# **CRYSTAL HORN FOR NEUTRINO BEAM FORMATION**

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### Abstract

In this paper a crystal horn for the neutrino beam formation is suggested. Crystal horn consists of a set of bent crystals which are placed axially symmetrically according to the primary beam axis. A portion of particles with small angular spread is captured into each crystal. It provides focusing of secondary particles outgoing from the target. Angular and momentum acceptance of such horn is investigated.

## **1 INTRODUCTION**

Nowadays a number of projects of long baseline experiments for neutrino oscillations study are developed at high energy proton synchrotrons [1]-[3]. Another opportunity to create intense neutrino beams can become construction of  $\mu^+\mu^-$  collider [4]-[5]. At proton synchrotrons neutrinos are produced in the pion and kaon decay. A large flux of the pions and kaons is created in collision of extracted proton beam with a nuclear target. For secondary particles - neutrino parents - focusing a magnetic horn is widely used. The horn is a coaxial magnetic lens which has high angular and momentum acceptance.

In this paper a crystal horn for the neutrino beam formation is suggested which is a set of bent crystals.

As was pointed out in [6] a channeling in a bent crystal can be used for solution of two important problems in designing of long baseline neutrino beam. First is an extraction of a beam from proton storage rings of colliders such as HERA and LHC on which the special systems for an extraction of a beam are not provided. Another problem is a deflection of a beam in directions which are not foreseen by existing systems of transportation of a extracted beam. A problem of a beam extraction from a collider can be decided with the help of so-called crystalline septum. The operation of such septa fabricated from silicon crystals is successfully tested on a number of accelerators. The greatest energy of a beam extracted by such fashion - 900 GeV - was reached in FNAL.

## 2 CRYSTAL HORN

Crystal horn consists (see Fig. 1) of a set of bent crystals which are placed axially symmetrically according to the primary beam axis. A portion of particles with small angular spread is captured into each crystal. At constant value of curvature radius of the crystals its length are increased with the distance from the beam axis increasing. It provides focusing of secondary particles outgoing from the target.



Figure 1: Particles outgoing from the target are deflected in the crystal horn.

# 3 POSITIVELY CHARGED PARTICLES PLANAR CHANNELING IN A BENT CRYSTAL

Let us consider the motion of a positively charged particle undergoing planar channeling in a bent crystal [7]. In a first approximation the planar channel potential has the form

$$U(x) = 4U_0(x^2/d^2),$$
 (1)

where  $U_0$  is the potential magnitude at the channel boundary, d is the distance between the crystallographic planes forming the channel, x is the displacement from the channel midplane. The presence in the channel of a transverse electric field causes bending of charged particle trajectories and thus, under certain conditions, particles will be kept inside the bent channel. For a given bending radius  $R_0$  particles at an energy  $\varepsilon$ , which satisfies an inequality

$$\beta \varepsilon < \beta_c \varepsilon_c = (4eU_0R_0)/d, \tag{2}$$

where  $\beta = v/c$ , v is the particle velocity will be captured into the channel. For a particle having energy  $\varepsilon \leq \varepsilon_c$  the transverse electric field on the equilibrium orbit  $E_0$  and the displacement  $x_0$  of this orbit off the channel midplane are

$$E_0 = \beta \varepsilon / eR_0, \ x_0 = d\varepsilon / 2\varepsilon_c.$$
 (3)

Here we neglect the dependence of the equilibrium orbit radius on the particle energy due to its shift from the channel midplane. For example, for the channel (110) of a silicon crystal bent at a radius  $R_0 = 40$  cm one has  $\varepsilon_c = 250$  GeV. We define the equilibrium orbit as that orbit for which the separation from the channel walls is everywhere constant. Introduce the electrical field index in the channel as  $n = (d \ln E/d \ln R)_{x=x_0}$ :

$$n = (2R_0/d)\varepsilon_c/\varepsilon,\tag{4}$$



Figure 2: Phase trajectories of particles channeling in a bent crystal.

where practically always  $n = R_0/x_0 \gg 1$ .

