# EXPERIMENTAL STUDY OF RESONANCES IN A HIGH INTENSITY SYNCHROTRON

T. Uesugi, S. Machida, Y. Mori, KEK, Ibaraki-ken, Japan

## Abstract

We studied experimentally the behavior of the half integer resonance in HIMAC synchrotron at NIRS(National Institute of Radiological Sciences). The experimental results were analyzed with a multi-particle simulation.

# **1 INTRODUCTION**

In a high intensity synchrotron, emittance growth or beam loss due to space charge effects is a serious problem. However, the detailed mechanism is not clear. In particular, the presence of space charge (incoherent) tune spread makes it difficult to understand in terms of resonance conditions. In 1968, Sacherer proposed a coherent model of the half integer resonance, based on the envelope equation with smooth approximation on external field and linear approximation on space charge field[1]. One of his results is that, if beam size growth is negligible, the condition of half integer resonance is represented with quadrupole mode coherent tune. If the beam size growth is considered, the resonance there arises a detuning effect to decrease space charge effect.

Based on the analytical prediction by Sacherer, we performed the experimental study of space charge effects and investigated beam size growth and detailed mechanism of beam loss, especially in the vicinity of a half integer resonance. The experiment was carried out in HIMAC synchrotron with  $He^{2+}$  coasting beam.

## 2 EXPERIMENT AT HIMAC

Table 1: The parameters of HIMAC synchrotron at injection energy.

| parameter       | value  |
|-----------------|--|
| circumference   | $2\pi R = 129.6$ m                                     |
| beam energy     | $K_{inj}$ = 6.0MeV/u ( $\gamma$ =1.06, $\beta$ =0.113) |
| repetition time | 3.3sec   |
| acceptance      | $(264\pi, 26.4\pi)$ mm-mrad                            |

Table 1 shows the parameters of HIMAC synchrotorn at injection energy. The quadrupole mode coherent tune was measured with a quadrupole beam monitor consist of four electrodes. Fig. 1 shows the intensity dependence of the quadrupole mode tune. That coefficients were  $\Delta \nu_{\rm q}/2 = 0.0028/10^{10} {\rm ppp}$ . Assuming that the horizontal emittance satisfies full acceptance, the vertical rms emittance were calculated to be  $3.13\pi {\rm mm}{\rm -mrad}$ .



Figure 1: Tune shift of quadrupole mode oscillation.

#### 2.1 Resonance cross experiment

Table 2: The operation parameters in resonance crossing, downward(left) and upward(right).

| experiment                         |            |            |  |
|------------------------------------|------------|------------|--|
| initial value $\nu_{0y}(0)$        | 3.567      | 3.458      |  |
| derivative $d\nu_{0y}/dt$          | -0.030/sec | +0.030/sec |  |
| simulation                         |            |            |  |
| initial tune $\nu_0(0)$            | 3.5157     | 3.4951     |  |
| derivative $d\nu_0/dt$             | -0.29/sec  | +0.29/sec  |  |
| incoherent tune shift $\Delta \nu$ | 0.0067     | 0.0078     |  |
| stopband width $\delta \nu$        | 0.01       | 0.01       |  |

We controled the current of defocusing magnet to bring the vertical tune to the 3.5 half integer, and observed the circulating current falling. A pair of qudrupole magnets (QDS) was also excited with  $-10\sim10A$  to control the strength of gradient error. Table 2 shows the conditions of resonance crossing downward and upward, where  $\nu_{0y}$  is the vertical betatron tune in zero-current limit. Horizontal tune was approximately constant at 3.36. For the comparison, we also took same data with low intensity beams.

We show typical beam loss wave-forms in Fig. 2. The figure shows that the qualitative behaviors of beam loss depend on the direction of resonance crossing. In the case that resonance was crossed downward, the beam loss occured gradually, while it occured rapidly in the other case. Those characteristic was independent of QDS current. The gradual loss behavior in downward case can be explained by the detuning effect with beam intensity decrease.

Fig. 2 also shows that the time of beam loss depends on the strength of QDS. We define critical tunes  $\nu_c$  by the unperturbed bare tune corresponding to the time at which beam loss began. Fig. 3 plots the critical tunes as functions of QDS current. The ambiguity of reading beam loss time



Figure 2: Typical beam loss wave-forms when resonance line is crossed (A)(B)downward or (C)(D)upward. In (B)(D), the beam current was decreased at an injection line. The QDS current is -4A or 0A.



Figure 3: Critical tunes of a half integer resonance as functions of QDS current.

gives systematic errors on  $\nu_c$  in the order of 0.001 in downward crossing case. The slope  $|d\nu_{0c}/dI_{qds}|$  was consistent with the expected value of stopband width of the half integer resonance caused by QDS current. The intensity dependence of critical tune shows the space charge effect on resonant beam loss.

