CHERENKOV RADIATION MECHANISM IN TWO-BEAM ACCELERATION PROBLEMS

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Abstract

Two-beam acceleration scheme with separated trajectories based on Cherenkov radiation in the bicylindrical waveguide with canal cut in the dielectric medium with the thin quasiconducting layer along the both axes is considered. The case of periodical sequence of the bunches is considered when a single-mode regime is installed as the frequency of the repetition rate of the bunches. The field distribution in the waveguide is researched. It is shown that sufficiently high intensities of the field are generated in the waveguide being suitable for the two-beam acceleration mechanism.

1 INTRODUCTION

We are considering relativistic electron beam reacceleration problem based on two beam interaction in the bicylindrical waveguide through the energy exchange between two beams while one of them (the leading beam) is generating Cherenkov waves in the medium the waveguide filled with and the second one (the accelerating beam) is being accelerated in this field. The bicylindrical waveguide permits one to implement the acceleration scheme with separated trajectories of these beams at the beams' simultaneous passing. The bicylindrical waveguide is represented as a regular waveguide with the cross section in the form of two partly overlaped circles, generally, with different radii. The theory of this structure is developed in [1], where a bicylindrical modes classification is suggested based on the analyse for the "limiting" form and the algorithms and programs are developed to calculate the bicylindrical eigen-modes. Based on these investigations we have chosen as a working mode the second bicylindrical mode in the bicylindrical waveguide with equal radii of both of cylinders.

The consideration of the periodical sequence of the bunches provides the Cherenkov waves generation in single mode regime, when the quasi-monochromatic Cherenkov wave with high amplitude is propagating along the waveguides at the frequency (mode) which is equal to the frequency of the bunches' repetition rate.

Some problems are arising at this consideration. The first problem is the polarization losses (Bohr's losses) [2] at the beam interaction with the medium. This effect is substantial one and brings to rapid energy losing and decelerating of the beam. To decrease the influence of these losses it is necessary that the canals should be cut in the medium for both beams - the leading and accelerating ones.

But Cherenkov effect takes place only in the medium and the radiation waves are attenuating if one removes from the boundary to the area, where the Cherenkov radiation condition isn't valid. Nevertheless the investigations which there carried out by us before for the case of the circle waveguide [3] show that the influence of the canal existence is not substantial in the S band of the wave, where the canal radius is about ten times smaller than the Cherenkov wavelength [4].

The next problem is the problem of the electrostatic charges having been accumulated on the inner surface of the canals when beam passes through them. To remove this charge we propose to cover the inner surfaces of the canals with a quasi-conducting thin layer. The problem is in the elucidation of this thin layer's influence on the Cherenkov wave forming in the waveguide.

The numerical method of the bicylindrical modes definition developed in (1-2) is not applicable, in particular, at the presence of two canals cut along the two axis in the medium the bicylindrical waveguide filled with to decrease the polarization losses, when the beams are interacting with this medium.

We suggest some extrapolation approaches based on the results received before (see above) analytically for circle waveguide loaded with the dielectric and the canal cut in the medium along the axes of the waveguide [3].

2 THE MAIN THEORETICAL STATEMENTS

We received the next expression for the longitudinal E_z component of the Cherenkov field of the periodical sequence of N identical cylindrical bunches with the radius r_0 and charge q moving at the velocity $v = v_z$ along the axis z, which is parallel to the generator of the cylinder and the charge distribution along the length of the bunch is a Gaussian with *rms* length $\sqrt{\overline{\xi^2}}$ and in the cross-section – uniform,:

$$E_z = 4\pi^2 q \sum_n \frac{\Psi_n(\vec{r})\int_0^{r_0}\Psi_n(\vec{r})rdr}{\varepsilon(\omega_n)} \; e^{-\frac{\omega_n^2\overline{\xi^2}}{4v^2}} \times \frac{\sin[N\omega_n d/2v]}{\sin[\omega_n d/2v]}\cos\frac{\omega_n}{v}\;[z - vt - (N-1)d] \quad (1)$$

In (1) $\vec{r}$ is the coordinate of the point in the cross-section of the waveguide. The own frequencies ω_n are defined by the formulae

$$\omega_n = \frac{\chi_n v}{\sqrt{\varepsilon\mu\beta^2 - 1}}; \ \left(\beta = v/c\right) \qquad (2)$$

which is expressed by the own values χ_n of the own functions (normalized) Ψ_n of the cross-section of the bicylindrical waveguide. The χ_n are defined from the boundary condition for the E-type of the waves

$$\Psi_n(\vec{r})\Big|_{\Sigma} = 0 \qquad (3)$$

on the contour Σ of the waveguide.

