ON THE OPTIMUM RF FREQUENCY FOR A LOW ENERGY SYNCHROTRON RADIATION SOURCE*

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Abstract

The rf frequency determines essentially the time structure of a storage ring synchrotron radiation source - bunch to bunch distance and bunch length - and influences some important technical parameters and boundary conditions of the rf system - cavity shunt impedance and the choice of the rf power source. For a low energy storage ring source the beam lifetime usually is dominated by the Touschek effect, which also depends implicitly on the rf frequency in a complicated way. It is shown that for a given average beam current and a fixed energy acceptance the Touschek lifetime increases with the rf frequency and approaches a nearly constant value at rf frequencies above a few hundred MHz for a 1GeV model storage ring. This demonstrates that under such conditions a low rf frequency (as sometimes suggested) cannot be the means to improve the Touschek lifetime.

1 INTRODUCTION

The optimisation of a synchrotron radiation (SR) source involves essentially the optimisation of the entire six-dimensional phase space of the electron beam. Much effort has been made in the last two decades to improve the transverse phase space parameters by inventing new lattice concepts for the modern high brightness SR sources. The longitudinal phase space on the other side has not seen innovations of comparable importance. One of the important parameters of the longitudinal phase space is the rf frequency f_{rf}, which determines the bunch to bunch distance and influences the bunch length and the bucket height. So far rf frequencies in the range from tens of MHz to about 800 MHz have been chosen for SR sources, mostly based on technical and practical arguments rather than on beam dynamical considerations. Among these practical arguments are the availability of adequate rf power sources, the size and shunt impedance of the cavity, or, in some cases, even the availability of surplus components. For frequencies up to typically 250 MHz tetrode amplifiers are commonly used whereas for higher frequencies klystron transmitters are more suitable. For example in the UHF range around 500 MHz, a large variety of TV-klystrons are commercially available.

For cavities of a given shape the size scales with the rf wavelength, which puts a lower practical limit on the rf frequency. On the other hand the shunt impedance per unit length is proportional to $\sqrt{f_{rf}}$. For frequencies well above 800 MHz the increasing thermal power densities in the cavity walls (excluding superconducting cavities here) lead to an upper limit for the rf frequency.

There are a few beam dynamical arguments in favour of a lower rf frequency and hence a longer bunch length: the beam spectrum covers only a smaller frequency band for the excitation of parasitic higher order modes (HOM's) of the cavity, which may be helpful to reduce the excitation strength for multi-bunch instabilities. If feedback systems are used to damp multi-bunch oscillations, the bandwidth of such a system, which is half the bunch frequency, can be reduced correspondingly. But these qualitative arguments did not play a significant role in the design of the present SR sources. One of the most important performance limitations for a low energy low emittance storage ring, however, is the Touschek lifetime, which also depends sensitively on the bunch current and bunch length. In this paper the issue of the optimum rf frequency is addressed with a detailed analysis of the rf frequency dependence of the Touschek lifetime.

2 TOUSCHEK LIFETIME

Following Le Duff [1] the Touschek lifetime can be expressed as

$$\tau_T = \frac{8\pi < \sigma_x\sigma_y > \sigma_l}{r_e^2 c N_b D(\epsilon)} \gamma^2 \left(\frac{\Delta E_{max}}{E}\right)^3 . \qquad (1)$$

In eq. 1 $< \sigma_x\sigma_y >$ is the average beam cross-section, σ_l the bunch length, $(\Delta E_{max}/E)$ the limiting energy acceptance, N_b the number of electrons per bunch and $D(\epsilon)$ the Touschek effect function

$$D(\epsilon) = \sqrt{\epsilon}\left[-\frac{3}{2}e^{-\epsilon} + \frac{\epsilon}{2}\int_\epsilon^\infty \frac{\ln u}{u} e^{-u}\,\mathrm{d}u + \frac{1}{2}(3\epsilon - \epsilon\ln\epsilon + 2)\int_\epsilon^\infty \frac{e^{-u}}{u}\,\mathrm{d}u \right]$$

$$\epsilon = \frac{1}{2\gamma^2}\frac{< \beta_x >}{\varepsilon_x}\left(\frac{\Delta E_{max}}{E}\right)^2 ,$$

where $< \beta_x >$ is the average horizontal beta-function, ε_x the horizontal emittance, and all other parameters have their usual meaning. The Touschek lifetime depends implicitly on the rf frequency through the quantities N_b, σ_l and $(\Delta E_{max}/E)$.

