METHOD FOR EFFICIENT ANALYSIS OF WAVEGUIDE COMPONENTS AND CAVITIES FOR RF SOURCES *

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Abstract

An efficient method for calculating scattering and dispersion parameters, resonant frequencies, and fields in a 2-D, arbitrarily shaped geometry is presented. This work is done to analyze and design high power waveguide components and cross-field RF devices. An arbitrary geometry is described by a piecewise planar boundary. The method is based on the scattering matrix technique. The simulated geometry is divided into regions. The boundary contour mode-matching method is used to obtain the scattering matrices for each region. Electromagnetic fields in each region are expanded in a series of plane waves. Due to the expansion, all integration in the mode-matching process is carried out analytically. The scattering matrices of the regions are combined using a generalized scattering matrix technique to obtain the scattering matrix and field distribution for the full geometry. Representation of the fields as a functional expansion is a useful feature of the method for further particle tracking in a cross-field device. The calculated results are compared with other electromagnetic models and excellent agreement is obtained.

1 INTRODUCTION

The mode-matching method, in combination with the generalized scattering matrix ($S-$matrix) technique, has turned out to be very efficient in simulating of the interaction of charged particles and electromagnetic structures, such as accelerating structures and cavities [1, 2]. In this paper we describe a method that uses the scattering matrix approach for design cross-field devices and planar waveguide components. The design of cross-field devices such as magnetrons and cross-field amplifiers requires methods for simulation of cavities that have an arbitrary 2D shape [3]. The methods should be able to calculate the dispersion parameters of a periodic structure, the resonant frequencies of a cavity, and the corresponding fields. Simulation of the devices includes tracking of electrons in fields represented as a sum of resonant modes. Functional expansion of the electro-magnetic fields is desirable for the efficient calculation of the particle dynamics. The use of the functional field expansion and the azimuthal periodicity of the device's cavity makes methods that are based on scattering matrices preferable for the simulation. Another application of the method is the design of planar high-power waveguide components. Short time of the simulation make the process of optimization of scattering parameters more efficient. Boundary contour mode-matching in a piecewise bounded 2D region is applied to obtain the scattering matrix and field amplitudes [4]. The Galerkin method is used for the mode-matching procedure. The geometry is divided into regions, and electromagnetic fields in each region are expanded in series of plane waves. The choice of plane waves as basis functions for mode matching [5] and the piecewise description of the geometry contour allow us to use, instead of numerical, analytical integration. Scattering matrices from the regions are combined using the generalized scattering matrix technique. Resonant and periodic conditions [1] are used to obtain resonant frequencies, dispersion parameters, and corresponding fields.

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2 MODE MATCHING

We divide the simulated geometry into regions. For each region we find relations between scattered waves as well as the fields-inside-region using non-orthogonal expansion of the fields (see *e.g.* [4, 5]).

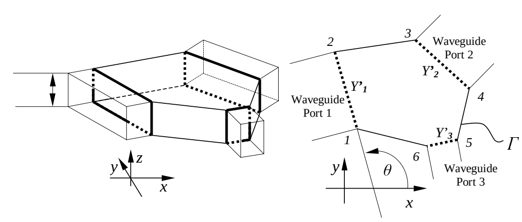

Figure 1: An inhomogenous waveguide region with planar sidewalls fed by rectangular waveguides (ports) through planar apertures Y'_w. The sidewalls are defined by points $s = 1, 2...6$. The ports are defined by points $(x_p, y_p), (x_{p+1}, y_{p+1})$, and $p = 1, 3, 5$. Γ - total sidewall surface including ports.

Geometry We will discuss the cylindrical geometry which is uniform in the z-direction. It consists of planar sidewalls and waveguide apertures, and planar top and bottom walls located at $z = 0$ and $z = L$ respectively, as illustrated on Fig. 1. The geometry in the x, y plane can be described by a set of points with coordinates (x_s, y_s) where $s = 1, ...N'$; here N' is the total number of sidewalls and apertures. Apertures that are connected to rectangular waveguides are described as ports with coordinates $(x_p, y_p), (x_{p+1}, y_{p+2})$. Here p is the index of the starting point for the correspondent port. In the particular case (shown on Fig. 1) the inhomogeneous region has three sidewalls, and three ports with index $p = 1, 3, 5$.

