MULTIPLE RESONANCE TREATMENT FOR LINEAR COUPLING

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Abstract

In this paper we discuss an analytical procedure to sum up all the members of a given resonance family expressing the joint influence in a single driving term. We discuss the compensation of linear coupling, pointing out the relation between the matrix-based and the resonance approach. The Henon map and a non linear lattice which reproduce the Antiproton Decelerator (AD) are used to test the efficiency of different schemes.

1 INTRODUCTION

Transverse, single particle dynamics in a synchrotron has been widely studied using an Hamiltonian perturbative approach [1]. The present work takes one step forward by summing all the members of a given resonance family and expressing the joint influence in a single driving term [2]. As an application we consider the problem of compensation of linear coupling making the bridge between the matrix and the Hamiltonian approach. The efficiency of different compensation schemes is discussed with regards to the Hénon map [2] and the AD [3].

2 THE SUMMING PROCEDURE

The summed-resonance driving term $C_{n_1,n_2,\infty}$ of a given resonance of order $N = n_1 + n_2$ is:

$$C_{n_1,n_2,\infty} = \sum_{p=-\infty}^{+\infty} C_{n_1,n_2,p} \qquad (1)$$

where:

$$C_{n_1,n_2,p} = \int_0^{2\pi} A(\theta) e^{-i(n_1 Q_x + n_2 Q_y - p)\theta} \mathrm{d}\,\theta \qquad (2)$$

is the single-resonance driving term with:

$$\begin{aligned} A(\theta) &= \frac{R^2}{\pi(2R)^{N/2}|n_1|!|n_2|!}\beta_x(\theta)^{|n_1|/2}\beta_y(\theta)^{|n_2|/2} \\ &\times \; e^{i[n_1\mu_x(\theta) + n_2\mu_y(\theta)]}\bar{K}(\theta) \end{aligned} \qquad (3)$$

The angle $\theta = s/R$ is the coordinate along the ring of average radius R and $Q_{x,y}$, $\mu_{x,y}$, $\beta_{x,y}$ are respectively the two components of the tunes, the phase advances and the beta functions.
Suppose $\bar{K}(\theta)$ is different from zero and constant in j short intervals $[\theta_i, \theta_i + \Delta\theta_i]$ in which $A(\theta) \simeq A(\theta_i)$ (thin lens approximation):

$$\begin{aligned} C_{n_1,n_2,\infty} &= \sum_{i=1}^{j} (\Delta C_{n_1,n_2,\infty})_i \qquad (4) \\ &= \sum_{p=-\infty}^{+\infty} \int_{\theta_i}^{\theta_i+\Delta\theta_i} A(\theta) e^{-i[\Delta - p]\theta} \mathrm{d}\theta \end{aligned}$$

where $\Delta \equiv n_1 Q_x + n_2 Q_y$.

The summation can be redefined making use of the shift $p = [\Delta] + k$ where k is an integer and $[\,\cdot\,]$ states for the standard integer value. Assuming $(\Delta - [\Delta])\Delta\theta_i \ll 1$, we get [2]:

$$\begin{aligned} C_{n_1,n_2,\infty} &= -\frac{\pi(\Delta - [\Delta])}{\sin[\pi(\Delta - [\Delta])]}\frac{R^2}{\pi(2R)^{N/2}|n_1|!|n_2|!} \qquad (5) \\ &\times \int_0^{2\pi} \beta_x^{|n_1|/2}\beta_y^{|n_2|/2} e^{i[n_1\mu_x + n_2\mu_y - (\Delta - [\Delta])\pi]}\bar{K} d\theta \end{aligned}$$

The standard-(single-)resonance driving term is [7]:

$$\begin{aligned} C_{n_1,n_2,p} &= \frac{R^2}{\pi(2R)^{N/2}|n_1|!|n_2|!} \qquad (6) \\ &\times \int_0^{2\pi} \beta_x^{|n_1|/2}\beta_y^{|n_2|/2} e^{i[n_1\mu_x + n_2\mu_y - (\Delta - p)\theta]}\bar{K} d\theta \end{aligned}$$

The figure 1 shows the ratio between $||(\Delta C_{n_1,n_2,\infty})_i||$ and $||(\Delta C_{n_1,n_2,p})_i||$ versus the distance from the resonance ($[\Delta] = 1$) [2].

