ESCAPE OF RELATIVISTIC ELECTRON BEAM FROM WAVE TRAP WITH ENERGY TRANSFER TO WAVE

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1 INTRODUCTION

Inclusion of resonant particles leads to many interesting and important results. In [1] resonant particles bring about ordering of the fields of oscillations in the "lattice" of wave packets in the coordinate-phase velocity (x, V_{ph}) space. Elsewhere [2] it has been shown that trapped particles can substantially renormalize the strength of three-wave interaction. Reflection of trapped resonant electrons from a localised potential perturbation in [3,4] causes a potential jump to form in the vicinity of perturbation. The presence of trapped particles also changes the dispersion relation of the waves [5]. The injection of electron beam into a plasma can give rise to a double layer in the beam-plasma mode and reflection of the electron beam from the plasma [6]. In this study we want to ascertain what other important nonlinear effects can be brought about by the presence of resonant particles in the beam-plasma interaction problem. When a cold electron beam relaxes in a plasma, upon reaching certain amplitudes ϕ_o the excited wave [7] traps the beam, forming vortices in the electron phase space (Fig. 1). In computer simulations [8] with definite initial wave amplitudes and beam densities the trapped electrons are distributed near the separatrix. Such a distribution also formed after a long time at the plasma boundary when a beam is injected into the plasma [9] (Fig. 2). This state may be unstable against the escape of a fraction of the electrons from the vortex with velocities V smaller than V_{ph} , i.e., with $V \approx V_{ph} - [2e(\phi_o+\phi(x))/m]^{1/2}$ (see Fig. 1), where $\phi(x)$ is the electrostatic potential of the wave. This behaviour of a nonlinear system may be accompanied by an increase in V_{ph} and decrease in ϕ_o . The latter assumption is supported by the inversely proportional amplitude dependence of the nonlinear correction to V_{ph} obtained by Fedorchenko et al. [5] for a beam-plasma system. Electrons escaping from the trap transfer energy and momentum to the wave and electrons trapped by the wave. In other words we shall show that in the electron phase space the vortex chain is unstable against the escape of part of electrons from the vortex with velocities smaller than the vortex velocity. This process is not the reverse of trapping: a fraction of the electrons are pumped from the flight region $V > V_{ph}$ to the flight region $V < V_{ph}$,transferring to the wave an energy

$$\Delta\varepsilon \approx (m/2)n_b[V_b^2 - (V_{ph}-V_{tr})^2], \quad (1)$$

which is ≈2 times that if the electrons were trapped by the field of the waves; here V_{tr} is the width, in the electron phase space, of the resonant interaction of the electrons with the wave and is proportional to the square root of the wave amplitude. That is, a fraction of the electrons, having slowed down as much as possible in the field of the wave, escape from the trap. This occurs, however, only for particular functions $V_{ph}(V_{tr})$, namely, $dV_{ph}/dV_{tr} > 1$, $dV_{tr}/dt > 0$ and $dV_{ph}/dV_{tr} < 0$, $dV_{tr}/dt < 0$. We shall show that the second case obtains in this problem, if electrons are distributed in the vicinity of the separatrix as a result of interaction with the wave.

2 ELECTRON BEAM INSTABILITY RELATIVE TO ESCAPE FROM WAVE TRAP

2.1 Nonrelativistic Electron Beam

Let us examine the instability of a wave with a trapped beam, whose electrons are of mixed phases and are distributed near the separatrix in a certain range of energies. Varying the amplitude and phase velocity of the wave, we shall demonstrate that the process develops in a self-consistent manner with $dV_{ph}/dV_{tr} < 0$, $dV_{tr}/dt < 0$ and $d\delta n/dt > 0$, where δn is the density of electron beams escaping from the trapping region with velocities $V < V_{ph}$.

For the purpose of describing the instability it is convenient to go over to action-angle variables. To do so we introduce the quantity

$$U = \begin{cases} \int_{\xi}^{\xi+2\pi/k} d\xi (V - V_{ph})k / 2\pi, & |U| > U_s, \\ \oint d\xi (V - V_{ph})k / 4\pi, & |U| < U_s, \quad \xi = X - V_{ph} t \end{cases} \quad (2)$$

In the steady state and with a slow variation of the amplitude the particle trajectories lie on surfaces U=const . Clearly, the instability depends strongly on the U distribution of trapped electrons relative to the separatrix, which corresponds to $\pm U_s$. If the trapped electrons are located deep in the potential well, there is a large threshold with respect to the variation of the amplitude and phase velocity of the wave for the electrons to escape from the trap. Without loss of generality, we place the trapped electrons in the form of a plateau in the vicinity of the separatrix in a U strip of width ΔU.

