficulties. A careful analysis of their properties is carried out. Using these new concepts, it is shown how to design isochronous machines for which the machine radius and the transition energy are independent of each other, while keeping the value of the dispersion function under control.

1 GENERAL FRAMEWORK

An important parameter of a circular machine is its radius. In machines operating at transition, appropriate lattices with low value of the dispersion function may be designed using wigglers, which are modules having dipole magnets with both positive and negative radius of curvature. Wigglers can be applied to proton machines of large radius and relatively low energy. Such a configuration is similar to that of electron rings. The underlying physical arguments are nevertheless different. In electron machines, the large radius and the wigglers are needed because of the effects of the synchrotron radiation. In proton machines, the large radius is necessary, for instance, to reduce the number of foil traversals to convert H^- ions into protons during the injection process. In that case, the wigglers affect the orbit dispersion, thus allowing small beam pipe aperture.

Having determined the overall machine geometry, two options are available, depending on the value of the slip factor

$$\eta = \frac{\Delta T/T}{\Delta \mathrm{p}/\mathrm{p}} = \gamma^{-2} - \gamma_{\mathrm{tr}}^{-2} \tag{1}$$

where γ is the relativistic factor and γ_{tr} the γ-value at transition energy: (*i*) isochronous (or quasi-isochronous) machine, in which the revolution frequency has no, or a weak dependence on momentum spread and $|\eta|$ is zero or small; (*ii*) non-isochronous machine, in which $|\eta|$ is far from zero.

The transition energy of a machine based on a FODO lattice is a function of the horizontal optical parameters through the approximate relation [1]

$$\gamma_{\mathrm{tr}} \approx Q_h \left[\frac{\sin \mu_h/2}{\mu_h/2}\right], \tag{2}$$

where Q_h, μ_h are the horizontal tune and the phase advance per cell respectively. Furthermore, the expression for the momentum compaction factor [1]

$$\alpha = \gamma_{\mathrm{tr}}^{-2} = \frac{\Delta C/C}{\Delta \mathrm{p}/\mathrm{p}} = \frac{1}{C} \oint \frac{\mathrm{D}(s)}{\rho(s)} d\,s \tag{3}$$

indicates that η is a function of D, Q_h, the bending radius ρ and the machine energy. This is a rather tight constraint in case of a large isochronous machine working at low energy.

2 ORBIT DISPERSION AND MOMENTUM COMPACTION

The constraint imposed by the choice of the bending radius has a critical effect on the value of the horizontal dispersion function D in the ring. Using Eq. (3) and assuming infinite curvature radius outside the dipoles and a constant value ρ_n in the n*th* bending magnet, the momentum compaction factor α can be written as

$$\alpha = \frac{1}{C} \sum_{n=1}^{N} \frac{1}{\rho_n} \int_{\mathrm{n}th\mathrm{bend}} \mathrm{D}(s) d\,s, \tag{4}$$

where C stands for the ring circumference. Thus

$$\alpha = \frac{1}{C} \sum_{n=1}^{N} \frac{L_n}{\rho_n} \bar{D}_n = \frac{1}{\bar{R}} \sum_{n=1}^{N} \frac{\theta_n}{2\,\pi} \bar{D}_n \tag{5}$$

$\bar{R} = C/2\,\pi$ being the mean radius of the machine, while L_n, θ_n stand for the length and bending angle and $\bar{D}_n$ is the mean value of the dispersion function at the location of the n*th* bending magnet, respectively. Since the sum of all bending angles equals $2\,\pi$, the mean dispersion $\bar{D}_{\mathrm{b}}^0$ over all bending magnets around the ring can be defined as

$$\bar{D}_{\mathrm{b}}^0 = \sum_{n=1}^{N} \frac{\theta_n}{2\,\pi} \quad \Longrightarrow \quad \bar{D}_{\mathrm{b}}^0 = \alpha\,\bar{R}. \tag{6}$$