Inhomogeneity of the electrical field in the channel gives rise to particle betatron oscillations about the equilibrium orbit

$$\eta = x - x_0 = a\cos(\sqrt{n+1}(l/R_0) + \phi), \qquad (5)$$

where  $l = \beta ct$  is the particle path length along the equilibrium orbit. Since  $n \gg 1$  the period of particle oscillation  $\lambda_0 = 2\pi R_0/\sqrt{n+1}$  does not depend on the bend radius and can be written as follows:

$$\lambda_0 = \pi d \sqrt{\varepsilon/2eU_0}.$$
 (6)

The boundary of the stable radial motion of particles in the channel is determinated by the maximum possible amplitude of betatron oscillations

$$a_m = d/2 - x_0.$$
 (7)

Correspondingly the maximum angle  $x'_m = (dx/dl)_m$ formed by the trajectory with the equilibrium orbit is

$$x'_{m} = \frac{d/2 - x_{0}}{R_{0}}\sqrt{n} = \frac{d/2 - x_{0}}{d/2}\theta_{ch},$$
(8)

where

$$\theta_{ch} = \sqrt{\frac{2eU_0}{mc^2\gamma}},\tag{9}$$

is the critical channeling angle. If the angle of a particle exceeds this value, such a particle is not captured into the channel. The pattern of the radial motion can be clearly depicted in the phase-space x,  $(R_0/\sqrt{n})x'$  (see. Fig. 2) [7].

In these coordinates a stable trajectory of a particle is represented by a circle:

$$(x - x_0)^2 + (R_0 x' / \sqrt{n})^2 = a^2,$$
 (10)

with radius a depending on initial conditions. The stable region in phase-space is restricted by a separatrix, which is the phase trajectory for  $a = a_m$ . The channeling particles make periodic motion on the phase plane on circles of

small radius. The particles not entrapped into a channeling regime perform labile motion transferring from one channel to another and moving in everyone channel in such a manner that their motion on a phase plane happens on arcs of circles of a large radius. As a consequence they are deflected less than the particles captured into the channel.

# **4 CAPTURE EFFICIENCY**

Let us consider the capture efficiency for bent channels.

1. If the angular spread of the beam is  $x'_0 \ll x'_m$ , then a portion of particles

$$\kappa = \frac{d - 2x_0}{d} = \frac{\varepsilon_c - \varepsilon}{\varepsilon_c} \tag{11}$$

is captured into the channel. It is clear that the efficiency  $\kappa$  of capturing of secondary particles increases with secondary particles energy  $\varepsilon$  decreasing. 2. If  $x'_0 \ge x'_m$  one has

$$\kappa \simeq \frac{S_{sep}}{S_{beam}} = \frac{\pi (d/2 - x_0)x'}{2dx'_m},\tag{12}$$

where  $S_{sep} = \pi (d/2 - x_0)^2$  is the phase-plane area encircled by the separatrix,  $S_{beam} = 2(R_0 x'_0 d)/\sqrt{n}$  is the phase-area of the beam related to one channel.

As follows from (2) the particles having rather large energy spread will be captured into the channel also. The distance  $\delta R$  between the orbits corresponding to the particles of energies  $\varepsilon$  and  $\varepsilon + \delta \varepsilon$  according to (3) and (4) is

$$\delta R/R_0 = 1/n(\delta \varepsilon/\varepsilon). \tag{13}$$

The momentum compaction factor

$$\alpha = (d\ln R)/d\ln\varepsilon \simeq n^{-1} \tag{14}$$

is very small since  $n \gg 1$ .

Thus the most notable feature of the particle motion in the bent crystals is a large value of which gives rise to an extremely small spatial separation of the orbits of particles with different energy.

The difference between the real and oscillatory potentials results in a dependence of frequency on the amplitude for the betatron oscillation (the nonlinearity of oscillations). However the basic properties of the motion considered above will hold in this case also.

#### **5** CONCLUSIONS

Length of magnetic devices used for neutrino beams formation can be essentially diminished at the expense of the use for control of charged beam parameters crystalline members. However at using crystal as a septum or a horn the crystal of length in some tens centimeters can be required. Thus an effect of dechanneling can bring to diminution of a share of a beam deflected in a given direction. Another serious problem is a radiation hardness of a crystal.

Note that in principle electromagnetic radiation accompanying channeling of secondary particles in bent crystals of the horn can be used for geometric parameters of neutrino beams diagnostics.

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