

MEASURING BETA-FUNCTIONS WITH K-MODULATION

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Abstract

The precise measurement of the local value of the beta-function at the place of a beam size monitor is necessary for the precise determination of the beam emittance. We developed a new method for the measurement of the beta-function by using of continuous square-wave modulation of the force of the quadrupole and by continuous tune tracking. Measurements were performed at LEP in order to evaluate the precision that can be achieved with this method in the LHC.

The paper describes the method and discusses in details the results obtained at LEP for colliding and non-colliding beams.

1 EMITTANCE MEASUREMENT

The emittance can be obtained from the measurement of the beam size from the following formula:

$$\sigma(s) = \sqrt{\varepsilon\beta(s)}$$

where $\sigma(s)$ is the transverse beam size, ε is the emittance, $\beta(s)$ is the beta-function and s is the longitudinal distance.

Therefore, in order to determine the emittance with high precision, the beta-function must also be known to high precision. We aim at a precision of 1%.

2 BETA-FUNCTION MEASUREMENT

The general principle of the beta-function measurement can be seen in fig.1. By changing the strength (k) of the quadrupole, the accelerator optics change, and therefore the tune change. The bigger the change of the strength, the bigger the tune change. The two are linked via the average value of the beta-function in the quadrupole:

$$\beta_{average} \approx 4\pi \frac{\Delta Q}{\Delta k \cdot l}$$

where ΔQ is the tune change, Δk is the change in quadrupole strength and l is the length of the quadrupole. The measurement of the tune is done by a PLL [1]. We could also have measured the tune with a continuous FFT, but chose not to, because a FFT has a quantization error of $1/(N\Delta T)$.

Apart from the internal noise in the PLL, the measurement of ΔQ is subjected to noise from various sources, like:

- A change in the closed orbit.
- Beam-beam interactions.

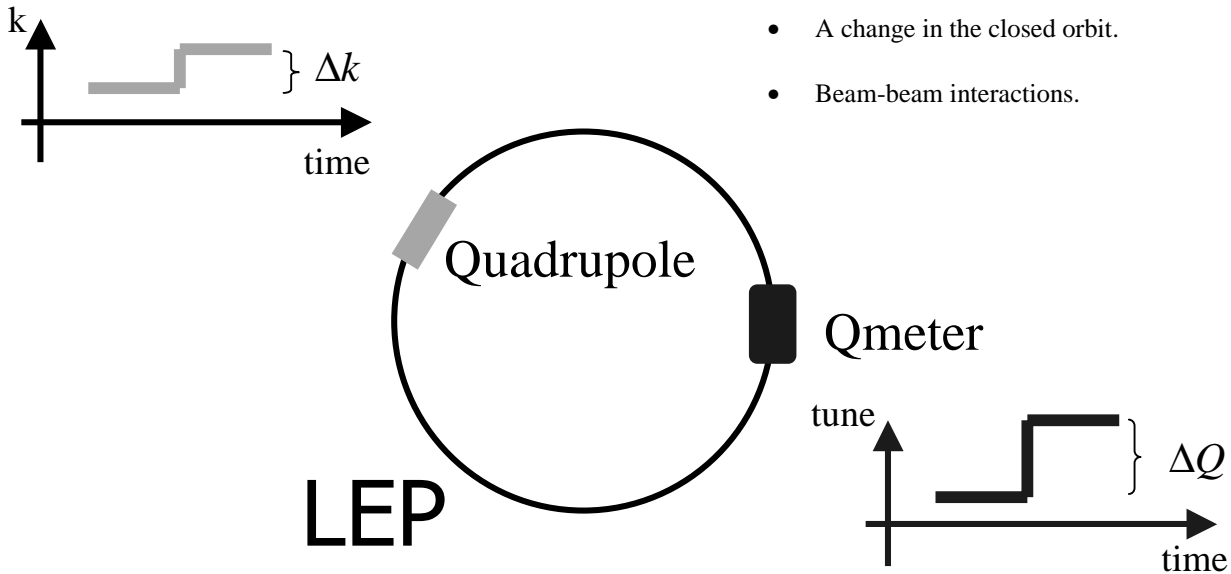


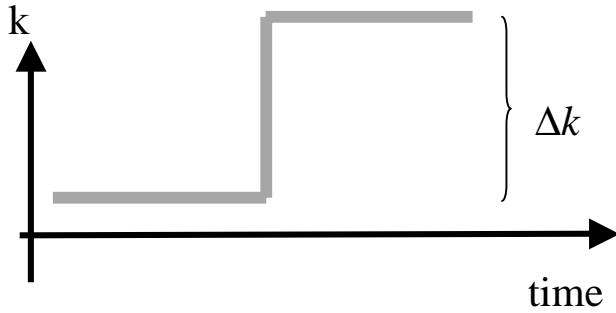
Figure 1: Static-k measurement. By changing the strength ($=\Delta k$) of the quadrupole, the tune ($=\Delta Q$) change. The ratio between them is determined by the value of the beta-function at the place of the quadrupole.

