

# MEASUREMENT OF SMALL TRANSVERSE BEAM SIZE USING INTERFEROMETRY

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## Abstract

The principle of measurement of the profile or size of small objects through the spatial coherency of the light is known as the van Cittert-Zernike theorem. We developed the SR interferometer (interferometer for synchrotron radiation) to measure the spatial coherency of the visible region of the SR beam, and we demonstrated that this method is able to measure the beam profile and size. Since the small electron beam emits a SR beam which has a good spatial coherency, this method is suitable for measuring a small beam size. In this paper, the basic theory for the measurement of the profile or size of a small beam via the spatial coherency of the light, a design of the SR interferometer, and the results of beam profile measurement, examples of small beam size measurements and recent improvements are described.

## 1 INTRODUCTION

The measurements of beam profile and size are two of the most fundamental diagnostics in an electron storage ring. The most conventional method to observe the beam profile is known as a beam profile monitor via imaging of the visible SR beam[1]. The resolution of this monitor is generally limited by diffraction phenomena. In the usual configuration of the profile monitor the RMS size of diffraction ( $1\sigma$  of the point spread function) is no smaller than 50  $\mu\text{m}$ . In the last 10 years, research and development in electron storage rings (especially in the area of emittance reduction) has been very remarkable. We can realise sub-diffraction-limited beam sizes in electron storage rings. So the above-mentioned profile monitor via imaging of the visible SR beam becomes useless in precise quantitative measurements of the beam profile and size. In the visible optics, opticians use an interferometer as the standard method to measure the profile or size of very small objects. The principle of measurement of the profile of an object by means of spatial coherency was first proposed by H.Fizeau [2] and is now known as the Van Cittert-Zernike theorem [3]. It is well known that A. A. Michelson measured the angular dimension (extent) of a star with this method [4]. Recently we developed the SR interferometer (an interferometer for SR beams) to measure the spatial coherency of the visible region of an SR beam, and as one of the results of investigations on the spatial coherence of

SR beams, we demonstrated that this method is applicable to measure the beam profile and size at the KEK Photon Factory [5]. Since the SR beam from a small electron beam has good spatial coherency, this method is suitable for measuring a small beam size. The characteristics of this method are: 1) we can measure beam sizes as small as 3 and 4 $\mu\text{m}$  with 1 $\mu\text{m}$  resolution in a non-destructive manner; 2) the profile is easy to measure using visible light (typically 500 nm); 3) the measurement time is a few seconds for size measurement and few tens of seconds for profile measurement. In this paper we describe the van Cittert-Zernike theorem, the design of the SR interferometer and examples of the profile and the beam size measurements

## 2 SPATIAL COHERENCE AND BEAM SIZE

According to van Cittert-Zernike's theorem, the profile of an object is given by the Fourier Transform of the complex degree of spatial coherence at longer wavelengths as in the visible light[3][6]. Let  $f$  denotes the beam profile as a function of position  $y$ ,  $R$  denotes distance between source beam and the double slit, and  $\gamma$  denotes the complex degree of spatial coherence as a function of spatial frequency  $\nu$ . Then  $\gamma$  is given by the Fourier transform of  $f$  as follows;

$$\gamma(\nu) = \int f(y) \exp(-2\pi i \nu \cdot y) dy, \quad \nu = \frac{2\pi D}{\lambda R}.$$

We can measure the beam profile and the beam size via spatial coherence measurement with the interferometer.

## 3 SR INTERFEROMETER

To measure the spatial coherence of SR beams, a wavefront-division type of two-beam interferometer using polarized quasi-monochromatic rays was designed as shown in Fig.1[6].

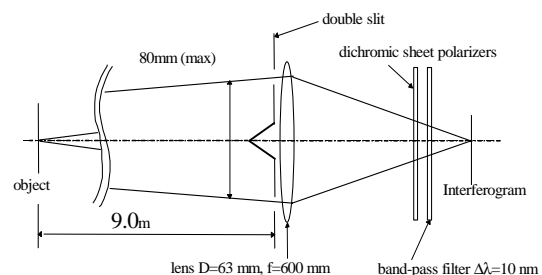


Fig.1 Outline of the SR interferometer.

