THE CURRENT LOADING AND NONLINEAR BEAM DYNAMICS IN LINAC

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Abstract

The dynamics of high intensity electron beam in a traveling wave structure is studied. The equations of particles motion in self-consistent RF field are devised. The possible trajectories in phase space for the particles with some initial conditions are analyzed. The recommendations for choice of RF structure parameters to optimize the efficiency of the electron acceleration are done.

1 INTRODUCTION

In the design of high-current linear electron accelerators it is important to maximize the efficiency and minimize the energy spread of the accelerated beam. A correct study of the dynamics of an intense low energy beam in a waveguide requires to consider both the change of the particle velocity and the change of the amplitude and the phase velocity of the wave which interacts with the beam in the accelerating structure. This nonlinear task must be solved in a self-consistent manner.

2 SELF-CONSISTENT EQUATIONS OF MOTION

Let's consider that the injected beam is bunched at a field frequency ω_b. The beam current $J(z,t)$ can be expanded into a Fourier series:

$$J(z,t) = J_0 \cdot \left(I_0 + Re\sum_{\nu=1}^{\infty} \tilde{I}_\nu(z) \cdot \mathrm{e}^{-i\nu\omega_b t} \right),$$

where J_0 is an effective constant current and I_ν is a relative value for ν-th current harmonic. The interaction of the beam with only the forward wave is suggested. In the filling time $t_f = L/c\beta_g$ a steady-state field distribution is established over the entire section length L , such that its dimensionless amplitude $\tilde{A}(\xi) = e\tilde{E}^+\lambda / mc^2$ is a function only of the longitudinal coordinate $\xi = z/\lambda$ and the particle energy $\gamma(\xi,\varphi_0)$ is function ξ and initial phase φ_0. In a self-consistent field the rate of energy gain for particles can be written as:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\xi} = \mathrm{Re}\tilde{A}(\xi)e^{i(\varphi-\varphi_0)} + F_c\, Im\sum_{\nu=1}^{\infty}\frac{\Gamma_\nu}{\nu}\tilde{I}_\nu\, e^{i\nu(\varphi-\varphi_0)} \quad (1)$$

The fields produced by the beam consist of the RF field and the Coulomb field. In the Eqs. (1) $F_c = 2J_0\lambda^2 / \pi J_\alpha b^2$ is dimensionless coefficient determined the longitudinal Coulomb field, b is a radius beam, $J_\alpha = 4\pi\cdot\varepsilon_0\cdot mc^3/e$, Γ_ν is a coefficient of shielding. The nonlinear phase can be found from the equation

$$\mathrm{d}\varphi/\mathrm{d}\xi = 2\pi\left(1/\beta_b(\xi) - 1/\beta(\xi,\varphi_0)\right) \quad (2)$$

We also assume that parameters of the accelerating structure are such that the waveguide impedance $R_n(\xi)=E^2\lambda^2/2P$, the wave phase velocity in the absence of the beam $\beta_{ph}(\xi)$ and the damping coefficient $w(\xi)=\alpha\lambda$ are slowly varying functions of the longitudinal coordinate ξ. In this case the equation for the RF field amplitude is:

$$\mathrm{d}\tilde{A}/\mathrm{d}\xi + w_1\tilde{A} = -\tilde{I}_1(\xi)B(\xi), \quad (3)$$

where $w_1 = w - \mathrm{dln}R_n / 2\mathrm{d}\xi$. The last equation can be derived by using the Vainshtein method [1], generalized to the case of a changeable impedance of the structure [2]. The dimensionless parameter $B(\xi) = \pm\dfrac{eJ_0R_n(\xi)}{2mc^2}$ determines the coupling of the beam with the wave in the accelerator: the upper sign corresponds to a structure with a positive dispersion (β_g>0), while the lower one corresponds to negative dispersion (β_g<0). The system of equation (1)-(3) describes the changes of the particle energy, the complex amplitude $\tilde{A}$ and phase φ of the self-consistent field in the accelerating structure.

