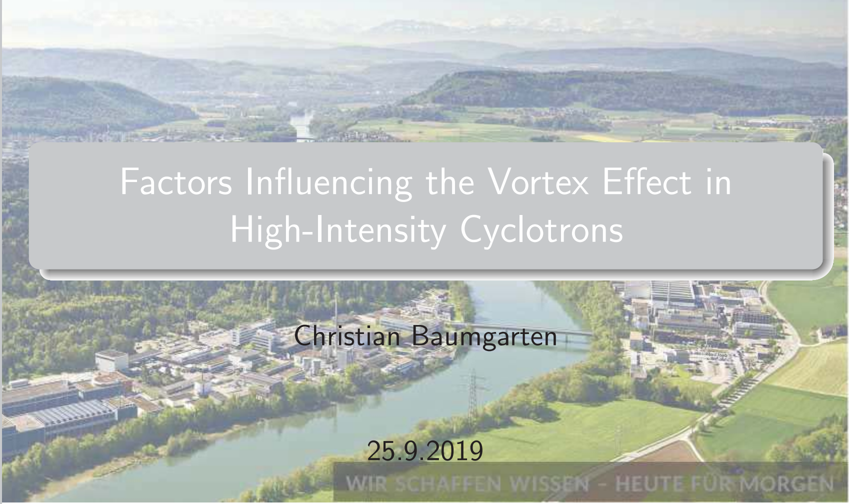

Factors Influencing the Vortex Effect in High-Intensity Cyclotrons

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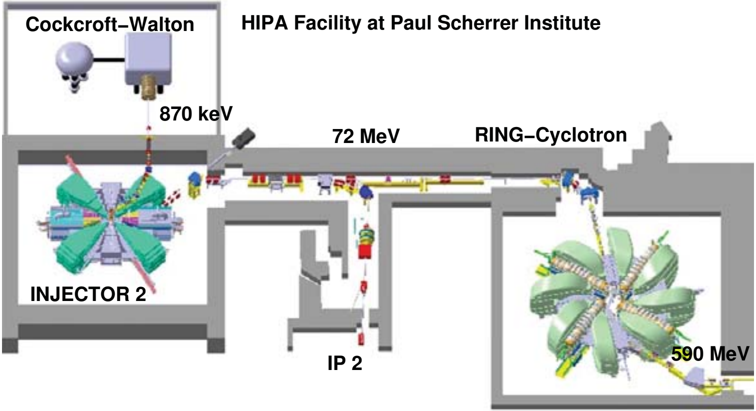
Outline

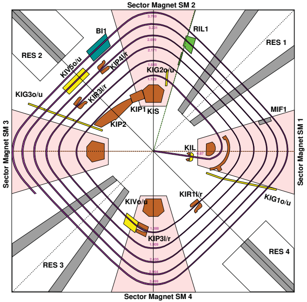
1. PSI / Injector 2
2. Visualizing the Vortex Effect
3. Matching
4. Simple linear (symplectic) model
5. Conditions for space charge induced "longitudinal focusing".
6. RF considerations.
7. Model versus OPAL [5, 6] simulations.
8. Summary.

PSI High Intensity Proton Acc. (HIPA)



Injector 2: First (and only?) cyclotron worldwide using the Vortex effect

- Axial injection with $E = 870\,\mathrm{keV}$.
- Considerable number of collimators (horz. + vert.)
- High accelerating voltages ($72\,\mathrm{MeV}$ after 80 turns).
- Max. Current so far $I \leq 2.7\,\mathrm{mA}$.
- 3rd harmonic resonators (formerly known as flat top) used for acceleration.
- Max. Current so far $I \leq 2.7\,\mathrm{mA}$.
- Various proofs of Vortex effect: bunch shape measurements [2].