In the vertical plane, the elliptical polarity of synchrotron radiation is opposite that in between the medium plane of the electron beam orbit. Therefore, there exists the  $\pi$  phase difference between the phases of the interferograms to correspond to the  $\sigma$ - and  $\pi$ -polarized components [5]. To eliminate the interferogram by  $\pi$ -polarized component, we must apply a polarization filter. A typical interferogram observed with the SR interferometer is shown in Fig.2.

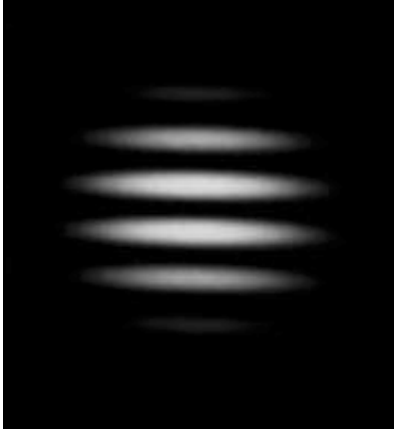


Fig. 2 A typical interferogram observed with the SR interferometer.

With this interferometer, the intensity of the interferogram is given by,

$$I(y,D) = (I_1 + I_2) \cdot \left\{ \text{sinc} \left( \frac{\pi \cdot a \cdot y \cdot \chi(D)}{\lambda \cdot f} \right) \right\} \cdot \left\{ 1 + \gamma \cdot \cos \left( k \cdot D \cdot \left( \frac{y}{f} + \psi \right) \right) \right\}$$

$$\gamma = \left( \frac{2\sqrt{I_1 \cdot I_2}}{I_1 + I_2} \right) \left( \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \right), \quad \psi = \tan^{-1} \frac{S(D)}{C(D)}$$

where  $y$  denotes position in the interferogram,  $a$  denotes the half-height of a slit, and  $f$  denotes the distance between secondary principal point of the lens and the interferogram[6].  $S(D)$  is the sine component and  $C(D)$  the cosine component of the Fourier transformation of the distribution function of the SR source.  $\chi(D)$  in this equation represents an instrumental function of the interferometer; this term has a cosine-like dependence, and comes mainly from two sources: 1) a cosine term in the Fresnel-Kirchhoff diffraction formula [6] which represents the angular dependence between the incident and diffracted light of a single slit; 2) reduction of effective slit height as double slit separation  $D$  increases. This term  $\chi$  is normally neglected in diffraction theory under the paraxial approximation, but we cannot neglect this term in the practical use of the interferometer.

#### 4 BEAM PROFILE MEASUREMENT

We can measure the beam profile by Fourier transform of the spatial coherence. Figure 3 shows the absolute

value the complex degree of the spatial coherence ( $|\gamma|$ , visibility) was measured by changing the double slit separation from 5 mm to 15 mm at the Photon Factory[5].

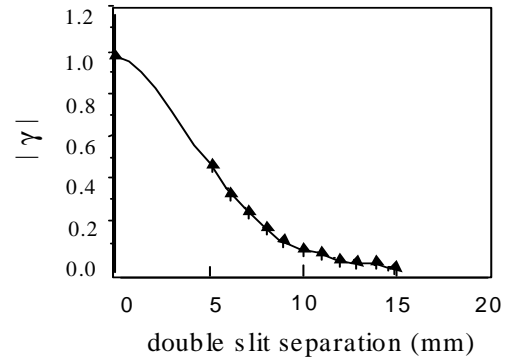


Fig 3. Result of  $|\gamma|$  at the Photon Factory

The result of beam profile by Fourier transform of the spatial coherence is shown Fig. 4.

