

# BEAM OPTICS DESIGN OF AN FFAG SYNCHROTRON

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## Abstract

We have calculated an orbit of a radial sector type Fixed Field Alternating gradient (FFAG) synchrotron. It is a scaled machine with a triple focusing structure. We also show a way to calculate linear optics of the machine with the aid of a synchrotron design code like SAD.

## 1 INTRODUCTION

A Fixed Field Alternating Gradient (FFAG) Synchrotron draws a growing attention recently because of its advantages such as a large acceptance, especially in horizontal and longitudinal directions, a possible fast repetition rate, for example 1 kHz. Although first proposal of FFAG was made in late 1950s [1] and a couple of electron accelerators using FFAG principle were made [2],[3], there has been almost no machine constructed since then. In order to re-establish design principle with the cutting edge technology, we have constructed a Proof of Principle (POP) FFAG synchrotron with 1 MeV output energy.

In that machine, a scaled radial sector type is adopted. It has a triplet focusing structure. This paper describes the orbit and optics design principle of a triplet radial sector FFAG synchrotron in general.

## 2 FFAG DESIGN

There are two types of FFAG, one is radial sector type and the other spiral sector one. The radial sector type employs normal and reversed bending magnets alternatively, which produce AG focusing with almost circular closed orbit as a total. When the number of unit cells is large; where we define an unit cell as a pair of two different types of bending magnets and a straight section in between, focusing and defocusing actions are derived from the main body part of a bending magnet which has gradient in strength in a radial direction. The less number of cells is, the edge focusing of the magnets becomes more significant. On the other hand, the spiral sector type relies primarily on the edge focusing to have AG focusing.

The radial sector type inherently tends to become a larger machine compared to the spiral one because of the reversed bending magnets which increase a ratio of average radius to bending radius (it is called a circumference factor). The vertical tune can be changed with relative strength of normal and reversed bending, whereas the spiral sector type does not have such a knob.

The edge focusing action in the spiral sector type becomes complex when large vertical aperture is required and the gap between magnet poles is widened. In particular, a gap in the inner orbit becomes large when the field gradient of magnet is shaped solely with the gap height without the help of distributed winding of coils. In fact, the technical design and a proper modelling of edge is a hard part for both types although it is less sensitive in the radial sector type.

Furthermore, in both types, free straight space is divided into pieces as the same number as magnets, which makes it difficult to be equipped with injection, extraction, and radio frequency devices. It becomes even more serious when we design a low energy and compact machine.

Our strategy is the following. Although the circumference factor is large in the radial sector type, robustness in the field shape and flexibility in optics are preferable. As for the shortly divided straight section, we adopted a triplet focusing scheme which combines two straight sections in a cell into one, resulting the length doubled. Moreover, the field cramping is expected between the normal and reversed bends with no free space, which reduces fringing field penetration.

## 3 ORBIT AND OPTICS

Since a closed orbit of FFAG depends on particle momentum, we need to find the orbit first. Then the optical property is determined in the vicinity of the closed orbit with linear approximation.

### 3.1 Orbit

Let us consider the orbit as shown in Fig. 1. That depicts only a half cell. There should be the other half having mirror symmetry at the center of a normal magnet.

Instead of normal and reversed bending magnets, let us call it as F and D magnets hereafter. In the horizontal direction, the normal bend gives horizontal focusing and the reversed one defocusing in the body region.

Although the field gradient in the body is shaped as

$$B/B_0 = (r/r_0)^k$$

we ignore the variation of bending field along with the orbit so that the bending radius is constant in the F and D magnets, respectively. The definitions of symbol are following.

Table 1: Definitions of symbol

symbol	definition
N	Number of cell
k	Field index
$\beta_F$	Opening angle of F/2 with respect to machine center
$\beta_D$	Opening angle of D with respect to machine center
$\theta_F$	Bending angle of F/2
$\theta_D$	Bending angle of D
$\rho_F$	Bending radius of F
$\rho_D$	Bending radius of D
LF	Path length of F/2
LD	Path length of D
r0	Orbit radius at F center
r1	Orbit radius at F exit