Plane waves Plane waves are one of the solutions of Maxwell's equations for a geometry that consist of two parallel infinite walls located at $z = 0$ and $z = L$. These waves do not have phase variation along a vector which is perpendicular to the direction of the power flow. Plane waves can be divided into two sets. Each set has either magnetic (hz-wave) or electric (ez-wave) field along $z-$axis. All fields have a harmonic $e^{j\omega t}$ time (t) dependence. Here ω is the angular frequency. Explicitly, we will write fields for the $hz-$wave as

$$
\begin{gathered}
H_z^h(x,y) = \sin(k_z z) e^{-jk_\perp(xC_\phi + yS_\phi)}, \\
\vec{E}_\perp^h(x,y) = \rho^h \sin(k_z z)(-\vec{x}S_\phi + \vec{y}C_\phi) e^{-jk_\perp(xC_\phi + yS_\phi)}, \\
\vec{H}_\perp^h(x,y) = -j\frac{k_z}{k_\perp}\cos(k_z z)(\vec{x}C_\phi + \vec{y}S_\phi) e^{-jk_\perp(xC_\phi + yS_\phi)},
\end{gathered}
$$

where $C_\phi = \cos\phi$ and $S_\phi = \sin\phi$; $k_z = \pi m L$, m is the mode index, $k^2 = k_z^2 + k_\perp^2$, where $k = \omega/c$, ϕ is the angle of $\vec{k}_\perp$ with respect to the $x-$axis, and $\rho^h = \omega\mu_0/k_\perp$. Similar expression could be written for $ez-$wave.

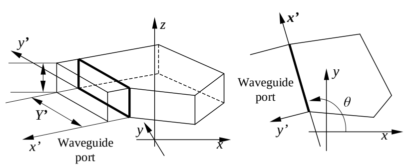

Figure 2: Layout of a waveguide port connected to a waveguide discontinuity. Note the local coordinate system (x', y', z). Dimensions of the port along the z and y' axes are L and Y' respectively.

Waveguide modes A rectangular aperture of a waveguide discontinuity can be described as a part of a rectangular waveguide with the same cross-section. Such a waveguide has its coordinate system (x', y', z), where x' is the waveguide axis as is shown in Fig.2. The waveguide fields can be separated into TE- and TM- modes with respect to the x' axis. Fields along the x'-axis in the waveguide with infinitely conducting walls with y'-size Y' can be described by basic scalar functions

$$
\begin{gathered}
\psi_{r(x')}^e(z,y') = \sin(k_{m(z)}z)\sin(k_{n(y)}y'), \\
\psi_{r(x')}^h(z,y') = \cos(k_{m(z)}z)\cos(k_{n(y)}y'),
\end{gathered}
$$

where $k_{m(z)} = \pi m/L$, $k_{n(y)} = \pi n/Y'$ and $n > 0$ for E-modes. Note that we use the same index m for waveguide modes and plane waves. The waveguide modes and plane waves with $k_z \neq k_{m(z)}$ do not interact. The electric fields transverse to x' we obtain from equation $\vec{E}_{r\perp} = \nabla_\perp\psi_{r(x')}^e + \nabla_\perp\psi_{r(x')}^h \times \vec{y'}$. Here $r = (m,n)$ is a combined mode index.

Field expansion The total electric and magnetic fields in a waveguide discontinuity (Fig. 1) are represented by a sum of N_l plane waves. We will choose weighting coefficients α in the sum to satisfy the boundary conditions. Fields on the boundary Γ of a geometry can be expressed as

$$
\vec{E}_\Gamma^g = \sum_q (\alpha_q^h \vec{E}_q^e + \alpha_q^e \vec{E}_q^h),\ \vec{H}_\Gamma^g = \sum_q (\alpha_q^h \vec{H}_q^e + \alpha_q^e \vec{H}_q^h).
$$