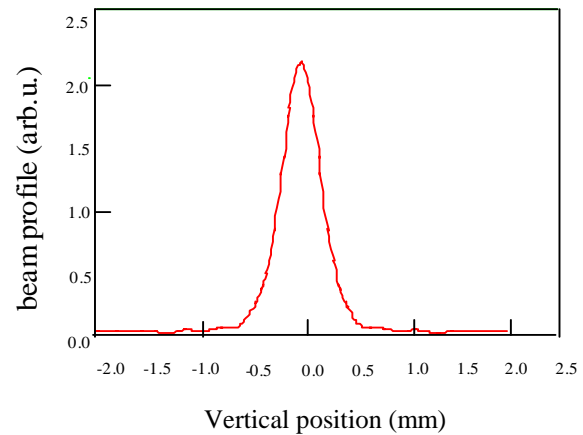


Fig. 4 Beam profile by the Fourier transform of the spatial coherence at the Photon Factory

#### 5 SMALL BEAM SIZE MEASUREMENT BY MEANS OF GAUSSIAN APPROXIMATION OF BEAM PROFILE

We often approximate the beam profile with a Gaussian shape. With this approximation, we can skip any phase measurement. The Fourier transform of even function (Gaussian) is simplified to a Fourier cosine transform. A spatial coherence is also given by a Gauss function. We can evaluate a RMS width of spatial coherence by using q least-squares analysis. The RMS beam size  $\sigma_{beam}$  is given by the RMS width of the spatial coherence curve  $\sigma_\gamma$  as follows:

$$\sigma_{beam} = \frac{\lambda \cdot R}{2 \cdot \pi \cdot \sigma_\gamma}$$

where  $R$  denotes the distance between the beam and the double slit. Experimentally, we must measure the  $|\gamma|$  (contrast of interferogram) as a function of slit separation  $D$ [6].

We can also measure the RMS. beam size from one data of visibility, which is measured at a fixed separation of double slit. The RMS beam size  $\sigma_{beam}$  is given by ,

$$\sigma_{beam} = \frac{\lambda \cdot F}{\pi \cdot D} \cdot \sqrt{\frac{1}{2} \cdot \ln\left(\frac{1}{\gamma}\right)}$$

where  $\gamma$  denotes the visibility, which is measured at a double slit separation of D[6].

With this technique, how small beam we can measure? In Fig. 5, it is shown a result of simulation. In this simulation, measuring condition of ATF is assumed ; the double slit separation is 50mm and distance between the source point and the interferometer is 7.4m. From this simulation, the beam size 4 $\mu$ m will gives the contrast 0.94 with 500nm and 0.91 with 400nm. The beam size 3 $\mu$ m will gives the contrast 0.97 with 500nm and 0.95 with 400nm. Since we can easily measure the intensity of light by 1% precision with CCD and image processor, we can measure a difference between the beam size 3  $\mu$ m and 4  $\mu$ m with a resolution better than 1 $\mu$ m.

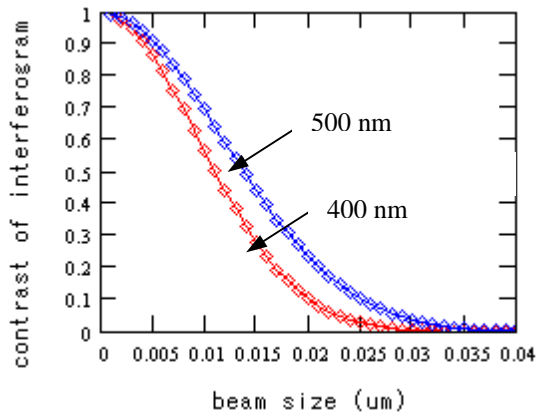


Fig. 5 Result of simulation about contrast of interferogram as a function of beam size .

## 6 SMALL BEAM SIZE MEASUREMENT

As mentioned in section 5, to assume the Gaussian profile, we can evaluate the beam size in the small-beam based on the degree of spatial coherence. We introduce two examples of small beam size measurements in this section.