Since orbits scale, r0 is a parameter that determines the magnetic field strength when the momentum of a beam is specified. Thus, ratios of  $\rho_F/r_0$ ,  $\rho_D/\rho_F$ , and  $r_1/\rho_F$  are derived accordingly. Those are,

$$\frac{\rho_F}{r_0} = \frac{\tan \beta_F}{\sin \theta_F + (1 - \cos \theta_F) \tan \beta_F}$$

$$\frac{\rho_D}{\rho_F} = \frac{\sin(\frac{\pi}{N} - \beta_F) - \cos(\frac{\pi}{N} - \beta_F) \tan(\frac{\pi}{N} - \beta_F - \beta_D)}{\sin(\theta_F - \frac{\pi}{N}) - [1 - \cos(\theta_F - \frac{\pi}{N})] \tan(\frac{\pi}{N} - \beta_F - \beta_D)} \sin \theta_F$$

$$\frac{r_1}{\rho_F} = \frac{\sin \theta_F}{\sin \beta_F}$$

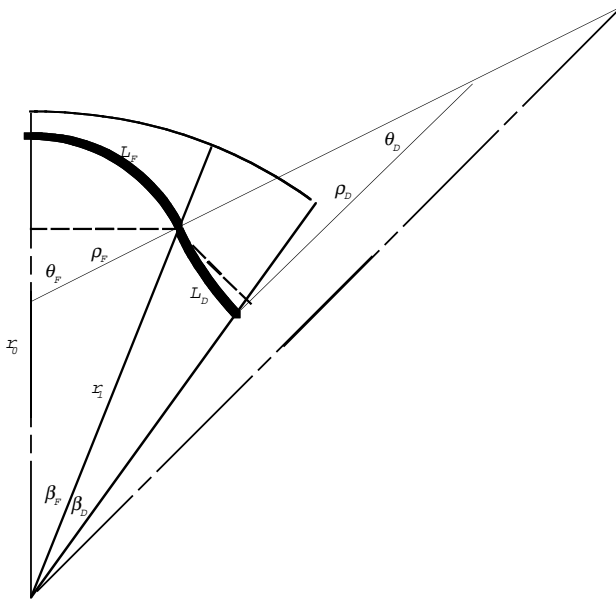


Figure 1: Closed orbit of a triplet focusing FFAG. Only a half cell; a half of F magnet, D magnet of one side, and a half straight section, is depicted.

### 3.2 Optics

Once an orbit is fixed, expanding fields to the first order (linear part only) in the vicinity of the orbit can approximate the optics. We employ the synchrotron optics code SAD.

First, the focusing and defocusing fields in the main body of magnet are approximated by local gradient of the magnet as,

$$k_1 L \equiv \frac{L}{B\rho} \frac{dB}{dr} \Big|_{r=r_0} = \frac{kL}{\rho r_0}$$

In fact, the local gradient should not be estimated at  $r=r_0$ , but somewhere at the average radius of local orbit in each magnet. However, we assume the orbit scallops is sufficiently small.

It is not small focusing effects at the edge, especially when the cell number is small. The edge focusing effects are estimated as a ratio of the angle with respect to the magnet face to the bending angle. At the both end of the F, it is,

$$\varepsilon_F = \frac{\theta_F - \beta_F}{2\theta_F}$$

At the F side in the D magnet,

$$\varepsilon_{D,1} = \frac{\theta_F - \beta_F}{\theta_D}$$

Finally at the exit of D to the straight section,

$$\varepsilon_{D,2} = -\frac{\frac{\pi}{N} - \beta_F - \beta_D}{\theta_D}$$

To be a meaningful solution,

$$\theta_F \leq \frac{\pi}{2} + \beta_F$$

Those are all the necessary input parameters for conventional optics calculation codes such as SAD.

## 4 PARAMETER SEARCH

In order to assure stability of AG focusing, there should be some relations among different parameters. For example, once we choose phase advance per cell, a relation between the number of cell and the field index k is fixed. Figure 2 shows when the horizontal phase advance is around 120 degrees and the vertical one is around 60 degrees.

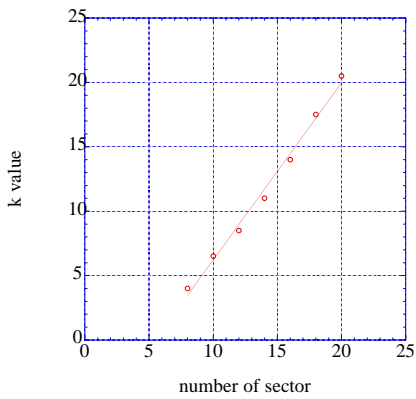


Figure 2: Once the phase advance of two transverse directions is determined, the relation of the field index  $k$  and the number of sector is unique.