3 K-MODULATION

3.1 Difference of static-k and k-modulation

K-modulation was introduced to gain precision due to averaging over many measurements, thereby removing random errors. K-modulation consists of a repeated change in the quadrupole strength.:

Static-k measurement:



Measurement with k-modulation:



Figure 2: Difference between static-k and k-modulation.

The error reduction resulting from the averaging is so big that we can afford to reduce the step size (= Δk). This makes the method less perturbing. K-modulation can even be used during physics with colliding beam.

Advantages of k-modulation:

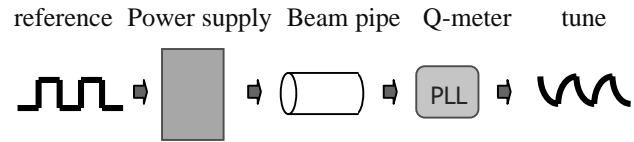
- Many measurements => increase precision
- Use small Δk => use during physics

Disadvantages of k-modulation:

- Compensate for dynamic effects (see chapter 3.2)
- Requires high (10^{-5}) precision in tune measurement

3.2 Compensation of dynamic effects

A change in the quadrupole strength will create a transient response. Many different equipment are part of the transient response:



In the static-k measurement, such transients could be ignored because datataking would only be done when the transients had died out. When using k-modulation , the transients might not have died out before the quadrupole strength changes again. Our tests used a k-modulation frequency of 0.25 Hz i.e. a time between up and down in quadrupole strength of 2 seconds. Since the transients takes ~ 1.5 seconds to die out, this left us ~ 0.5 seconds to measure the stable response of the tune. In figure 3 is shown the response of the tune to a k-modulation cycle (this cycle is the average of very many cycles). The blue curve is the tune response and the pink curve is the Fourier component of the fundamental frequency. From this curve we established the ratio between the Fourier component and the stable response of the tune. This ratio we called the form factor (here 1.27544). When measuring shorter series of k-modulations with different amplitudes of Δk , we could then calculate the stable response by dividing the Fourier coefficient of the fundamental frequency with the form factor.

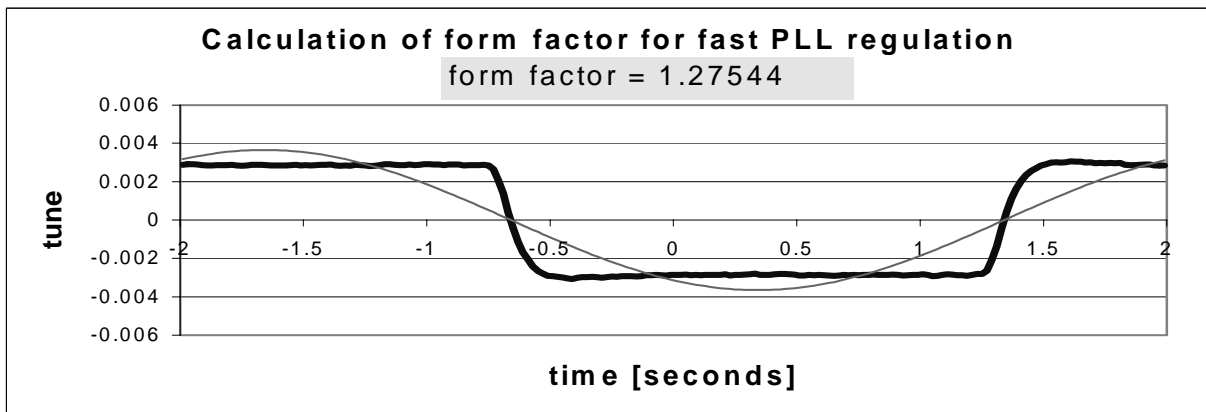


Figure 3: The tune response to k-modulation. The pink curve is the fundamental frequency of the tune response.