To lower the value of $\bar{D}_{\mathrm{b}}^0$, a possible strategy consists in introducing dipoles with a negative value of the radius of curvature. If M_{w} extra bends of length L_{w} are inserted in the lattice, $M_{\mathrm{w}}/2$ with negative (positive) bending radius $-\rho_{\mathrm{w}}$ (ρ_{w}), respectively, the expression for α reads

$$\alpha = \frac{\bar{\mathrm{D}}_{\mathrm{b}}^1}{\bar{R}_{\mathrm{w}}} + \frac{M_{\mathrm{w}}\theta_{\mathrm{w}}}{4\,\pi \bar{R}_{\mathrm{w}}} \left(\bar{\mathrm{D}}_{\mathrm{w}}^+ - \bar{\mathrm{D}}_{\mathrm{w}}^-\right) \tag{7}$$

in which $\bar{D}_{\rm w}^{+}$ and $\bar{D}_{\rm w}^{-}$ stand for the mean dispersion over the dipoles with positive and negative curvature respectively, $\theta_{\rm w}$ is the deflection angle produced by each additional dipole, and $\bar{R}_{\rm w} = C_{\rm w}/2\,\pi$, where $C_{\rm w} = C + L_{\rm w} M_{\rm w}$ represents the new value of the circumference length. Assuming that $\bar{D}_{\rm w}^{+} = -\bar{D}_{\rm w}^{-} = \bar{D}_{\rm b}^{1} > 0$, the mean dispersion reads

$$\bar{\rm D}_{\rm b}^{1} = \frac{\alpha \bar{R}_{\rm w}}{1 + \dfrac{M_{\rm w}\theta_{\rm w}}{2\,\pi}}. \tag{8}$$

Hence $\bar{\rm D}_{\rm b}^{1}$ is reduced according to

$$\bar{D}_{\rm b}^{1} = \bar{D}_{\rm b}^{0} \frac{1+\Delta}{1+\Delta\dfrac{\rho_{\rm w}}{\bar{R}}} \quad \text{with} \quad \Delta = \frac{M_{\rm w}\theta_{\rm w}}{2\pi} \tag{9}$$

In the next section, optical modules (wigglers) based on negative curvature bending magnets are presented. All the optics computations have been carried out using the *BeamOptics* program [2].

3 WIGGLER MODULES

To ease the matching with standard FODO lattices, the starting point to build a wiggler module has been a FODO-structure made of three identical cells of length L. For varying the momentum compaction with respect to the standard structure, alternating sign bending magnets (which explains the name given to such a module in analogy to the wiggler magnets used in electron machines) are interspersed with quadrupoles such that the overall bending angle is zero. This property allows the insertion of wiggler modules between arcs made of standard FODO cells. Two variants of the wiggler have been studied, with different numbers of free parameters.

3.1 Two-parameters wiggler module

Two free parameters can be used to determine the optical properties of the wiggler: the strength of the quadrupoles and the bending angle ψ of the dipoles. In the thin lens version of the wiggler module, the bending dipoles are located at $L/4, 5L/4, 7L/4, 11L/4$ from the first half defocusing quadrupole, generating the sequence of bending angles $\psi, -\psi, -\psi, \psi$. The module length $3L$ is considered as a scaling parameter.

The Twiss parameters for the wiggler module are those of a FODO cell of length L [2], while the dispersion function $D_{\rm in}$ and its derivative $D'_{\rm in}$ at the entrance of the wiggler, along with α are given by the formulae

$$D_{\rm in} = -\psi \frac{f[1-6(f/L)]}{1-12(f/L)^2} \qquad D'_{\rm in} = 0$$
$$\alpha = -\frac{\psi^2}{6}\frac{1-48(f/L)^2}{1-12(f/L)^2},$$

with $f/L = 1/4\sin(\mu/2)$, f being the focal length of the quadrupole. Figure 1 shows the optical functions for a wiggler module. The sign of α can be easily determined as a function of f/L. The dispersion function oscillates around zero keeping the same sign as the deflection angle of the dipoles.

