OPTIMISATION OF THE DYNAMIC APERTURE OF THE DIAMOND STORAGE RING LATTICE

A. Wolski, Daresbury Laboratory, Warrington, WA4 4AD, UK

Abstract

The strong focusing required for high brightness in a third generation light source presents challenges in maintaining a good dynamic aperture. A traditional approach to improving dynamic stability has relied on fixing the tunes of the lattice away from major resonances, to minimise the strengths of the driving terms. We present here the results of an alternative approach, based on controlling the phase advances over different sections of the lattice, to reduce the magnitude of damaging terms in the single-turn map. This has yielded good results in the case of the storage ring for DIAMOND [1], the UK third generation light source.

1 DIAMOND LATTICE DESIGN CHALLENGES

Users of synchrotron light sources demand high brightness photon output, good stability and reliability of operation. For the storage ring lattice, these properties depend on a low natural emittance, and a large dynamic aperture to ensure efficient injection and good beam lifetime. Unfortunately, the strong focusing required to reach low emittance leads to a large negative chromaticity of the lattice, which must be corrected with strong sextupoles to reduce the effects of the head-tail instability. The nonlinearities introduced by the sextupoles tend to reduce the dynamic aperture.

Significant effort is needed to resolve the conflicting requirements for low emittance and large dynamic aperture. For DIAMOND, we are currently working on a 3 GeV, 24 cell double-bend achromat storage ring, with either four-fold or six-fold symmetry and a circumference around 500 m. User consultations have led to a target emittance of 3 nm rad, and we are aiming for a dynamic aperture of ±20 mm horizontally and ±10 mm vertically in the long straights, including magnetic field and alignment errors. Presently, we are working for a momentum acceptance of 4%, though lifetime studies could lead to a requirement for larger acceptance.

2 LINEAR PROPERTIES

Traditionally, a double-bend achromat is tuned to give zero dispersion in the straight sections. The natural emittance of the lattice is determined by the beta and dispersion functions, at the entrance to the dipoles. The minimum emittance that can be achieved under the zero-dispersion condition is [2]:

$$\varepsilon_{\min} = \frac{2\pi^3 C_q \gamma^2}{\sqrt{15} N_b^3 J_x} \qquad (1)$$

where C_q is the quantum constant, γ the relativistic factor, N_b the number of bending magnets in the lattice, and J_x the horizontal damping partition number.

If some dispersion is allowed in the straight sections, then the minimum emittance is reduced by a factor three. Analytic expressions can be given for the required values of the beta and dispersion functions at the entrance to the dipoles to achieve the minimum [3]. Using expression (1) for a 3 GeV 24-cell lattice gives a theoretical minimum emittance of 1.9 nm rad with zero dispersion in the straights, or 0.64 nm rad if dispersion is allowed.

The drawback with reducing emittance by allowing dispersion is that the dispersion at the location of the chromatic sextupoles is reduced, so their strength needs to be increased. This increases the nonlinear effects in the lattice, and reduces the dynamic aperture. In principle, we can achieve the target emittance for DIAMOND without allowing dispersion in the straights. In practice, however, engineering constraints make it difficult to achieve the optimum conditions for low emittance, and lead to the necessity of allowing some dispersion to reach an emittance of 3 nm rad.

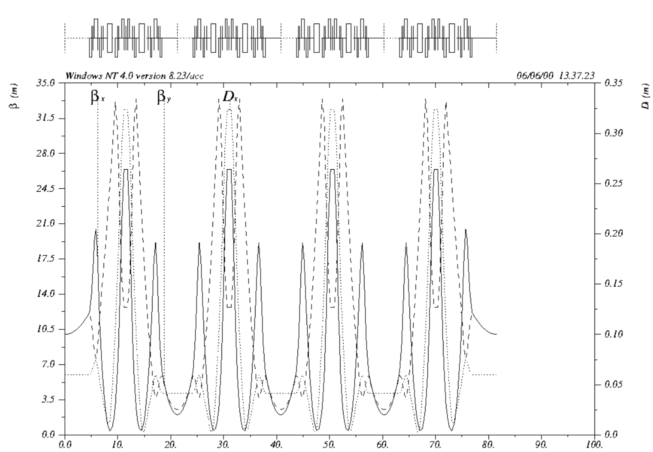

Figure 1: Lattice functions in a six-fold symmetry version of the DIAMOND storage ring lattice.

The beta functions and horizontal dispersion for a typical six-fold symmetric 24-cell lattice are shown in Figure 1. The natural emittance of this lattice is 2.5 nm rad, and some dispersion (0.06 m in the long straights, 0.04 m in the short straights) has been allowed to achieve this.