In multi-bunch operation (assuming that all buckets are equally populated) $N_b = I/(e_0 f_{rf})$, where I is the average beam current. Following [2] the bunch length can be

* Work supported by the Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie and by the Land Berlin.

written as

$$\sigma_l = c\,\sigma_{\scriptscriptstyle E}\sqrt{\frac{\alpha E}{2\pi f_{rf} f_0}\left\{\frac{1}{e_0 V_{rf}(1-(1/q)^2)^{1/2}}\right\}}\,, \quad (2)$$

with the overvoltage factor, given by $q = e_0 V_{rf}/U_0$, where V_{rf} is the total accelerating cavity voltage, U_0 the electron energy loss per turn and $\sigma_{\scriptscriptstyle E}$ the relative energy spread.
The energy acceptance $(\Delta E_{max}/E)$ of a storage ring is limited longitudinally by the rf system or transversely by nonlinear properties of the lattice or by the vacuum chamber aperture in dispersive ring sections. The rf acceptance $\delta_{rf} = (\Delta E_{max}/E)_{rf}$ can be written as

$$\begin{aligned} \delta_{rf}^2 &= \frac{U_0 f_0}{\pi \alpha f_{rf} E} F(q) \quad (3) \\ \text{with} \quad F(q) &= 2\left(\sqrt{q^2-1} - \arccos(1/q)\right) . \end{aligned}$$

In case of a rf-limited energy acceptance the Touschek lifetime depends in a complicated way on the rf frequency as well as on the rf voltage, and it is impossible to derive a closed expression for $\tau_{\scriptscriptstyle T}(f_{rf})$, the quantity of interest here.

For a practical storage ring the energy acceptance of the transverse motion $(\Delta E_{max}/E)_\perp$ is usually limited to a few percent. Therefore the best one can do is to provide a longitudinal acceptance equal to the transverse acceptance, $\delta_{rf} = (\Delta E_{max}/E)_\perp$. Now, for a fixed rf acceptance d_{rf} =const, the required rf voltage can be obtained from eq. 3 as a function of the rf frequency for a given storage ring.
For a large overvoltage factor, $q \gg 1$, $F(q)$ in eq. 3 can be approximated by $F(q) \approx 2q - \pi$. Substitution of $V_{rf}(f_{rf})$ into eq. 2 gives $\sigma_l(f_{rf})$, and consequently the Touschek lifetime can be evaluated as a function of the rf frequency using eq. 1, yielding less complicated expressions

$$\begin{aligned} V_{rf} &= \frac{\pi\, U_0}{2 e_0}\left(\frac{E\alpha}{f_0 U_0}\delta_{rf}^2\, f_{rf} \;+\; 1\right) \quad (4) \\ \sigma_l &= \frac{c\;\sigma_{\scriptscriptstyle E}}{\pi}\sqrt{\frac{E\,\alpha}{\delta_{rf}^2\,E\,\alpha\,f_{rf}^2 \;+\; f_0\,f_{rf}\,U_0}} \\ \tau_{\scriptscriptstyle T} &= \frac{8\;<\sigma_x\sigma_y>\;\delta_{rf}^3\;e_0\;\gamma^2\sigma_{\scriptscriptstyle E}}{I\;D(\epsilon)\;r_e^2}\;\times \\ &\quad \sqrt{\frac{E\;\alpha\;f_{rf}}{\delta_{rf}^2\;E\;\alpha\;f_{rf}\;+\;f_0\;U_0}} . \end{aligned}$$