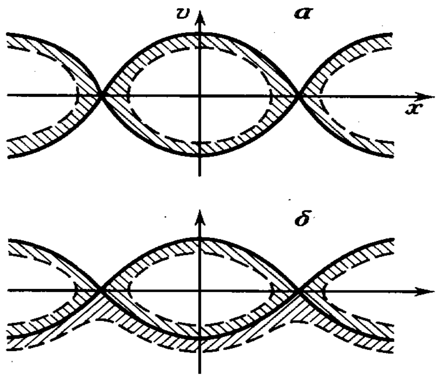

Fig. 1. Phase portrait of electrons interacting with a monochromatic wave and located near the separatrix: a) all of the electrons are trapped by the wave; b) part of the electrons escaped from the trap.

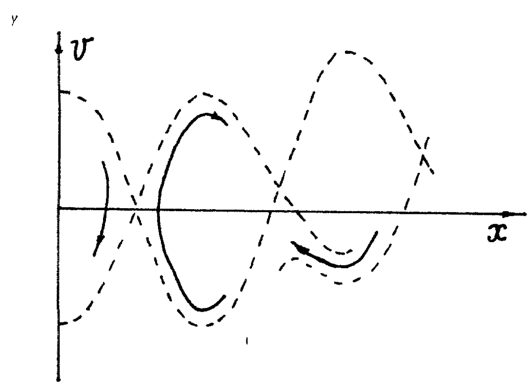

Fig. 2. Phase portrait of electrons interacting with a wave field and located near the separatrix for the case of electron beam injection in the plasma

When in (2) we substitute the equation $\varepsilon = mV^2 - 2e\phi(x, t)$ for the trajectories of electrons in the field $\phi(x, t)$ of a wave, we obtain the expressions for connection of energy with U.

To study the evolution of a wave with trapped electrons, i.e., to study the wave instability determined above, using the nonlinear dispersion relation and the energy balance equation we find the sign and absolute value of dV_{ph}/dV_{tr} and $\delta n/dV_{tr}$.

We construct the nonlinear dispersion relation and the energy balance equation by using the Poisson, Vlasov and Maxwell equations

$$\partial^2\phi/\partial x^2 = 4\pi e(\delta n_e + \delta n_b), \qquad (3)$$

$$-\partial E/\partial t = 4\pi j, \qquad (4)$$

$$\partial f/\partial t + V\partial f/\partial x = (e/m)E\partial f/\partial v, \qquad (5)$$

where δn_e and δn_b are the perturbations of the plasma and beam electron densities. Multiplying (4) by $E/4\pi$ and (5) by mV^2 and integrating the latter over V, upon equating the parts of the resulting equations

$$E^2/8\pi + \varepsilon_p + \varepsilon_b = E_o^2/8\pi + \varepsilon_p^o + \varepsilon_b^o, \qquad (6)$$

we obtain the energy balance equation for the wave field, the plasma electrons ε_p, and the beam electrons ε_b.

Let us now find the approximate nonlinear correction to the linear dispersion relation, expanding the perturbation of the electron densities in powers of the ratio of $e\phi$ to the electron energy. Since for plasma electrons the wave excited as a result of beam-plasma instability evolves adiabatically, from the continuity equation we find an expression for the perturbation of the electron density:

$$\delta n_e = n_e(x) - n_o = n_o\{[1 + 2e\phi(x)/mV_{ph}^2]^{-1/2} - 1\} \approx \approx -n_o e\phi(x)/m\,V_{ph}^2 \qquad (7)$$

We also find

$$n_b(x) = \int dV f_b = \int dU\, f_b(d\varepsilon/dU)[2m(\varepsilon + e\phi)]^{-1/2}, \qquad (8)$$

and the expression for the perturbation of electron beam density

$$\delta n_b(x) \approx -[e\phi(x)/2(2m)^{1/2}]\int dV\,(f_b/\varepsilon^{3/2})(d\varepsilon/dU). \qquad (9)$$

Since the beam electrons are close to the separatrix, from (2) we find $d\varepsilon/dU = 2mV_{tr}$, disregarding to electron energy spread,

$$\delta n_b \approx -\left(e\phi(x)/m\sqrt{2}\right)\times \left\{\frac{n_b/V_{tro}^2 \text{ as in Fig. 1a}}{\left(n_b - \delta n\right)/V_{tr}^2(t) + \delta n \;/V_{tro}^2 \text{ as in Fig. 1b}}\right. \qquad (10)$$

where $V_{otr} = V_{tr}(t{=}0)$. Substituting (7) and (9) into (3) and ignoring the change in the wave shape during the escape of part of electrons from the trap, we obtain the nonlinear correction to linear dispersion relation

$$\partial V_{ph}/\partial V_{tr} = -\left(n_b/n_o\sqrt{2}\right)\left(V_b/V_{tr}\right)^3 \qquad (11)$$

The amplitude corresponding to the amplitude of beam trapping by the wave field $V_{tr} = (\pi/4)(V_b - V_{ph}\})$ is found from (11)

$$\partial V_{ph}/\partial V_{tr} \approx -3 \qquad (12)$$

From (12) it follows that the phase velocity of wave depends markedly on the amplitude.