The calculations having been executed using the formulae (1,2) show that will be observed very high intensity of the Cherenkov fields: about 150 MV/m for the beam parameters of YerPhI test facility LINAC-20 (~3X10^9 particles per bunch, N=3X10^4 bunches per pulse, with the repetition rate ~3GHz, i.e. d=10cm between the neighboring bunches). To estimate the real expected values of the fields it is necessary that some important factors should be taken into account: the problem of the polarization losses (Bohr's losses) and the problem of the electrostatic charges on the inner surface of the canals. The approach to the solution of this problem is based on the features of the bicylindrical functions. In Fig.1 it is illustrated the second bicylindrical E-mode in the equal-radii bicylindrical waveguide. On the left cylinder there is shown the first E - mode for the corresponding circular waveguide. The calculations for the bicylindrical mode are executed for the waveguide parameters $r_1 = r_2 = 4.1528\ cm$ and the distance between the centers $L = 7.4743\ cm$ corresponding to the weak coupling between the cylinders. The field distributions of the E_z component of the second bicylindrical mode (dotted) and the first cylindrical modes (thick) on the frequency of the repetition rate 2.7672 GHz are very close to one another.

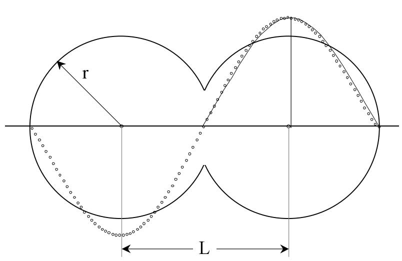

Figure.1.

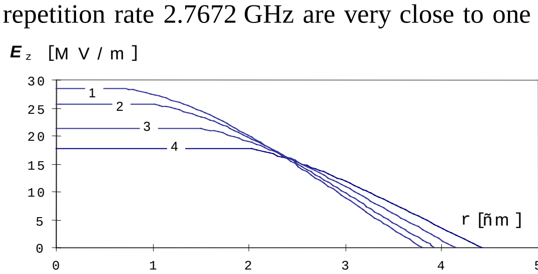

Figure 2.

As it was noticed above the method developed in [1] to define the bicylindrical modes isn't valid for the case of non-uniform cross-section (with canals). Therefore we suggest that the results received analytically for the circle waveguide [3] should be used. In Fig.2. there is brought the circle waveguide resonant radii dependence on the canals radii. As one can see, the distribution of the field accelerating component E_z in the cross-section is a quasiuniform (quasiconstant) one in the areal of the canals and decreases when approaching to the waveguide boundary. Such behavior of the field is very convenient for the acceleration. The resonant values of the radii (a) provide the single mode regime on the chosen (the second) mode with the high value of the field. These values depend on the radii of the canals (b). In Fig.3. there are illustrated four different values of the waveguide radii for the four different values of the canals radii:

1. b=0,7cm, a=3,802 cm; 2. b=1 cm, a=3,909 cm;
3. b=1,5 cm, a=4,138 cm; 4. b=2 cm, a=4,412 cm.

$$S = S_0 + f\left(S_{\text{can}}\right) \qquad (4)$$

The strict analytical expressions for the field permit one to construct the dependence (4) among the areas of the waveguide cross-sections without the canals (S_0) and at the canal's existence (S). The strict analytical expressions permits one to approximate this dependence, which turns be a quasilinear one:

$$f(x) = 1{,}854\,x - 0{,}0437\,x^2 + 0{,}0013\,x^3 \qquad (5)$$

The next step we have to do is extrapolation of the relationship (4) which is available for the circle waveguide for the case of the bicylindrical one owing two canals. We suggest the following extrapolation formula

$$S = S_0 + (2 - \delta^{\nu})\ f(x) \qquad (6)$$

where $f(x)$ is the same function we have introduced above, δ is the relation of the overlapped part of the circles to the whole value of the cross-section, ν is close to 1 for the case of the equal radii bicylindrical waveguide. To define the value for any other case one will have to carry out additional investigations each time. For $\delta = 1$ (full over-lapping) we have single circle waveguide with the cross-section S_0 and if $\delta = 0$ we have two separated circles with the twice more value for the cross-sections area. The accuracy of the extrapolation formula (6) is confirmed by two factors: the close distributions of the fields in bicylindrical and circle waveguides when the latter are calculated by strict analytical formulae and the low values (very close to zero) of the field in overlapped small area for the case of thin coupled quasicylinders as well.

Formula (6) permits one to define the "resonant" values for the bicylindrical waveguides cross-section which provides the single-mode regime on the frequency of the repetition rate of the bunches for the Cherenkov modes generation with high value of the field. As we have noticed above the field intensity made be estimated by the formula (1) for the uniform filling because the canals' sizes are much smaller than the wavelength at the single-mode regime (about 10 cm)

To consider the influence of the conducting layer on the Cherenkov field formation we also based on the strict analytical formulae for the circle waveguide. At first we'll consider Cherenkov radiation forming in the transparent dielectric ring. This consideration shows that at the reducing of the ring thickness Cherenkov spectrum is shifted into the high frequences and is restricted when the Cherenkov radiation condition $\beta^2\varepsilon > 1$ stops to be satisfied. Therefore it is more convenient to describe the permittivity of thin layer in the form where the Cherenkov radiation isn't be generated: $\varepsilon = 1 + i\, 4\pi\sigma/\omega$, σ is the conductivity.