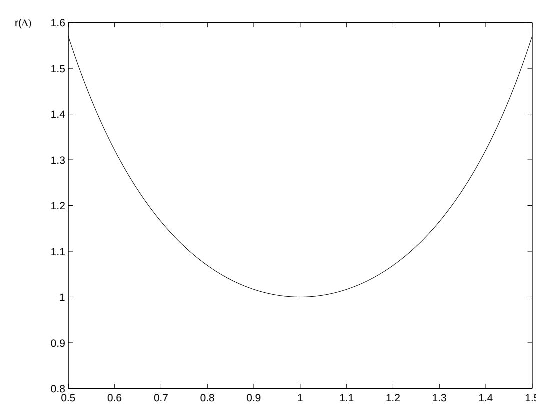

Figure 1: Ratio $r(\Delta)$ between $\|\Delta C_{n_1,n_2,\infty}\|$ and $\|\Delta C_{n_1,n_2,1}\|$ versus Δ.

As the working point approaches the closest resonance the two estimations converge to the same value for the modulus.

3 LINEAR BETATRON COUPLING

Analyses of betatron coupling can be broadly divided into two categories: the matrix approach [4], [5], [6] that decouples the single-turn matrix to reveal the normal modes and the Hamiltonian approach [7], [8] that evaluates the coupling in terms of the action of resonances using a perturbation method.

3.1 *The coupled Hénon map*

We performed a numerical analysis on the Hénon map, a hyper-simplified lattice model whose phase-space trajectories show some of the expected characteristic of a realistic lattice map (non linearities, regions of regular and stochastic motion etc.). In this application the linear coupling is generated and corrected by $1+4$ thin skew quadrupoles [1]. The compensation for both sum and difference resonance is achieved by solving the 4-equations system in the four unknowns k_i the coefficients of which are given by the driving terms [2].

Table 1 shows a comparison between the strengths of the 4 correctors (k_{2-5}) when compensating the single-turn matrix using the MAD program [9]), the two infinite families of sum and difference resonances (for the same $\theta = 0$) and the closest sum and difference resonances to the working point. The last two compensations have been obtained making use of the AGILE program [10].

Table 1: Compensator strengths (k_{2-5}) in presence of the coupling source k_1.

k (m^{-2})	Matrix	Summed	Single
k_1 (source)	0.5	0.5	0.5
k_2	-0.051	-0.050	0.559
k_3	0.034	0.033	0.554
k_4	-0.319	-0.313	0.476
k_5	-0.275	-0.275	0.117

Moreover we studied the dynamics aperture defined as follows:

$$D = \left[\int_0^{\frac{\pi}{2}} [r(\theta, N)]^4 \sin(2\theta) \mathrm{d}\theta \right]^{\frac{1}{4}} . \qquad (7)$$

where N is the number of turns and $r(\theta, N)$ is the last stable initial condition along θ before the first loss (at a turn number lower than N) occurs.

The results for the three studied optics for short and medium term tracking, are quoted in Table 2.

[1] Lattices with only solenoids or with both type of coupling elements give the same kind of results

Table 2: Dynamic aperture values. The errors are estimated to be 2% for N=5000 and 4% for N=20000.

D (m)	Uncoupled	Summed	Single
N=5000	0.0406	0.0412	0.0372
N=20000	0.0405	0.041	0.037

3.2 *The Antiproton Decelerator (AD)*

The comparison between the summed and single resonance compensation has also been performed for the case of a real machine [3], the Antiproton Decelerator (AD) at CERN [11]. The AD has been designed to decelerate an antiproton beam from 3.5 GeV/c (the momentum at which the antiprotons are produced) down to 100 MeV/c (the momentum favored for the foreseen experiments). The compensation of the blow-up of the phase-space volume occupied by the beam during the deceleration is assured by cooling at several energies levels both the transverse and longitudinal emittances. In particular, at low energies use is made of an electron cooling system which exploits the action of a solenoid field generating linear coupling between the transverse degrees of freedom of the single particle motion. The coupling effect is not desirable mainly for two reasons. First, the AD working point is quite close to the main diagonal of the frequency diagram. The effect of the difference resonance driving term [2] on the beam dimensions (which should remain below a given threshold in order to optimize the electron cooling performance) can not be neglected. Second, in presence of coupling the tunes can approach each other up to a minimum distance (which corresponds to C^- in the single-resonance theory). Such a stopband seriously limits the space for monoeuvring around the AD working point unless a coupling compensation is applied. Four correctors (two skew quadrupoles and two solenoids) have been foreseen to compensate the AD linear coupling. After a careful choice of their position around the machine lattice, the two compensation strategies have been tested in order to maximize the area of the stability diagrams (see below). As in the case of the Hénon map the results are quite different: the matrix (summed resonance) compensation allows a better restoring of the unperturbed situation leading to an improvement of the dynamic aperture with respect to the single resonance compensation of about 10 %. We present the result of numerical investigation on the stability. The figures 2,3,4 have been obtained by a tracking procedure iterating the symplectic map representing the lattice over N turns , for each initial condition in the physical plane. If the orbit is still stable after the last turn, the stable initial condition is plotted in the stability (x, y) diagram. An analogous analysis has been also performed for