To determine the real evolution of the system we must now to find the sign of $d\delta n/dV_{tr}$ from (10). For this purpose we calculate the quantities of (6) as follows:

$$\begin{aligned} &\varepsilon_p = \\ &= \left(mn_o/2\lambda\right)\iint dx dV V^2 \delta\left(V - e\varphi/mV_{ph}\right) = \\ &= \left(E^2/8\pi\right)\left(\omega_p/kV_{ph}\right)^2, \end{aligned} \quad (13)$$

$$\begin{aligned} &\varepsilon_b = \left(m/2\lambda\right)\times \\ &\times \int dx \int_0^\infty dV \left\{ \begin{aligned} &\left(V_{ph}^2 + V^2\right)\begin{bmatrix} f\left(V_{ph}+u\right) \\ + f\left(V_{ph}-u\right) \end{bmatrix} + \\ &+ 2V_{ph}V\left[f\left(V_{ph}+u\right) - f\left(V_{ph}-u\right)\right] \end{aligned} \right\} \end{aligned} \quad (14)$$

From (13)-(14), with allowance for the smallness of V_{tr}/V_{ph} and $\Delta U/V_{tr}$, we obtain expression for change in the particle and wave energies with changes by δV_{tr}:

$$\begin{aligned} &\delta\left(E^2/8\pi + \varepsilon_p\right) \approx mn_o V_{tr}^3 4\delta V_{tr}/V_b^2, \\ &\delta\varepsilon_b \approx mV_{ph}\left(n_b \delta V_{ph} - U_s \delta n\right) \end{aligned} \quad (15)$$

From (6) and (15) we obtain

$$\left(V_{tr}/n_b\right)\left(d\delta n/dV_{tr}\right) \approx -\pi/2 \quad (16)$$

which means that with time the amplitude of the wave decreases, its phase velocity increases, fraction of the electrons with velocities $V<V_{ph}$, i.e., with $U \approx U_s$ escape from the trap.

2.2 Relativistic Electron Beam

One can obtain for the case of relativistic electron beam the expressions similar to (11), (12), (16)

$$\begin{aligned} &\partial V_{ph}/\partial V_{tr} = -\left(n_b/n_o\sqrt{2}\right)\left(V_b/V_{tr}\gamma_b\right)^3 \approx -3 \\ &\left(V_{tr}/n_b\right)\left(d\delta n/dV_{tr}\right) \approx \\ &\approx -\left(\pi/6\right)\left(dV_{ph}/dV_{tr}\right) \approx -\left(\pi/2\right) \end{aligned} \quad (17)$$

Here γ_b is the relativistic factor of the beam. One can see that the relations for both cases, for relativistic and for nonrelativistic beams, are closed.

3 CONCLUSION

In summary, we have studied the instability of a nonlinear wave with a trapped beam, formed as a result of the development of beam-plasma instability so that some sizeable fraction of trapped electrons is located close to the separatrix. It has been shown that the trapped electrons are unstable with escape from the trap with velocities less V_{ph}, resulting in an additional transfer of energy to the wave by the relaxing beam. To make this possible: the wave is unstable against an increase in phase velocity and against a concomitant decrease in wave amplitude. The necessary condition that a considerable fraction of trapped electrons are located close to separatrix. A similar picture of evolution of plasma instability was observed in the computer simulations of Refs. [8-9]. Jones et al. [8] investigated the time problem while Okuda et al. [9] investigated the spatial problem. The latter [8] demonstrated the formation of a chain of vortices in the electron phase space when an electron beam is injected into the plasma at its boundary. Jones et al. simulated the relaxation of a cold beam distributed in a long system. At fixed initial wave amplitudes and beam densities after vortices are formed in the electron phase space from beam electrons trapped by the wave, the vortices are unstable against the loss of electrons with velocities smaller than the vortex velocity. In this case the vortex velocity increases and its amplitude decreases. Similar electron beam behaviour has been observed at back wave excitation by beam.

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