### **3 MULTI-PARTICLE SIMULATION**

We developed a multi-particle simulation in vertical dimension. The dispersion effect, choromatic effect, and the thickness of qudrupole magnets are neglected. The strength of error magnets and beam intensity were chosen taking after those of experiments at  $I_{rmqds}$ =-4A. However, initial tune is nearer to 3.5 and their derivatives are faster than experiments in order to save the calculation time. The simulation parameters are listed in Table 2.

Fig. 4,5 show the simulation result for a uniformly and a parabolically charged beam, respectively. It is shown in (A) that the beam loss occured gradually in the case of decreasing tune, and rapidly in the other case, as oberved in the experiments. According to Figs. 4(B),5(B)), the growth of beam sizes before starting beam loss was also gradual and rapid in the cases of decreasing and increasing tune.

The rms beam size agreed with the matched solution of the envelope equation with fixed external field strength there(broken line), in each charge distribution. Fig. 4(D),5(D) shows the 99%, 90% and 75% beam sizes devided rms beam size, which are indices of beam distribution. In both distribution, they are almost constant until beam loss starts, so that the change in distribution is expected very small.

Here we compare the simulation results with experimental ones, quantatively. Since the aperture limit is a little larger than that of HIMAC synchrotron, We calculated the maximum of full beam size about by

$$y_{\max} = \tilde{Y}_m \frac{y_{99\%}(0)}{y_{rms}(0)} \tag{1}$$

where  $Y_m$  is the maximum value of matched beam size with respect to s and  $(y_{99\%}(0), y_{rms}(0))$  are the 99% and rms beam size observed at s=0. The critical tune in simulation was defined by the time at which  $y_{max}$  exceeds 8.15, which corresponds to the aperture of HIMAC. The results are  $3.512 \sim 3.515$  in the case of downward approach, depending on distribution, and 3.499 in ohe other case. The former is consistent with the experimental value 3.513, however the latter is significantly nearer than experiment 3.496.

We observed the tunes of randomly picked up five particles and qudrupole mode coherent tune (Fig. 4(C),5(C))). In the uniformly charged beam, the incoherent tunes agreed with the tune calculated with the matched solution  $\tilde{y}_0(s)$  of rms envlelope equation, by

$$\nu = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)} \tag{2}$$

where  $\beta(s) = \sqrt{\tilde{y}(s)E}$  is betatron amplitude and s is the longitudinal coordinate. In the case of downward approach, the decrease in tunes were decelerated, and they never cross the half integer.

## 4 DISCUSSION

Both experimental and simulation result showed that the beam loss occurs gradually when the tune approaches a half integer in the decreasing direction, and rapidly in the other direction. In addition, the growth of the beam size was also occured gradually when the tune approaches a half integer in decreasing direction, and rapidly when it approaches in increasing direction. Those qualitative characteristics agreed with Sacherer's analysis.

The rms beam size before beam loss began agreed with the matched solution of envelope equation of without changing focusing strengths with time. The condition of beam loss relates not to the rms beam size but the full size, so that the criteria depends on distribution. However,



Figure 4: Simulation results for a uniformly charged beam. Horizontal axis corresponds to turn number. (A)Beam loss wave-form. (B)The rms size of simulated beam(waving line) together with the matched solutions of envelope equation(the other). (C)Observed tunes; black circle shows the half of quadrupole mode coherent tune and white characters show the tunes of single particles. The dashed line is unperturbed tune, which is identical with  $\nu_0$ . (D)The 99%, 90% and 75% beam sizes devided by rms beam size.

the simulation results tells that the full beam size grows in proportional to the rms size. Therefore, we can estimate the threshold of beam loss with finding the matched solution of envelope equation.

As to our experiments, the threshold of beam loss was consistent with that of simulation, in the case of decreasing tune. However in the other case, it was lower than simulation. This may mean that the charge distribution is much more wide than simulation because of the injection just below the excited half integer tune. Other possible reasons of that disagreement are the tune spread due to chromatic effect ( $\sim$ 0.04), due to nonlinear external field, and effects of higher order resonance on charge distribution.

In the process of resonant beam size growth, both incoherent tunes and a half of quadrupole mode coherent tune did not necessarily reach the half integer value, as the particular case of uniform beam shows. Therefore, Therefore, the condition of half integer resonance in the presence of space charge effect can not be represented simply in terms of tune.

In summary, we investigated the behavior of a beam experimentally near a half integer tune, with the help of multiparticle simulation. Both experimental and simulation result showed:



Figure 5: Simulation results for a parabolically charged beam.

- 1. Analysis of a half integer resonance using rms envelope equations gives fairly good explanation to an actual beam in a synchrotron, where the external field employs AG focusing scheme unlike uniform focusing which the analysis is based on.
- 2. The resonant behavior of rms beam size is independent of detailed charge distribution. However, the beam loss criteria depends directly on the distribution of large amplitude particles.
- 3. The beam loss due to a half integer resonance occur when the betatron tunes are near a half integer. However, simple measures such as coherent and incoherent tune are not sufficient to describe the resonant beam growth quantatively.

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