EXCITATION OF A SOLITARY PRECURSOR OF WAKE-FIELD TYPE BY ELECTRON BEAM

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1 INTRODUCTION

In [1] the excitation of electromagnetic soliton has been investigated at wakefield generation, which can not be described as envelope. The soliton is formed after electromagnetic pulse. The solitons are nonlinear wideband electromagnetic pulses. Many papers have been published on solitary perturbations and their applications (see, for example, [2, 3]).

At large intensity of laser radiation the qualitative changing of interaction of this radiation with plasma is realised. Namely, the possibility of solitary perturbation formation appears. Really, experiments have shown that if the dispersion relation of excited oscillations is linear, then the solitary perturbation formations are possible.

There were many attempts to construct analytical solution in type of electromagnetic solitary perturbation. In this paper similar solitary perturbation is investigated analytically.

2 PROPERTIES OF A SOLITARY PERTURBATION, PROPAGATING WITH LIGHT VELOCITY

In magnetized plasma one mode, propagating under angle θ to the magnetic field $H_o \to \infty$ has following dispersion relation

$$\omega = ck\omega_p \cos\theta \Big/ \left(\omega_p^2 + c^2k^2\right)^{1/2} \qquad (1)$$

Here ω_p is the electron plasma frequency; ω, k are frequency and wavevector; c is the light velocity. One can see that for $\omega_p << c\,k$ the dispersion relation is close to linear one $\omega \approx c\,k\cos\theta$. Hence, one can assume that a solitary perturbation can be formed on this mode. Let us derive a nonlinear equation, describing this solitary perturbation.

Let us consider plane metallic waveguide filled by plasma. The waveguide is of dimension **a** along axis **y**; the perturbation propagates with velocity $\vec{V}_s$ in (x, z) under angle θ to axis **z**. We consider the solitary perturbation of electric potential φ of small amplitude, $-\varphi_o$. From Maxwell equations one can derive the equation for electric field $\vec{E}$

$$\begin{aligned}&\Delta\vec{E} - \left(\vec{V}_c\vec{\nabla}\right)^2 \vec{E}\Big/c^2 + \\ &+ 4\pi e[\vec{\nabla} n - \left(\vec{V}_c\vec{\nabla}\right) n\vec{v}\big/c^2] = 0\end{aligned} \qquad (2)$$

Here $n, \vec{v}$ are density and velocity of plasma electrons. For latter determining we use the kinetic equation for electron distribution function f_e

$$\begin{aligned}&\partial f_e/\partial t + (\vec{v}\vec{\nabla})f_e - \\ &-\left(e/m_e\right)\left(\vec{E} + \left[\vec{v}\times\vec{B}\right]/c\right)\partial f_e/\partial\vec{v} = 0\end{aligned} \qquad (3)$$

As we consider strong magnetic field $\vec{H}_o \to \infty$, directed along z axis, then the plasma electrons propagate along this axis. Because we derive solution in kind of stationary soliton, propagating with velocity $\vec{V}_s$, we use dependence of f_e on coordinate and time $\vec{r} - \vec{V}_s t$. In this case equation (3) has following form

$$\begin{aligned}&-(\vec{V}_s\vec{\nabla})f_e + v_z\partial_z f_e + \\ &+\left(e/m_e\right)\left(\partial_z\varphi\right)\partial f_e/\partial v_z = 0,\end{aligned} \qquad (4)$$

where $E_z = -\partial\varphi/\partial z$. From Vlasov equation for electrons (4) one can obtain expressions for electron density perturbation $\delta n = n - n_o$ and their z component of current

$$\begin{aligned}&\delta n = -n_o\left(e\varphi\cos^2\theta/m_e V_s^2\right)\times \\ &\times\left(1 - 1{,}5\, e\varphi\cos^2\theta/m_e V_s^2\right), \\ &j_z = -eV_s\delta n/\cos\theta\end{aligned} \qquad (5)$$

Using (5), from (2) we derive nonlinear equation for small amplitudes

$$\varphi'' = [k_\perp^2 - \left(1 - V_s^2/c^2 \cos^2\theta\right) \times \\ \times \left(1 - 1{,}5 e\varphi \cos^2\theta / m_e V_s^2\right) \times \\ \times \omega_p^2 \cos^2\theta / V_s^2]\varphi / \left(1 - V_s^2/c^2\right) \tag{6}$$

"'" is the space derivative along direction of perturbation propagation, $k_\perp$ is the transversal wavevector. Integrating (6), one can obtain equation for $\phi = e\varphi\cos^2\theta / m_e V_s^2$

$$(\phi')^2 = \phi^2\omega_p^2[k_\perp^2/\omega_p^2 + 1/c^2 - \\ -\cos^2\theta / V_s^2 + \phi(\cos^2\theta/V_s^2 - \\ -1/c^2)]/(1 - V_s^2/c^2) \tag{7}$$

Using $\phi'\big|_{\phi=-\phi_o} = 0$, one can derive the expression for velocity of solitary perturbation in type of electric potential well

$$V_s \approx \\ \approx \frac{c\cos\theta\left[1 + \phi_o c^2 k_\perp^2 / 2\left(c^2 k_\perp^2 + \omega_p^2\right)\right]}{\left(1 + c^2 k_\perp^2/\omega_p^2\right)^{1/2}} \tag{8}$$

One can see that the solitary perturbation is formed on two modes with dispersion relations $\omega \approx \omega_p k/k_\perp$ and $\omega \approx ck\cos\theta$. The velocity of the solitary perturbation grows with amplitude.