3 NONLINEAR PROPERTIES

The chromatic sextupoles are positioned in the conventional places in the lattice. Thus, in each cell, one sextupole for correcting horizontal chromaticity is at the centre of the achromat, where there is dispersion and a large horizontal beta function. Two sextupoles for correcting vertical chromaticity are placed either side, where the vertical beta function is large and the horizontal beta function is small. Families of harmonic sextupoles can be positioned around the quadrupoles at either end of the straight sections. An advantage to allowing dispersion in the straight sections is that the harmonic sextupoles have a (weak) chromatic effect, and can be useful, for example, for correcting higher order chromaticities.

As a design principle, we aim to reduce the nonlinear effects of the sextupoles by adjusting the phase advance to give conditions for cancellation of the higher order geometric terms. This has been applied, for example, in the case of SPEAR 3 [4]. For a section of n periods, cancellation can be achieved by a phase advance of $2\pi/n$ between the sextupoles. This can readily be seen by representing the map for individual elements using Lie operators. Thus, we write the map for a sextupole as[1]

$$S = \mathrm{e}^{:x^3 - 3xy^2:}$$

and for a phase advance horizontally, vertically (μ_x, μ_y) as

$$R = \mathrm{e}^{:-\mu_x J_x - \mu_y J_y:}$$

where $J_x = \frac{1}{2}\left(x^2 + p_x^2\right)$, and similarly in the vertical plane. The map for a periodic section of n sextupoles may then be written

$$M = \prod_{m=0}^{n-1} \left(\mathrm{e}^{:x^3 - 3xy^2:} R \right)$$

The sextupole kicks may be 'lumped' at the end of the map, by repeatedly inserting identity factors RR^{-1}, and making use of the similarity transformation:

$$A^{-1} \mathrm{e}^{:f:} A = \mathrm{e}^{:A^{-1} f:}$$

thus:

$$M = R^n \prod_{m=0}^{n-1} \mathrm{e}^{:R^{m-n} x^3 - 3R^{m-n} xy^2:}$$

We note that the effect of a phase advance on x^3, for example, is just

$$R^{-1} x^3 = \left(x \cos \mu_x - p_x \sin \mu_x \right)^3$$

The sextupole kicks may be grouped into a single Lie operator using the Baker-Campbell-Hausdorff formula

$$\mathrm{e}^{:f:} \mathrm{e}^{:g:} = \mathrm{e}^{:f + g + \frac{1}{2}[f,g] + \cdots:}$$

Finally, we apply the identity

$$\sum_{m=0}^{n-1} \cos^a \left(\frac{2\pi m}{n} \right) \sin^b \left(\frac{2\pi m}{n} \right) \equiv 0$$

and find that the third order terms in the generator cancel, so we are left with

$$M = R^n \mathrm{e}^{:f_4:}$$

where f_4 is a polynomial of lowest order four in the phase space variables. In the special case that the phase advance is π horizontally and 2π vertically between two sextupoles (the $-I$ transformer), there are no extra terms generated by the Poisson bracket in the Baker-Campbell-Hausdorff formula, and the map is completely linearised. Even if the map is not made completely linear, cancellation of the lowest order nonlinear terms can give significant improvement in the dynamic aperture.

A real lattice differs from the idealised model of the above treatment in two important respects. First, the sextupoles all have length greater than zero, and second, different families are interleaved. The above treatment generalises to include the case where the sextupoles are interleaved, but for sextupoles of non-zero length, the generator includes a second order momentum term. This prevents cancellation of the lowest order nonlinear terms.

The question we need to address is whether the cancellation principle gives real benefit for the dynamic aperture in practical cases. To investigate this issue, we constructed four lattices as follows:

- The AT lattice has six-fold symmetry, full chromatic correction and four families of harmonic sextupoles. The tune point is chosen to avoid strong resonances, but without concern for cancellation of higher order geometric terms.
- The PI lattice has six-fold symmetry, only one family of thin sextupoles, arranged to give a $-I$ transformer, and adjusted to give zero horizontal chromaticity.
- The PICC lattice has six-fold symmetry, full chromatic correction, and a phase advance tuned to give good geometric cancellation. There are four families of harmonic sextupoles.
- The PIFF lattice is derived from the PICC lattice, but with four-fold symmetry. The tune is not adjusted, thus there is incomplete geometric cancellation.

We investigated the dynamics for each lattice by looking at the horizontal phase space plots, comparing the results of tracking in MAD 8.23, MERLIN 2 and MARYLIE 3.0 [5]. MERLIN 2, a C++ class library for beamline

[1] This is a map appropriate for a thin sextupole of unit strength. The chromatic effect can easily be seen by replacing x by $x + \eta\delta$, where η is the dispersion, and δ the momentum deviation, treated as a parameter.

simulation[2], is capable of fast symplectic tracking, and allows us to write code specifically for dynamic aperture calculations; we therefore used MERLIN 2 for determining the dynamic aperture of each of the above lattices. Finally, we used MARYLIE 3.0 to investigate the nonlinear terms in the generator.