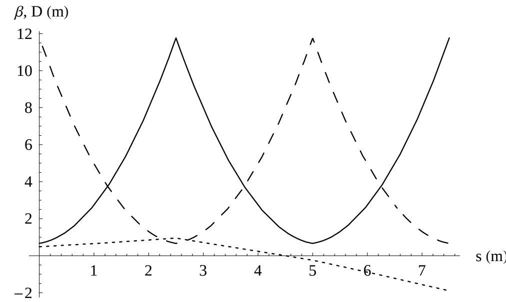

Figure 1: Optical functions for half a wiggler module with two parameters (β_h solid, β_v dashed, D_h dotted line). In this case $\psi = 0.03$ rad, $f = 1.4$, $L = 5$ m.

3.2 Four-parameters wiggler module

In this variant of the wiggler module, two additional dipoles are used. The free parameters are the quadrupole strength, the deflection angles of the two independent dipoles, and the position of the additional bending magnets. The dipoles are located at $L(1-\chi)/4, 3L/4, (5+\chi)L/4, (7-\chi)L/4, 9L/4, (11+\chi)L/4$ from the first half defocusing quadrupole generating the following sequence of bending angles $\psi, \lambda\psi, -(\lambda+1)\psi, -(\lambda+1)\psi, \lambda\psi, \psi$. The additional free parameters are λ (a real quantity) and χ ($0 \le \chi \le 1$). Also in this case it is possible to find a closed form expression for $D_{\rm in}$, $D'_{\rm in}$ and α, for instance

$$D_{\rm in} = -\psi^2 \frac{f[1-2(3+\lambda)(f/L)]}{1-12(f/L)^2} \qquad D'_{\rm in} = 0$$
$$\alpha = -\frac{\psi}{6}\frac{[a(\lambda) - 3b(\lambda)(f/L) - 24c(\lambda)(f/L)^2]}{1-12(f/L)^2}$$

which holds for $\chi = 0$. The coefficients are defined as $a(\lambda) = 1+\lambda+\lambda^2$, $b(\lambda) = 2\lambda+\lambda^2$, and $c(\lambda) = 2+2\lambda+\lambda^2$.

The analysis of the sign of α is more involved in this case due to the presence of the parameters λ and χ. The detailed computations can be found in Ref. [3].

4 DESIGN OF AN ISOCHRONOUS RING USING WIGGLERS

The scenario studied at CERN for a Neutrino Factory [4] is based on proton drivers to produce the intense proton beam needed to generate neutrinos. Preference has been to use the tunnel of the previous ISR whose mean radius is 150 m. An earlier scheme, considered proton drivers based on an isochronous lattice [5]. The beam kinetic energy is 2 GeV so that $\eta = 0$ yields $\gamma_{\rm tr} = 3.132$ and $\alpha = 0.102$. For an isochronous FODO lattice, the mean betatron function would be about 48 m (due to $\gamma_{\rm tr} \approx Q_h$) and the mean dispersion function $\bar{D}_{\rm b}^{0}$ would be approximately 15 m.

An isochronous lattice based on a FODO cell interspersed with wiggler modules would break the relationship between $\gamma_{\rm tr}$ and the optical parameters, thus reducing the excursion of the dispersion function.

A four-parameters wiggler module has been chosen with $\lambda = -0.464$ and $\chi = 1$. The value $\chi = 1$ implies that three bending magnets are combined-function (dipole-quadrupole) magnets. Hence, the positions of the bending magnets are at $0, 3L/4, 3L/2, 9L/4, 3L$ from the first element. To achieve dispersion matching at the junction FODO/wiggler, the wiggler and FODO dipole bending angles ψ and ϕ have to be related by an expression that, in the thin lens approximation and for $\chi = 1$, reads

$$\psi = \frac{\phi}{2} \frac{[1-8(f/L)][1-12(f/L)^2]}{1-(4+\lambda)(f/L)-2(2+\lambda)(f/L)^2}. \qquad (10)$$