Approximately from $\Delta\xi = \phi_o / \phi'\big|_{\phi=-\phi_o/2}$ one can find the expression for width of the solitary perturbation

$$\Delta\xi = \\ = \frac{\left(2m_e/e\varphi_o\right)^{1/2} 2c\omega_p \left(c^2k_\perp^2 + \omega_p^2\sin^2\theta\right)^{1/2}}{\left(c^2k_\perp^2 + \omega_p^2\right)k_\perp} \tag{9}$$

and at $\sin\theta << ck_\perp/\omega_p << 1$ $\Delta\xi \approx 2c^2\left(2m_e/e\varphi_o\right)^{1/2}/\omega_p$. One can see that the width of the the solitary perturbation $\Delta\xi$ decreases with its amplitude φ_o , and there are three parameters for control of the solitary perturbation properties: $k_\perp, \omega_p, \theta$.

Using (8), one can solve the equation (7) in type

$$\phi = -\phi_o / ch^2\left[\xi\left(\phi_o\eta\right)^{1/2}/2\right] \tag{10}$$

where ξ is the coordinate along the direction of perturbation propagation $\eta = k_\perp^2\left(c^2k_\perp^2 + \omega_p^2\right)/\left(c^2k_\perp^2 + \omega_p^2\sin^2\theta\right)$..

3 EXCITATION OF A SOLITARY PERTURBATION BY AN ELECTRON BEAM

Up to this time we have considered the stationary soliton. Taking into account the electron beam and time derivative in Vlasov equation should lead to growth with time of soliton amplitude. Hence from Vlasov equation for distribution function of electrons f_e one can obtain

$$\partial_t f_e^{(o)} + (V - V_s/\cos\theta)\partial_z f_e^{(1)} \approx 0 \tag{11}$$

which is obtained by assuming smallness and a slow time and space variation of ϕ . Here $f_e^{(o)}$ is the quasistationary distribution function of electrons, $f_e^{(1)}$ is the perturbation of electron distribution function, determined by nonstationary of potential $\phi(t)$. Integrating (11) over velocity one can get the next order correction to the space derivative of the electron density. This expression must be equal to n_b', the space derivative of the electron beam density perturbation, which follows from the space derivation of Poisson's equation. The latter quantity is found from the hydrodynamic electron equations in linear approximation which read at $V_b \approx V_s/\cos\theta$

$$\partial_t^2 n_b = -n_{bo}(e/m)\partial_z^2\varphi \tag{12}$$

From (11) , (12) and Poisson equation follows

$$\partial_t^3\varphi = -(n_{bo}/2n_o)V_s^3\varphi''' \tag{13}$$

From (6), (7), (13) one can derive

$$\partial_t^3\phi = \\ = -\frac{(n_{bo}/2n_o)\phi'(\phi_o + 3\phi)\omega_p^2 V_s\cos^2\theta}{\left(1 + \omega_p^2\sin^2\theta/c^2k_\perp^2\right)} \tag{14}$$

A solution of (14) is obtained by an appropriate extension

$$\phi = \phi_o(t)\mu[\xi - \int_{-\infty}^{t} d\tau\delta V_s(\phi_o(\tau))], \tag{15}$$

where from (10) $\mu(\xi)=1/\mathrm{ch}^2(\xi\sqrt{2}/\Delta\xi(\phi_o))$. In (15) a change δV_s of the soliton velocity V_s due to the interaction with the electron beam is taken into account. From (14), (15) one can derive for δV_s and growth rate $\gamma=\partial \ln\phi_o / \partial t$ of amplitude ϕ_o following expressions

$$\delta V_s \approx V_s (n_{bo}/n_o)^{1/3},$$
$$\gamma \approx \tag{16}$$
$$\approx \frac{\omega_p \cos^2\theta (n_{bo}/n_o)^{1/3}(1.5e\varphi_o/mV_s^2)^{1/2}}{(1+\omega_p^2\sin^2\theta / c^2 k_\perp^2)}$$

Note the appearance of $(n_{bo}/n_o)^{1/3}$ in both expressions as in the linear electron-beam-plasma instability.

4 CONCLUSION

Thus a beam-plasma-type interaction between the electron beam and electron solitary perturbation leads to its growth in metallic magnetized plasma-filled waveguide in coincidence with a similar result obtained in [4-24] for solitary perturbations in drifting plasmas and in beam-plasma systems.

From (8), (9) and (16) the properties of solitary perturbation are followed at $\theta=0$

$$V_s \approx$$
$$\approx \frac{c\left[1+\phi_o c^2 k_\perp^2 \Big/ 2\left(c^2 k_\perp^2+\omega_p^2\right)\right]}{\left(1+c^2 k_\perp^2 \big/ \omega_p^2\right)^{1/2}}$$

$$\Delta\xi = \left(2m_e / e\varphi_o\right)^{1/2} 2c^2\omega_p \Big/ \left(c^2 k_\perp^2+\omega_p^2\right) \tag{17}$$

$$\gamma \approx \omega_p (n_{bo}/n_o)^{1/3}(1{,}5e\varphi_o / m_e V_s^2)^{1/2}$$

Also the properties of solitary perturbation are interesting for $k_\perp = 0$

$$V_s \approx c,\ \ \Delta\xi = \left(2m_e / e\varphi_o\right)^{1/2} 2c^2 \Big/ \omega_p ,$$

$$\gamma \approx \left(\omega_p / c\right)(n_{bo}/n_o)^{1/3}(1{,}5e\varphi_o / m_e)^{1/2} \tag{18}$$

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