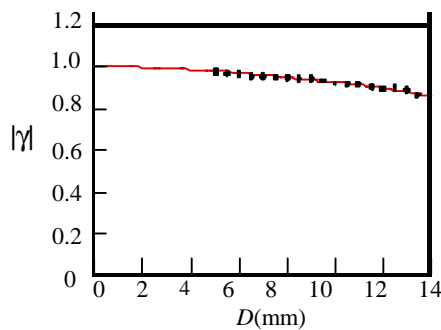


Fig. 6 Absolute value of the complex degree of spatial coherence in the vertical direction. Dotted line denotes

measured  $|\gamma|$ , and solid line denotes the best-fit beam size of  $16.5 \pm 0.6\mu$ m.

One is the result of vertical beam size measurement at the AURORA[6] and other is the vertical and the horizontal beam size measurement the ATF damping ring at KEK[6].

Figure 6 shows the result of  $|\gamma|$  as a function of slit separation with a least-squares fitting by a Gaussian profile at the AURORA. The obtained beam size from this fitting is 16.5  $\mu$ m.

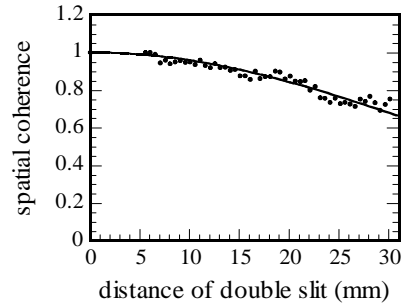


Fig. 7. Absolute value of the complex degree of spatial coherence in the vertical direction at ATF damping ring. Dotted line denotes measured  $|\gamma|$ , and solid line denotes the best-fit beam size of  $14.7 \pm 0.6\mu$ m.

The result of  $|\gamma|$  as a function of slit separation for vertical direction with a least-squares fitting by a Gaussian profile at the ATF damping ring is shown in Fig. 7. The result of  $|\gamma|$  as a function of slit separation for horizontal is shown in Fig. 8. A least-squares fitting of the  $|\gamma|$  in Figure 9 includes a field depth effect for the horizontal direction [6]. The obtained beam size from these fitting is 14.7  $\mu$ m in the vertical and 39 $\mu$ m in the horizontal.

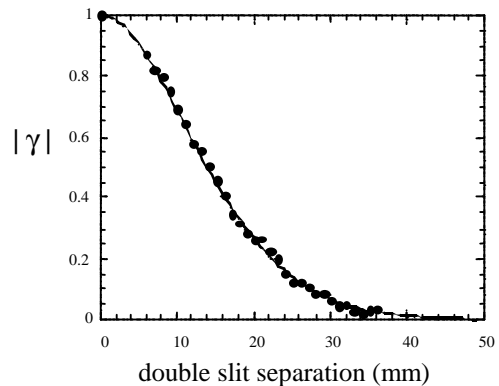


Fig. 8. Absolute value of the complex degree of spatial coherence in the horizontal plane at the ATF damping ring. The dotted line denotes measured  $|\gamma|$  and the solid line denotes the best-fit value of  $39 \pm 1\mu$ m.

## 7 AUTOMATIC BEAM-SIZE MEASUREMENT IN KEK-B FACTORY

As described in section 5, with a Gaussian beam profile approximation again, we can estimate the RMS beam size

from one data of visibility of one interferogram, which is measured at a fixed separation of double slit. Using this method, we can easily measure a beam size automatically from an analysis of interferogram taken at fixed separation of double slit  $D$  [7]. To find the visibility  $\gamma$  from the interferogram, we use the standard Levenberg-Marquart method for non-linear fitting. After the image processing of the interferogram, the results are relayed to a computer in the control room to display and further analysis. Figure 9 shows an example of the display panel for LER. A same panel is also displayed for HER. The interferogram, best fit curve and beam size trend graphs for vertical and horizontal directions are shown in the panel. By this automatic beam-size measurement system, we can measure the vertical and horizontal beam size in every second and which are extremely useful for beam tuning.