3 INTEGRALS OF MOTION AND LAWS OF CONSERVATION

Let us write the equation for the beam energy gain averaged over all injection phase φ_0 of the particles:

$$\frac{\mathrm{d}\gamma_c}{\mathrm{d}\xi} = \frac{1}{2}\mathrm{Re}\left(\tilde{A}\cdot\tilde{I}_1^*\right). \quad (4)$$

Here $$\gamma_c = \overline{\gamma(\xi,\varphi_0)} = \frac{1}{2\pi}\int_0^{2\pi}\Pi(\varphi_0)\gamma(\xi,\varphi_0)\mathrm{d}\varphi_0;$$

$\Pi(\varphi_0)$ is initial function of the particles distribution. The Coulomb part of field is equal to zero past averaging. Eliminating the complex value of the first current harmonic for Eqs. (3) and (4) we can find an equation relating the modulus of complex amplitude for RF field, ($A = |\tilde{A}|$), and the beam energy γ_c in the waveguide:

$$\mathrm{d}A^2/\mathrm{d}\xi + 2w_1A^2 = -4B\mathrm{d}\gamma_c/\mathrm{d}\xi \quad (5)$$

This equation expresses conservation of energy. With pronounced current loading, in which case the damping at the wall can be neglected for a constant impedance structure, we find:

$$4B\gamma_c + A^2 = H_1 \qquad (6)$$

The total power flux of RF field and beam is equal to constant H_1 for every cross-section of the waveguide.

Let us multiply left and right parts of equation (1) by $d\varphi/d\xi$ and average the result over all injection phase φ_0 of the particles:

$$2\pi\overline{\frac{d\gamma}{d\xi}\left(\frac{1}{\beta_b}-\frac{1}{\beta}\right)}=\frac{1}{2}\,\mathrm{Im}\left\{\frac{d}{d\xi}\left(\tilde{A}\tilde{I}_1^*\right)+w_1\left(\tilde{A}\tilde{I}_1^*\right)\right\}-$$

$$-\frac{1}{4}F_c\sum_\nu\frac{\Gamma_\nu}{\nu^2}\frac{d\left|\tilde{I}_\nu\right|^2}{d\xi}$$

In the simplest case, β_{ph}=cte, w_1=0 , from above equation (7) one gets:

$$\overline{f(\gamma)}-\frac{1}{2}\,\mathrm{Im}\left(\tilde{A}\tilde{I}_1^*\right)+\frac{1}{4}F_c\sum_\nu\frac{\Gamma_\nu}{\nu^2}\left|\tilde{I}_\nu\right|^2=H_2, \qquad (8)$$

where $f(\gamma)=2\pi\left(\gamma/\beta_b-\sqrt{\gamma^2-1}\right)$, H_2 is a new constant. It is the second law of conservation.

At last, let us multiply the equation (1) by ($d\varphi/d\xi$-1) and average the result over all injection phase φ_0. If initial beam modulation is absent we can find third law of conservation:

$$\overline{\left(d\varphi/d\varphi_0-1\right)\cdot\gamma(\xi)}=H_3 \qquad (9)$$

The two integrals (6) and (7) relate the beam energy, the amplitude of the self-consistent field, $A(\xi)$, and the harmonics of the beam current. The third integral can be used to determine the phase and energy spread of the beam.

4 EQUATIONS FOR TRAIN OF BUNCHES

The nonlinear beam dynamics in self-consistent field was solved for the case of constant impedance structure in Ref.[3] and for variable impedance structure in Ref. [4]. The basic assumption used in [3], [4] is that the beam can be treated as a train of well-grouped bunches. For large entrance amplitude of RF field $A(0)$ this assumption is good because the beam is bunched in the initial part of the waveguide. The equations system (1)-(3) is become simpler for the train of bunches. Let us introduce the difference of phases $\psi=\alpha_A-\alpha_I$ where α_A is a phase of complex amplitude $\tilde{A}=Ae^{i\alpha_A}$ and α_I is a phase of complex amplitude for the first current harmonic $\tilde{I}_1=I_1e^{i\alpha_I}$. Using the new phase ψ the equations (4), (2) and (3) become:

$$\begin{cases}\dfrac{d\gamma_c}{d\xi}=\dfrac{1}{2}AI_1\cdot\cos\psi,\\[2ex] \dfrac{dA}{d\xi}=-w_1A-I_1B\cdot\cos\psi,\\[2ex] \dfrac{d\psi}{d\xi}=2\pi\left(\dfrac{1}{\beta_b}-\dfrac{1}{\beta}\right)+\dfrac{I_1B}{A}\cdot\sin\psi.\end{cases} \qquad (10)$$

For the well-grouped bunches, when the bunches have a small phase size, $I_1\approx 2$ and (10) coincide with the self-consistent equations system for point bunches in [4]. For the case when the Coulomb term of the field and the damping RF field at the wall can be neglected the integral of motion (8) can be written as:

$$f(\gamma_c)-A(\xi)\cdot\sin\psi=H_2 \qquad (11)$$

This expression and the energy conservation law (6) are relating the bunch energy γ_c , its phase ψ and RF field amplitude A for every cross-sections of the waveguide. In the cylindrical system of the coordinates where A is radius, ψ is azimuth angle and γ is longitudinal coordinate the expressions (6) and (11) can be represent by the 3D second-order surfaces (fig.1.). The line of intersection for these surfaces is the solution of the equations system (10).

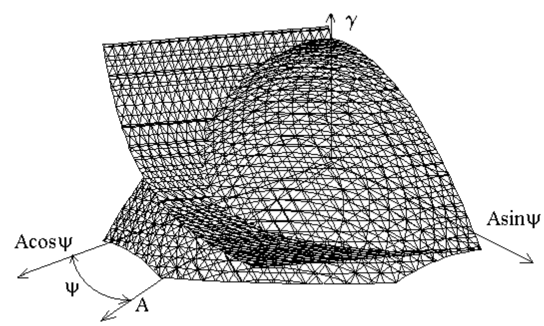

Figure 1.

5 PARTICLES PHASE TRAJECTORIES

When the amplitude A and the phase ψ of the self-consistent field are expressed in terms of the H_1 and H_2 values, the result is an equation whose solution yields the behavior of the bunch energy γ_c as a function of the longitudinal coordinate ξ :

$$\left(\frac{d\gamma_c}{d\xi}\right)^2+2U(\gamma_c)=H_1, \qquad (12)$$

where $U(\gamma_c)=\frac{1}{2}\{f(\gamma_c)-H_2\}^2+2B\gamma_c$.

The expression (12) is similar a nonlinear system Hamiltonian that consists of the «kinetic energy» and the «potential energy» U. The equation (12) shows that the shape of the «potential well» is governed by the H_2 and B. The value of H_1 determines the possible range of the bunch energy gain. The general behavior of the function

U (γ) for a waveguide structure with a positive dispersion when $\beta_{ph}< 1$ is shown on fig.2. For the structure with $\beta_{ph}< 1$ the function $U(\gamma_c)$ can have either a single minimum or two minimums and a single maximum. If $\beta_{ph}\geq 1$, the «potential well» has a single minimum only.

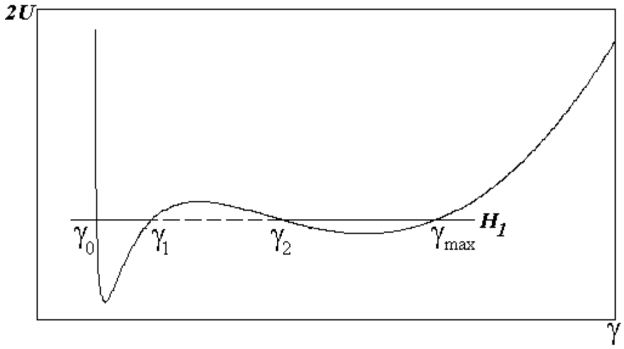

Figure 2.