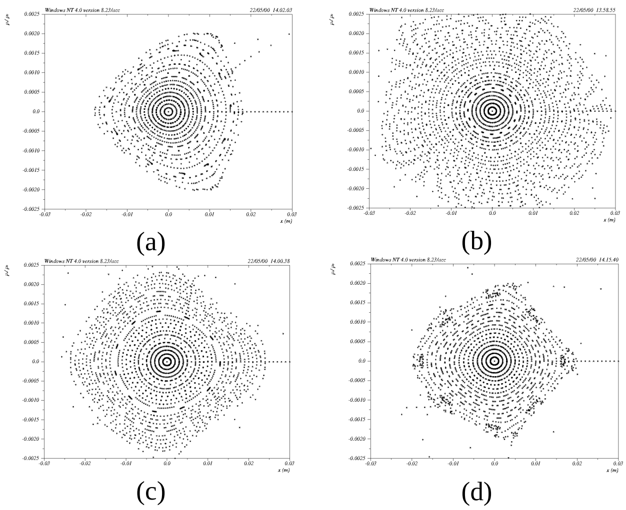

Figure 2: Horizontal phase space plots for (a) AT lattice, (b) PI lattice, (c) PICC lattice and (d) PIFF lattice. All plots are on the same scale, with the horizontal axis from –30 mm to +30 mm. The observation point was at the centre of the long straight, where β_x is 10 m.

The horizontal phase space plots are shown in Figure 2. It is clear that the dynamics in the PI lattice are very regular up to large amplitude, indicating good cancellation of the nonlinear terms in the generator; it should also be remembered that this is the only lattice with no harmonic sextupoles. The AT and PIFF lattices show relatively poor nonlinear cancellation. The most practical lattice, which combines full chromatic correction with reasonable nonlinear properties, is the PICC lattice, which has been designed using the geometric cancellation principles outlined above.

The dynamic aperture of each of the four case-study lattices is shown in Figure 3. As expected, the PI lattice has a very large dynamic aperture, and the horizontal dynamic aperture of the PICC lattice is significantly greater than either the AT or the PIFF lattices. Vertically, however, the PICC lattice is not so good, although the reasons for this are not immediately clear.

The third order terms in the generators for the single-turn maps are shown in Figure 4. The geometric cancellation in the PI and PICC lattices is evident; the large terms in the PI lattice at the right hand side of the plot are related to the vertical chromaticity, which of course is not corrected in this lattice.

[2] Thanks to Nick Walker of DESY for making MERLIN 2 available.

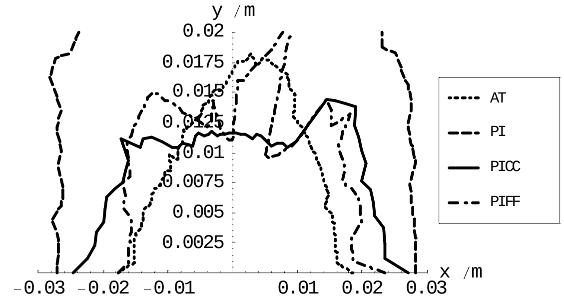

Figure 3: Dynamic apertures of the case-study lattices.

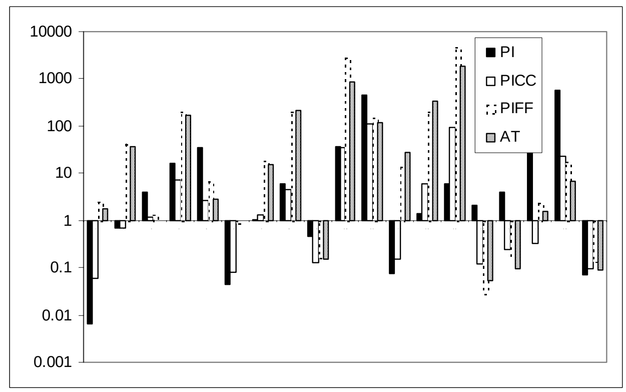

Figure 4: Third order terms in the generators for the single-turn maps for each of the four case-study lattices.

Although we do not present the results here, we also find that the PICC lattice has good stability with respect to momentum deviation, and the dynamic aperture is well maintained in the presence of magnet field errors.

Further improvements may be possible. For example, detuning the lattice may lower the chromaticity, reducing the required strengths of the chromatic sextupoles. In addition, we are looking at how the dynamic aperture depends on higher order terms in the generator.

4 CONCLUSION

Our results suggest that the geometric cancellation design principle can be successfully applied to improve the dynamic aperture of strongly nonlinear lattices.

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