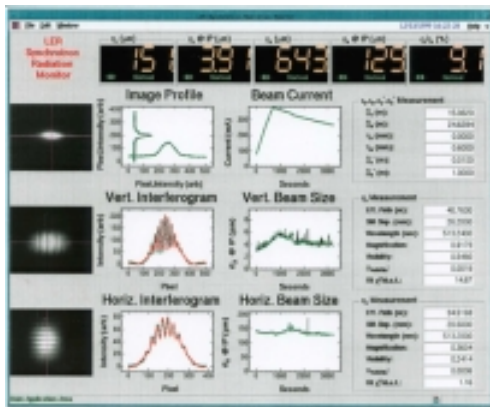


Fig. 9 SR Monitor panel in control room, showing LER vertical and horizontal beam sizes.

## 8 Recent improvements

In this section, It is described that some topics from recent improvements.

### 8-1 Linearity of CCD camera

Since we measure a contrast of interferogram in the beam size measurement, the linearity of the CCD camera is very important. In here, it is shown that some results of linearity measurements of CCD camera. The linearity of the CCD camera is measured by using accurately calibrated neutral-density filters. The optical density of these filters are calibrated by spectro-densitometer within 0.1% error in spectrum range from 400nm to 650nm. Since stability of the intensity of SR beam is less than 0.1% in a few minutes measurement at the Photon Factory, we used SR beam as the incident light. The measurement is performed in the intensity range from zero to saturation level.

A result of linearity measurement for commonly-used CCD camera is shown in Fig. 10. To see this figure, CCD camera is not always linear in the region which is far from its saturation.

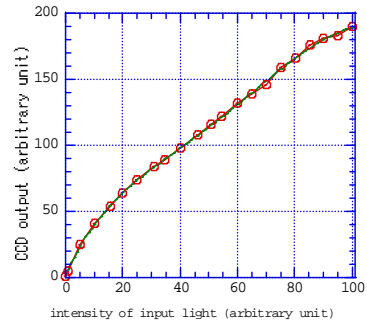


Fig. 10 A result of linearity measurement of commonly-used CCD camera.

Another result of linearity measurement for CCD camera is shown in Fig. 11. This camera is sold as a camera for quantitative measurement of intensity.

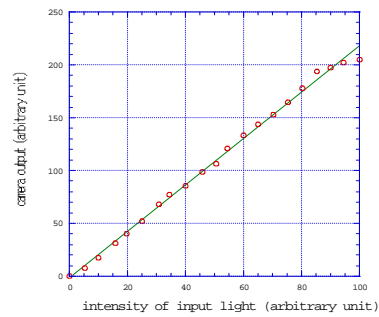


Fig. 11 A result of linearity measurement of CCD camera for quantitative measurement.

To see Fig. 11, the CCD camera which is sold for quantitative measurement has a good linearity. Right now, we check the linearity of the CCD camera before installation in the interferometer.

### 8-2 Effect of floor vibration

The vibration of floor of the accelerator building is not negligible smaller for interferometer measurements. To see an effect of floor vibration to interferometer, we measured the beam size as a function of exposure time of the CCD camera. A result of measurement at the Photon Factory is shown in Fig. 12.

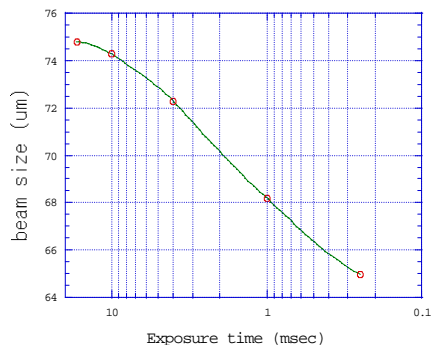


Fig.12 Beam size as a function of CCD exposure time.

From this result, decrease of the beam size is observed at shorter exposure time of CCD camera. From the view point of the accelerator physics, the beam size without an effect of floor vibration is more correct one, but an instantaneous beam size is often inconvenient for accelerator users, so we apply a exposure time 15msec.