If we only look for the f_{rf} dependency, we get

$$\begin{aligned} V_{rf} &\sim f_{rf}(1 + C/f_{rf}) \\ \sigma_l &\sim \frac{1}{f_{rf}}\sqrt{\frac{1}{1+C/f_{rf}}} \\ \tau_{\scriptscriptstyle T} &\sim \sqrt{\frac{1}{1+C/f_{rf}}} , \end{aligned}$$

where $C = f_0\,U_0/(\alpha\,E\,\delta_{rf}^2)$ is a constant given by the machine parameters. In all three expressions the term C/f_{rf} can be neglected for larger frequencies. Thus we expect that V_{rf} growths in proportion to f_{rf}, σ_l decreases with $1/f_{rf}$ and $\tau_{\scriptscriptstyle T}$ becomes independent of f_{rf}, as shown in Figure 1.

3 CURRENT DEPENDENT EFFECTS

So far we have assumed that the bunch length is determined only by the natural energy spread of the beam. Already at rather low threshold currents, however, turbulent effects for example will lengthen the bunch, and this in turn will increase the Touschek lifetime. Following [3] the lengthening of the bunch can be described by

$$\sigma_l^3 = \frac{\alpha\,R^3\,I_b}{Q_s^2\,\sqrt{2\pi}\,E}\left|\frac{Z(\omega)}{n}\right|_{eff} , \quad (5)$$

where Q_s is the synchrotron tune, $I_b = f_0\,e_0\,N_b$ the average bunch current, R the ring average radius and $|Z(\omega)/n|_{eff}$ describes the effective impedance of the vacuum chamber. Looking only to the f_{rf} dependent terms we obtain

$$\sigma_l^3 \sim N_b/Q_s^2 . \quad (6)$$

For constant momentum acceptance and $q \gg 1$, as in the previous case, we get

$$d_{rf} \sim \sqrt{V_{rf}/f_{rf}} = const \qquad \text{and} \qquad Q_s \sim \sqrt{V_{rf}\,f_{rf}} .$$

Taking into account, that for a constant average beam current $N_b\,f_{rf}$ =const, the bunch length is proportional to

$$\sigma_l^3 \sim \frac{N_b}{Q_s^2} \sim \frac{N_b}{V_{rf}\,f_{rf}} \sim \frac{1}{V_{rf}\,f_{rf}^2} \sim \frac{1}{f_{rf}^3} .$$

For the Touschek lifetime we finally get the relation

$$\tau_{\scriptscriptstyle T} \sim \frac{\sigma_l}{N_b} \sim \frac{1}{N_b f_{rf}} = const.$$

As in the case without bunch lengthening effects, we expect for larger values of the rf frequency a constant Touschek lifetime. These two cases will be discussed in the next chapter, using an explicit example.

4 NUMERICAL EXAMPLE

As a specific example, these parameters have been determined numerically for the recently proposed SESAME ring [4], using the machine parameters given in Table 1.
Calculations for the case without bunch widening effects have been performed applying eq. 4. The program ZAP [3] was used for current dependent simulations, including Potential Well Distortion (PWD), Turbulent Bunch Lengthening (TBL) and Intra Beam Scattering (IBS). The later effect does almost not influence the Touschek lifetime in our example, because of both, the high energy and the large natural emittance of our model ring.
Figure 1 shows the required rf voltage, the bunch length