Here $\vec{E}_q^e$, $\vec{H}_q^e$ are the magnetic and electric field for the ez plane wave, and $\vec{E}_q^h$, $\vec{H}_q^h$ are the fields for the hz plane wave, $q = (m,l)$ is a combined mode index, and the Γ is the total transverse surface, including ports (Fig.1). Index $l = 0, 1...N_l - 1$ describes discretization in the direction $\vec{k}_\perp$, and $k_z = \pi m/L$. The direction-angle of each l-th wave is $\phi_l = 2\pi l/N_l$. In contrast to modes in the rectangular waveguide sections, the plane waves are not orthogonal along the boundary. The waveguide discontinuity is fed by rectangular waveguides (ports). Tangential fields in the w-th port are

$$
\vec{E}_t^w = \sum_r (a_{rw}^e + b_{rw}^e)\frac{\vec{E}_{rw\perp}^e}{\sqrt{N_{rw}^e}} + \sum_r (a_{rw}^h + b_{rw}^h)\frac{\vec{E}_{rw\perp}^h}{\sqrt{N_{rw}^h}},
$$

$$
\vec{H}_t^w = \sum_r (a_{rw}^e - b_{rw}^e)\frac{\vec{H}_{rw\perp}^e}{\sqrt{N_{rw}^e}} + \sum_r (a_{rw}^h - b_{rw}^h)\frac{\vec{H}_{rw\perp}^h}{\sqrt{N_{rw}^h}},
$$

where $r = (m,n)$ is the combined index for waveguide modes, N_{rw}^h and N_{rw}^e are normalization constants for the waveguide modes. We normalize each waveguide mode to unit power. The boundary conditions which need to be imposed to resolve the fields in the geometry are

$$
\vec{E}_{t\Gamma}^g = \begin{cases} \vec{E}_t^w \text{ for } \Gamma = Y_w' \\ 0 \text{ else,} \end{cases} \tag{1}
$$

$$
\vec{H}_{t\Gamma}^g = \vec{H}_t^w \text{ for } \Gamma = Y_w'. \tag{2}
$$

Here w is the port index, and $\vec{E}_{t\Gamma}^g$ and $\vec{H}_{t\Gamma}^g$ are tangential electric and magnetic fields inside the discontinuity.

Mode matching procedure The following mode matching procedure is general. It is valid for the plane wave expansion as well as for other types of expansions. First, we enforce electric field continuity by integrating both sides of (1) with plane wave magnetic fields $(\vec{H'}_{q'}^e)^*$ and $(\vec{H}_{q'}^h)^*$ over the boundary Γ. For simplicity we drop h and e indexes. We define the product of "plane wave - plane wave" integration as

$$
A_{qq'}^{11} = \int_\Gamma \vec{E}_q \times \vec{H}_{q'}^* \cdot \vec{n}\, ds', \tag{3}
$$

and

$$
A_{pq'}^{21} = \int_{Y_w'} \vec{E}_p \times \vec{H}_{q'}^* \cdot \vec{n}\, ds'. \tag{4}
$$

Here $\vec{n}$ is normal to Γ, and $p = (r,w)$ is a combined index for all the waveguide modes in the ports. For the case of a piecewise planar boundary and the plane wave expansion, there are three types of plane wave-plane wave integrals and four plane wave - waveguide integrals. Each integral is evaluated analytically. Using the products (3) and (4) we can write the boundary conditions (1) in matrix form as

$$A^{11}\vec{\alpha} = A^{21}(\vec{a} + \vec{b}), \tag{5}$$

where $\vec{\alpha}$ is the vector of weighting coefficients and $\vec{a}$, $\vec{b}$ are vectors of incident and reflected waves, respectively. Second, we enforce magnetic field continuity by integrating the complex conjugate of (2) with the waveguide electric fields $E_{p'}$. The left part of the continuity condition we write using (4) as $[(A^{21})^*]^T\vec{\alpha}$. We simplify the right part using orthogonality of the modes in a waveguide. Due to the chosen normalization, $A^{22}_{pp'} = \int_{Y'_w} \vec{E}_p \times \vec{H}^*_{p'} \cdot \vec{n}\, ds' = \delta_{pp'}$. Therefore $A^{22} = I$ is the identity matrix. The condition of magnetic field continuity becomes

$$A^{22}(\vec{a} - \vec{b}) = I(\vec{a} - \vec{b}) = [(A^{21})^*]^T\vec{\alpha}. \tag{6}$$