From the technical point of view, the size of magnet has a realistic limit. In other words, the orbit excursion, which is defined as a difference between the outer orbit and inner one, does not depend on the size of the machine or other parameters. Roughly speaking, once the orbit excursion  $\Delta r$  is fixed, the outer orbit radius and the  $k$  value has as simple relation.

$$r_{\max} = \frac{\Delta r}{\left[ 1 - \left( \frac{p_{\text{ext.}}}{p_{\text{inj.}}} \right)^{\frac{1}{1+k}} \right]}$$

where the ratio of injection and extraction momenta is typically around 3. Through the  $k$  and  $r_{\max}$ , the maximum field strength can be plotted as a function of the number of sector. For example, Fig. 3 shows the relation when we assume  $\Delta r=0.5\text{m}$ ,  $p_{\max}=0.644\text{GeV}/c$  (kinetic energy of 200MeV proton.)

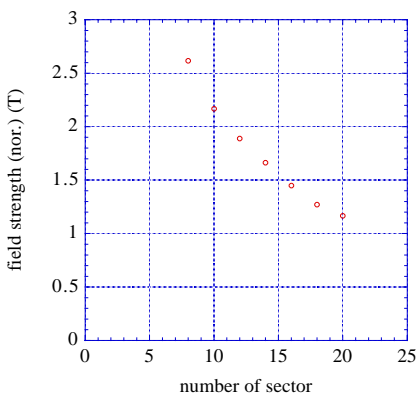


Figure 3: Relation of Fig. 2 gives the  $k$  value as a function of the number of sector. Then, once the orbit excursion is fixed, here it is 0.5m, the field strength is determined.

In the same manner, the length of straight is determined as a function of the number of sector. It is interesting that

the length is almost independent of the sector number as shown in Fig. 4.

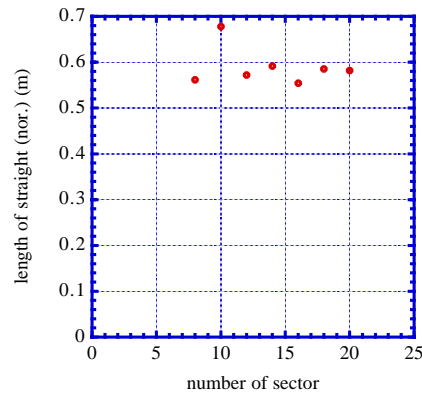


Figure 4: Length of straight is almost constant independent of the number of sector when the orbit excursion is fixed.

## 5 EXAMPLE

Here is an example of FFAG design. The 200MeV FFAG synchrotron is designed with  $N=12$ ,  $k=9$ , and the radius is around 5m. The beta and dispersion functions for three cells are depicted in Fig. 5.

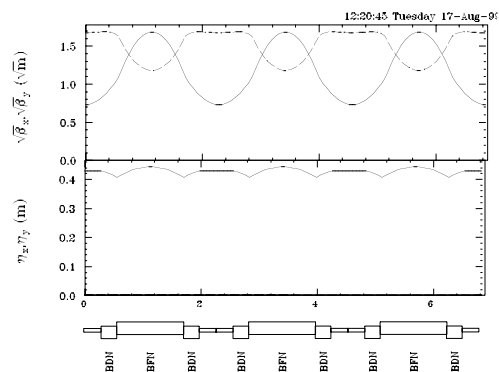


Figure 5: Beta and dispersion functions of a triplet focusing FFAG with  $N=12$  and  $k=9$  as an example. Solid line is a horizontal beta function and dashed one is a vertical.

## REFERENCES

- [1] K. R. Symon, et. al, Physical Review, Vol.103, No.6, 1956, p.1837.
- [2] F. T. Cole, et. al., Review of Scientific Instruments, Vol.28, No.6, 1957, p.403.
- [3] D. W. Kerst, et. al., Review of Scientific Instruments, Vol.31, No. 10, 1960, p.1076.