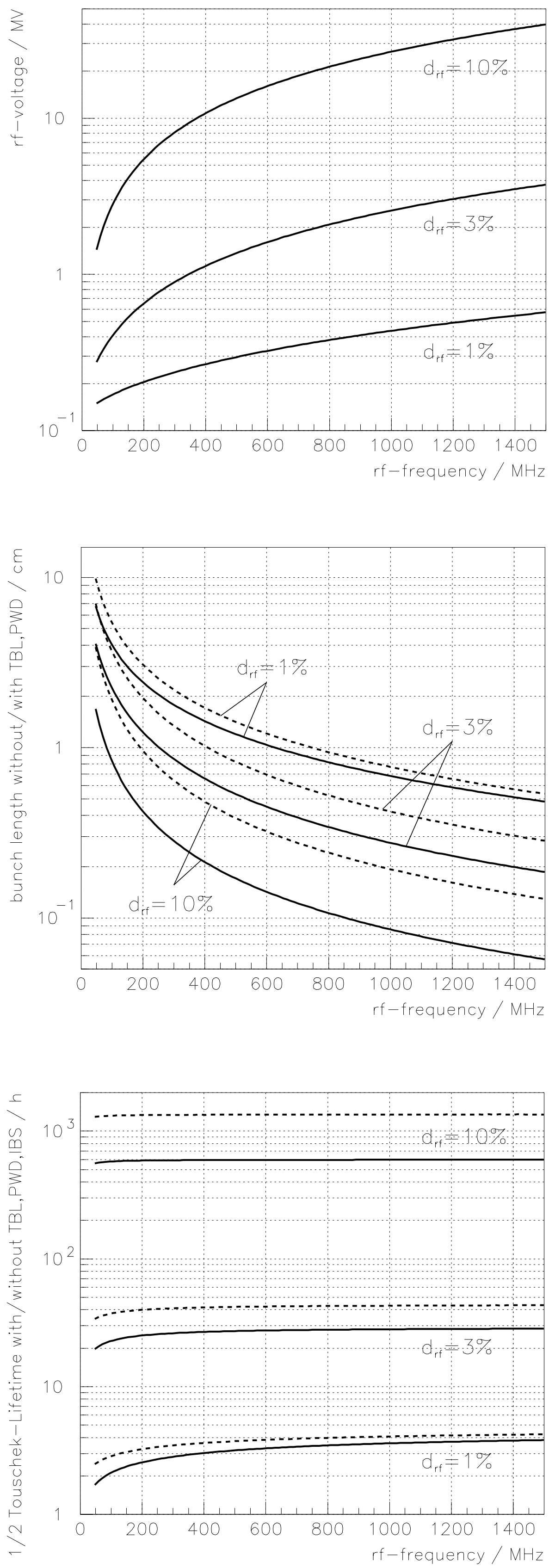

Figure 1: rf voltage, bunch length and Touschek lifetime vs rf frequency, (full line = without / dashed line = with bunch widening effects).

Energy E / GeV	1.0
Energy loss per turn U_0 / keV	120.0
momentum compaction α	0.005
Bending radius ρ / m	1.779
circumference C / m	100.8
revolution frequency f_0 / MHz	2.974
average current I / mA	700
emittance ε_x / nrad m	50
beam size $<\sigma_x\sigma_y>$ / m^2	$4.5 \cdot 10^{-8}$
mean beta-function $<\beta_x>$ / $<\beta_y>$ / m	8.0 / 10.0
eff. broadband impedance $\|Z/n\|_{\|}$ / Ω	3.0

Table 1: Machine parameters for the SESAME storage ring for 1% coupling, including two 7.5 T Multipole wigglers.

and the resulting Touschek lifetime for different rf acceptances. It can be seen, that although the Touschek lifetime increases moderately with the rf frequency in the lower frequency-range, it stays essentially constant at higher frequencies. Thus an rf frequency of around 500 MHz, which has often been adopted for practical reasons, is in fact a reasonable choice in terms of Touschek lifetime as well. A significant improvement in Touschek lifetime can only be achieved by increasing the energy acceptance, hence the lattices for several 3rd generation SR sources have been designed for a transverse acceptance in the 3% range. If still larger energy acceptances are envisaged an rf voltage well above a few MV will be necessary, as it scales with the square of the rf acceptance in this regime (see eq. 4). With rf cavities installed in a typical straight section length of only a few meters, rf voltages of this magnitude can no longer be generated economically with normal conducting cavities. In this case superconducting cavities may provide the only solution.

5 CONCLUSION

For a given SR storage ring source and a fixed rf acceptance, e.g. a value chosen to equal the transverse acceptance, the Touschek lifetime does not improve by lowering the rf frequency, rather it decreases moderately at lower frequencies and is essentially constant for rf frequencies above a few hundred MHZ. Under such conditions an rf frequency around 500 MHz, which has often been adopted for practical reasons, is a reasonable choice in terms of Touschek lifetime as well.

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