[2] Because of the proximity of the working point to the main diagonal of the frequency diagram, the summed and single resonance theories gives close values of C^-. See figure 1.

the Hénon map case [2].

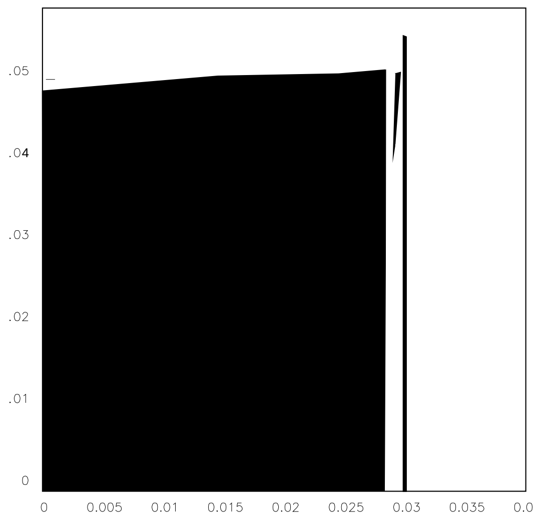

Figure 2: Stability domain of the uncoupled case for $N = 5000$.

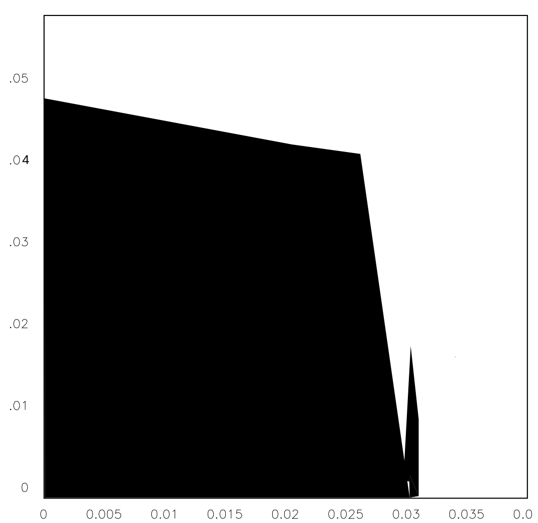

Figure 3: Stability domain of the summed-resonance compensation for $N = 5000$.

The comparison points out that the summed resonance compensation allows a more efficient restoration of the uncoupled optics.

4 CONCLUSION

A general method which sums up all the resonances within a given family is discussed. This techniques has been tested studying the problem of linear coupling compensation. We both considered the Hénon map and a lattice which reproduce the main features of AD at CERN. The results indicate that the summed-resonance compensation (numerically shown to be equivalent to the matrix one) is the more beneficial for stability and dynamical aperture.

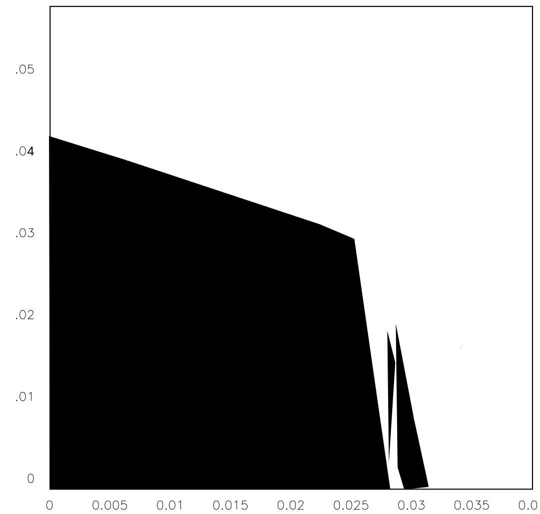

Figure 4: Stability domain after the single-resonance compensation for $N = 5000$.

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