Scattering matrix and fields in the waveguide discontinuity By eliminating $\vec{\alpha}$ from (5) and (6), we obtain the ratio between amplitudes of the incident and reflected modes as

$$\vec{b} = (I + C)^{-1}(I - C)\vec{a}, \tag{7}$$

where $C = A^{21}(A^{11})^{-1}((A^{21})^*)^T$. The resulting matrix $S = (I + C)^{-1}(I - C)$ is the general scattering matrix of the geometry. We substitute (7) in (6) and obtain

$$\vec{\alpha} = [A^{21}(A^{11})^{-1}]^T(S + I)\vec{a}. \tag{8}$$

The result is a linear dependence of the weighting coefficients of the plane waves on the incident amplitudes of waveguide modes.

Properties of the A^{11} matrix Matrix A^{11} is Hermitian, and has zeros along the diagonal. The matrix becomes ill-conditioned with increasing discretization number N_l. Due to the ill-conditioning, the numerical calculation of the inverse matrix $(A^{11})^{-1}$ is difficult. We changed basis functions to a linear combination of the plane waves in order to make the matrix A^{11} well defined. For the particular implementation we choose a transformation that is based on an expansion of a Bessel function (see [5]). An element of the matrix that represents the transformation is $T_{\beta l} = \frac{(j)^{(\beta - N_\beta)}}{N_l}\cos\left(\frac{2\pi|l - N_l|}{N_l}\right)$, for $l > N_l$, and for $l \le N_l$ $T_{\beta l} = \frac{(j)^{(\beta - N_\beta)}}{N_l}\sin\left(\frac{2\pi|l - N_l|}{N_l}\right)$, where l is the index of the plane wave and $\beta = 0, 1 \ldots 2, N_\beta - 1$ is the index of the Bessel function. In numerical simulation, typical numbers are $N_\beta = 7$ and $N_l = 16$. The matrix for the fields inside the discontinuity is $A'^{11} = TA^{11}(T^*)^T$. And the matrix of "outside coupling" is $A'^{12} = A^{12}(T^*)^T$. The method works well for practical geometries, but it has limitations due to nonorthogonality of functions in the field expansion. The matrix A'^{11} become ill-conditioned with increasing $N_\beta(>20)$ or with decreasing ω. To cope with the problem we renormalize matrix T with increasing of N_β and use cylindrical waves expansion for low ω (see [4]).

3 CALCULATION OF ARBITRARY GEOMETRY

Using mode matching we obtained the scattering matrix (7) and the relation between external waves and internal fields (8) for each region. Now, using generalized scattering matrix technique (see *e.g.* [1, 6]) we can calculate external scattering parameters and fields for whole geometry. The numerical implementation of this technique is straightforward and requires much less computer time than the calculation of the scattering matrix for each region.

4 NUMERICAL RESULTS

A computer code has been developed based on the finite plane-wave series expansion. The code is part of a cross-field device, computer-aided design program. The number of plane waves depends on frequency and varies from 16 to 40. As an example we consider a waveguide T-junction. The size of the junction is 1 cm $\times$ 1cm. The fields have no z-variation. The solution has been verified by the finite-element software package $HFSS$, developed by Ansoft corporation. Fig. 3 shows the amplitudes of some typical scattering parameters, calculated by contour mode-matching (curves) and $HFSS$ (symbols). It can be seen that there is no noticeable discrepancy in the amplitudes. It is worth mentioning that two modes propagate in the waveguides at frequencies above 15 GHz.

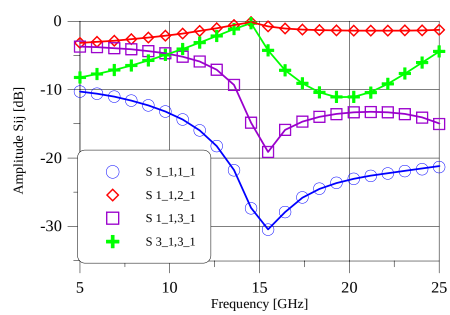

Figure 3: Amplitudes of scattering parameters for 1cm$\times$1cm E-plane T-junction. Lines – contour mode matching, symbols – finite element code $HFSS$.

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