### 8-3 Deformation of SR extraction mirror

The extraction mirror for SR beam deformed by strong irradiation of SR. The actual rays due to this deformation propagate different optical paths compare with ideal rays. So, Two optical paths of actual rays those come to double slit give a different separation from ideal ray's one. We must know true separation of two rays at the location of double slit. To measure wavefront error and true separation of two rays, we applied the Hartmann screen test[8]. In the Hartmann screen test, the wavefront is sampled by a number of rays normal to it, ray deviation at observation plane can be obtained. We used a 100-holes square-array screen as shown in Fig.13. The interval of hole is 5mm. The square-array screen is fixed on a X-Y moving stage.

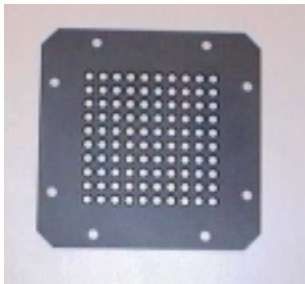


Fig.13 A 100-hole square-array screen.

The setup of the wavefront-error measurement at the Photon Factory is shown in Fig. 14. With this setup, if we measure the dot positions of the Hartmann pattern on the observation plane with 0.1mm resolution, we can measure the wavefront with  $\lambda/6$  (in here,  $\lambda$  is 633nm) precision.

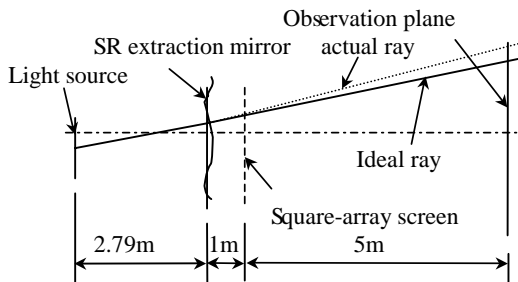


Fig. 14 Measurement setup.

A typical Hartmann pattern observed in the Photon Factory is shown in Fig. 15. Reconstructed wavefront-error is also shown in Fig. 15. To determine the true separation between the two rays at the location of double slit, we use a single-hole screen as shown in Fig. 16. The paths of two ideal rays are probed by scanning the single-hole screen in the plane which perpendicular to the optical axis.

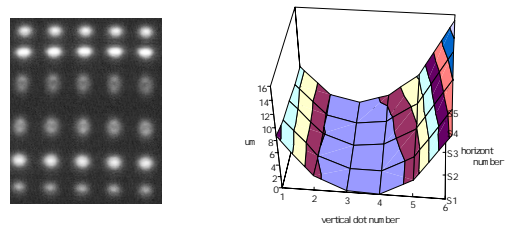


Fig. 15 Typical Hartmann pattern observed in the Photon Factory and reconstructed wavefront.

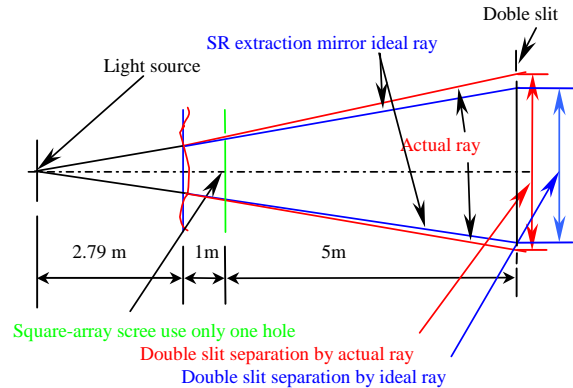


Fig. 16 Setup of determination of true double slit separation by scanning a single-hole screen.

## 7 CONCLUSIONS

The SR interferometer was developed to measure the spatial coherency of the visible region of the SR beam, and we demonstrated that this method is able to measure the beam profile based on the van Cittert-Zernike theorem. Using a Gaussian beam profile approximation, we can measure  $\mu\text{m}$  range very small beam size with the resolution less than  $1\mu\text{m}$ . With automatic analysis system, the SR interferometer is conveniently used as a beam size monitor. The measuring interval is about 1sec.

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