For the case of conducting layer with thickness δ on the inner surface of the canal with the radius b, in the waveguide with the radius a we receive the following expressions for the E_z component of the Cherenkov waves of the periodical sequence of N bunches

$$E_{zi} = -\frac{2q\sqrt{1-\beta^2}}{(b-\delta)v}\sum_{\lambda} A_i(\omega_\lambda, r)\frac{2I_1(k r_0)}{k r_0}F\left(\frac{\omega}{v}\right)\Big/\frac{dD}{d\omega} \quad (7)$$

$$F\left(\frac{\omega}{v}\right) = 2e^{-\left[1-\left(\frac{w_1}{\omega_1}\right)^2\right]\frac{\pi\overline{\xi^2}}{d^2}} e^{-\frac{w_1}{v}|z-vt|}\frac{1-e^{-N\frac{w_1 d}{v}}}{1-e^{-\frac{w_1 d}{v}}}\cos\frac{\omega_1}{v}\left(z-vt+\frac{w_1\overline{\xi^2}}{2v}\right) \quad (8)$$

for the first circle mode at the frequency of the repetition rate of the bunches: $\omega_\lambda = 2\pi v/d$. Here $A_i(\omega_\lambda, r)$ - characterize the field distribution in three regions: in the dielectric (i=1), in the conducting layer (i=2) and in the canal (i=3) and own frequencies ω_λ are obtained from the equation $D = 0$.

The marks are made on (8): $\mathrm{Re}\,\omega \to \omega$, $\mathrm{Im}\,\omega \to w$ and if $w \to 0$ then $\lim\limits_{w\to 0} F\left(\frac{\omega}{v}\right) = 2N e^{-\frac{\omega^2\overline{\xi^2}}{4v^2}} \cos\frac{\omega}{v}(z - vt)$.

3 THE NUMERICAL ANALYSIS

Thin conducting layer is able to decrease essentially the amplitudes of the fields end to bring to zero the efficiency of the Cherenkov radiation mechanism in two beam accelerating problem. Indeed, the calculations having been carried out using the formulae (6-7) for the well conducting layer ($\varepsilon'' \sim 10^7$) with the thickness 10 Å show the inadmissable low level for the fields amplitude. On the other hand in order to remove the static charge, accumulated on the surface of the canal one can use some quasi-conducting material.

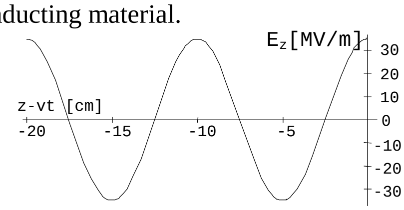

Figure 3.

The picture of the field is represened in the Fig. 2. This field is created by 3000 bunches, $3\cdot 10^9$ electron per bunch, for the permittivity $\varepsilon' = 1$ and $\varepsilon'' = 10^3$ and the layer thickness is 70 Å, the single mode regime is set with the amplitude ~ 35 MV/m.

In Fig.4 the field distribution in the cross section is brought:

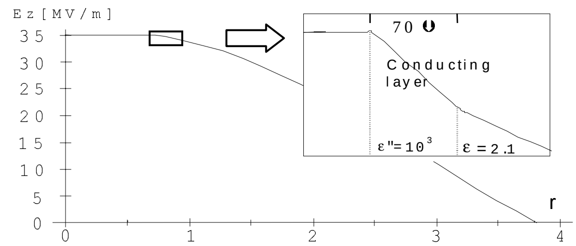

Figure 4.

In the conducting layer the field is decreasing more rapidly than in the dielectric and is qasiuniform in the canal. The Fig. 4 is increased 10^6 time in the quasi-conducting layer

Thus quasiconducting layer with a thickness of 70 Å permits one to receive a quasimonochromatic wave with an amplitude (35 MV/m) high enough. Such a layer permits one to remove electrostatic charges.

The dependence of the interference factor on the number of the bunches which describes the signal amplifying is brought in Fig 4

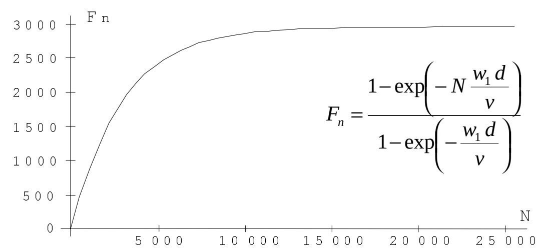

Figure 5.

It is seen that the existence of quasiconducting layer brings to rapid satiation of the field.

4 CONCLUSION

The investigations presented above show the efficiency of the Cherenkov mechanism in electron re-acceleration TBA scheme. There will be considered the thermal balance problem in the case of the beam interacting with the dielectric with quasi-conducting layer as well.

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