The model of ideally grouped bunches gives a good description of the actual interaction of a modulated beam with an accelerating structure, provided that the phase stability of the bunches is not disrupted. In analyzing the beam dynamics it is important to know the behavior of the bunch phase in the course of the interaction of the bunch with RF field. It is convenient to use the energy-phase (γ_c, ψ) plane. At low energies, the particles have longitudinal stability if the bunch phase ψ lies in the interval $(0,\pi)$. As γ_c increases , the phase can go outside this interval for a certain time, but the trajectories on the (γ_c, ψ) plane should remain closed (the bunches are captured by the wave) if an acceptable energy spead of the beam is to be achieved.

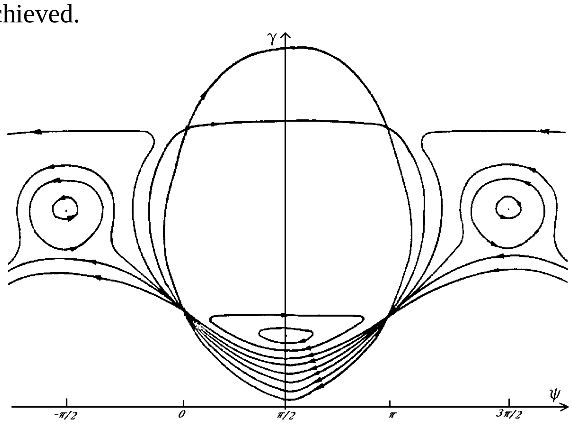

Figure 3

The behavior of γ_c as function of ψ can be found by means of the analysis for «potential well» U and the intersection line of H_1 and H_2 surfaces. The projection of this lines for different H_1 on the phase space (γ_c,ψ) is shown in fig.3. for the waveguide when $\beta_{ph}<1$ .For this case there are always closed trajectories near two minimums of the function $U(\gamma_c)$. From the large number of trajectories there is a single closed phase trajectory for which the efficiency, $\eta=4B(\gamma_{max}-\gamma_0)/A_0^2$, has maximum. The analysis of the phase portrait is shown that the «slip» of the bunch with respect to the wave makes it possible to achieve the amplitude A=0 for this trajectory. The phase ψ is equal to zero in this point. The last condition can be found from expression $\mathrm{tg}\psi=\left(f(\gamma_c)-H_2\right)/\left(\mathrm{d}\gamma/\mathrm{d}\xi\right)$, when $A\rightarrow 0$. If A and ψ are to approach zero, the following two conditions must hold:

$$\gamma_{max}=H_1/4B\,, \tag{13}$$

$$f(\gamma_{max})=H_2 \tag{14}$$

If γ_{max} is substituted into (14), we can find the value of the phase velocity at which the efficiency is maximized. However, the accelerating section cannot be optimized in terms of the efficiency for all values of β_{ph} found in this manner. The reason is that for small values of γ_c when $\beta_{ph}<1$ the equation $U(\gamma_c)=H_1$ can have two additional roots γ_1 and γ_2 which lie between γ_0 and γ_{max} . In this case the maximum beam energy is γ_1 , rather than γ_{max} (fig.2). For a given H_1 and a given H_2 there exists a limiting value $\beta_{ph,c}$ above which these intermediate roots do not exist. The nature of phase trajectories is very sensitive to the initial injection conditions, i.e., to the choice of H_1 and H_2, when a value of β_{ph} is near $\beta_{ph,c}$. In order to find the optimum efficiency conditions for the accelerating waveguide section using (14) we must always analyze the «potential well» $U(\gamma_c)$.

6 CONCLUSION

In the paper the self-consistent equation of the particles motion in the traveling wave structure has been solved. It is shown that the conservation laws relate the beam current and energy with RF field amplitude and phase. The phase space trajectories for the particles with some initial constants H_1 and H_2 was been analyzed. The recommendations for choice of the phase velocity β_{ph} and the coupling parameter B to optimize the efficiency of the electron acceleration was been given.

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