3.3 Obtained resolution in tune measurement

In order to disturb physics the least possible, the value of ΔQ is minimised. We characteristically used values of ΔQ of 0.005. Because we aimed at a measurement precision of 1%, the amplitude of the tune-noise should be less than 0.01% i.e. $5 \cdot 10^{-5}$ in tune. Fig. 4 shows the Fourier spectrum of the noise of the PLL, which tracks a constant tune:

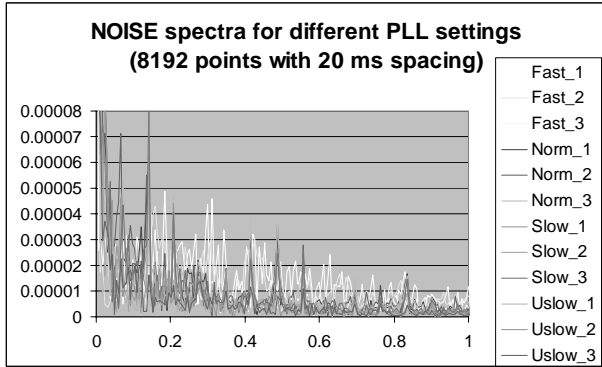


Figure 4: Fourier components of noise

The noise at 0.25 Hz (the k-modulation frequency) was always less than 0.00005 for any mode (Fast, Normal, Slow or Ultraslow) of the PLL.

4 RESULTS

The following table shows results from an machine development (MD). The theoretical value comes from MAD. The MD started with two series of k-modulation, then followed three 1000 turns [2] measurement (The 1000 turns measurement is the bench mark for beta-function measurements), then followed 4 static-k measurements, and finally -after many hours- came two series of k-modulation:

Table 1: MD results

Method	β value	σ
Theory (QS249)	181.7 m	
k-modulation before	161.1 m	0.43
1000 turns	166.5 m	2.86
static-k	164.4 m	5.62
k-modulation after	162.0 m	0.06

All four measurements agrees to each other within 2σ .

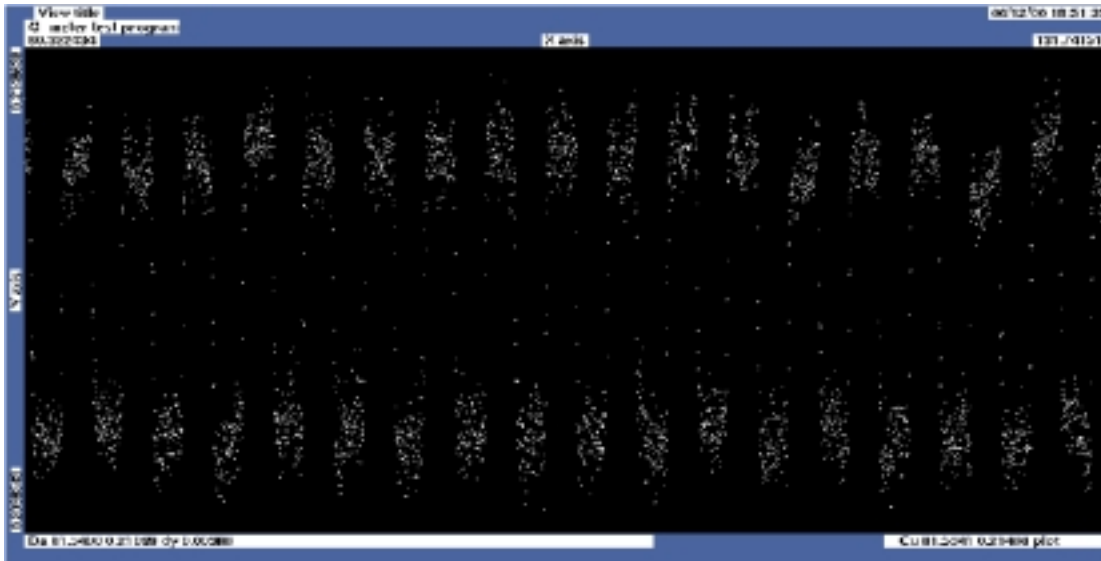


Figure 5: An example of raw tune measurements with k-modulation

5 CONCLUSION

β -function measurement with k-modulation is precise to better than 1%. It is a robust method that measures as well during collisions as with a single beam. The values of the beta-function from k-modulation measurements compare to within 4% to the 1000 turns measurements. We propose to measure β -functions with k-modulation in LHC. We expect to get even better precision by increasing the k-modulation frequency. An increased frequency will increase the number of averages and be further away from the noise of the slow tune drifting.

6 ACKNOWLEDGEMENTS

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7 REFERENCES

- [1] Buur G., Collier P., Lohmann K. D. & Schmickler H., Dynamic tune and chromaticity measurements in LEP. CERN SL 92-15 DI
- [2] Applications of the 1000-turns Orbit Measurement System at LEP. P.Castro