The machine consists of eight super-periods, each made of one FODO cell 9.45 m long, followed by four wigglers 28.35 m long each. The ring circumference is 982.8 m. The bending angle of each FODO dipole is $\phi = \pi/8 = 0.393$ rad, yielding for the wiggler $\psi = 0.287$ rad and the remaining angles are $0.287, -0.133, -0.308, -0.133, 0.287$ rad, respectively. The integrated focusing strength is 0.401 m^{-1}. Figure 2 shows the optical functions for a FODO cell and a wiggler module. The constraint imposed by the requirement $\eta = 0$ yields a dispersion curve not symmetric with respect to zero, oscillating in the wiggler between -8 m and 3 m, the minimum being attained at the defocusing quadrupole location. Figure 3 shows the overall ring geometry.

It is worth mentioning that an isochronous lattice with

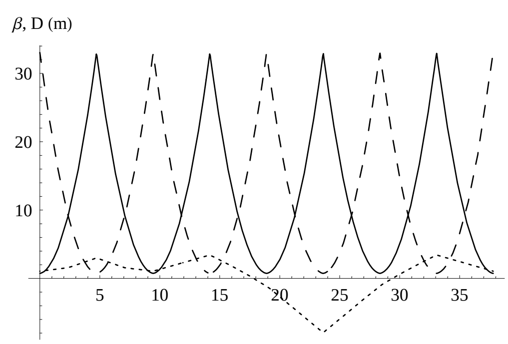

Figure 2: Optical functions for the isochronous, thin lens, lattice with wigglers (β_h solid, β_v dashed, D_h dotted line). A single FODO cell with half wiggler cell is shown.

similar values of the optical functions can be obtained without using wigglers, but only negative radius of curvature dipoles (see Ref. [5] for more details). In Fig. 4 the optical functions for such a lattice are shown.

5 CONCLUSION AND OUTLOOK

A number of optical modules based on FODO cells and alternating-sign bending magnets have been presented, and their properties discussed in detail. They allow the tight

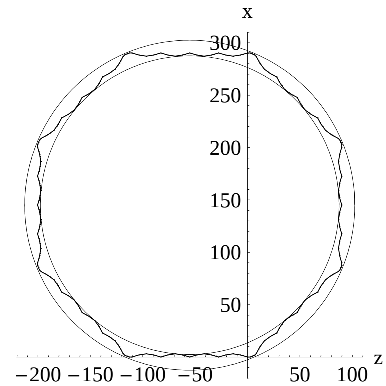

Figure 3: Geometry of the isochronous, thin lens, FODO lattice with wigglers. The inner and outer circles represent the walls of the existing ISR tunnel

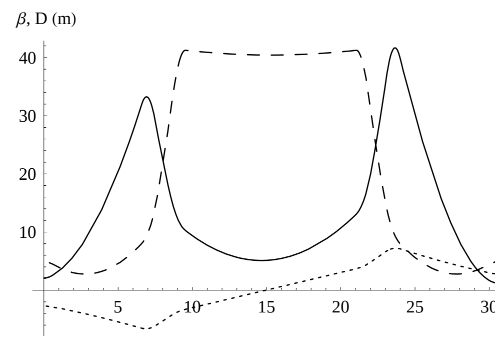

Figure 4: Optical functions for half a super-period of the isochronous, thick lens, lattice with negative curvature dipoles (β_h solid, β_v dashed, D_h dotted line).

relationship between Q_h, D, ρ, and machine energy to be broken, thus enabling the design of large isochronous machines with reasonable values of the dispersion function. As an example, a lattice for an isochronous proton driver for the Neutrino Factory fitting the ISR tunnel has been presented.

The modules described in this paper allow the value of $\gamma_{\rm tr}$ to be decreased (having $\alpha > 0$). However, it is also possible to design similar modules with $\alpha < 0$ in order to increase $\gamma_{\rm tr}$ [3].

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