Visualization I

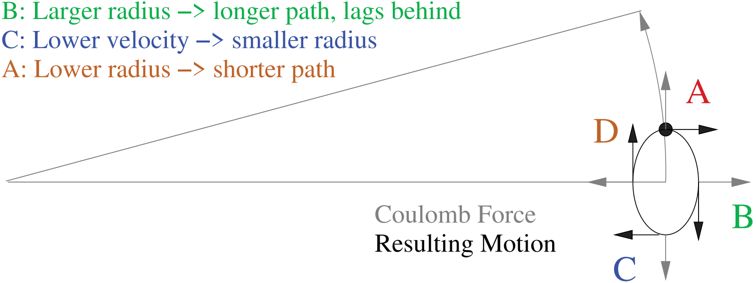
Cause and (orthogonal) Effect of Space Charge in Vortex Motion

A: Higher velocity –> larger radius

B: Larger radius –> longer path, lags behind

C: Lower velocity –> smaller radius

A: Lower radius –> shorter path



Visualization II ($E \times B$-Drift)

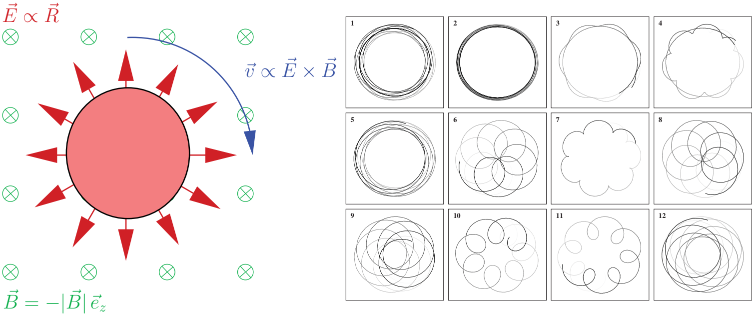
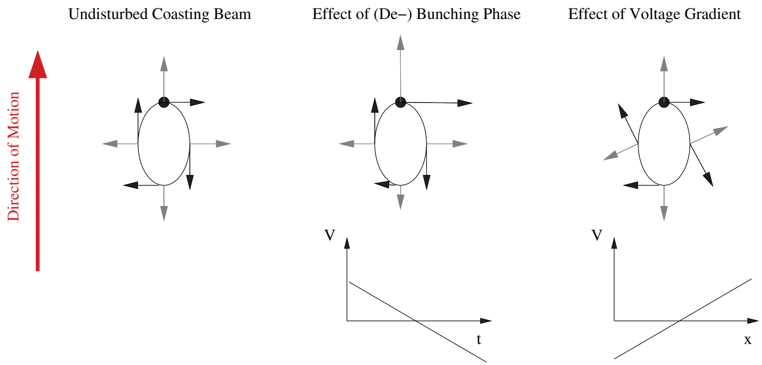
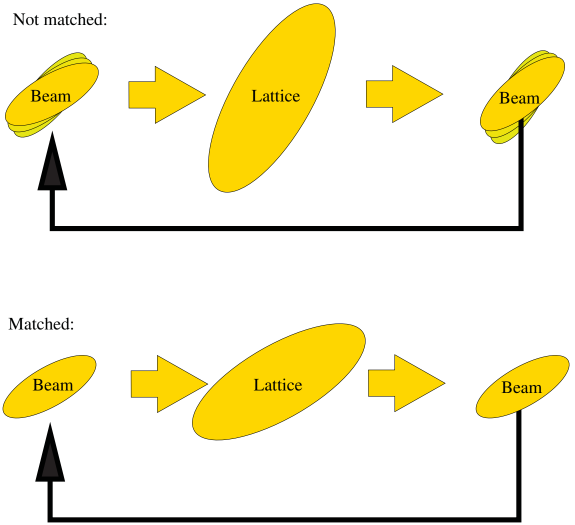


Figure : Left: Scetch of the principle. Right: More “realistic” orbits for various starting conditions.

Possible (linear) RF-Effekts



Coasting beam: Matching without Space Charge



Coasting beam: Matching without Space Charge

- Hill-Type equation of motion: $\psi' = \mathbf{F}(s)\,\psi$.
- One-turn transfer matrix **M** defined by lattice (Bend::Drift:Bend::Drift...).
  M is symplectic: $\mathbf{M}\,\mathbf{J}\,\mathbf{M}^T = \mathbf{J}$.
- $\mathbf{M} = \exp(\mathbf{F}\,s)$ is exponential of Hamiltonian matrix **F** [3].
- CFC (const foc. channel): $\psi' = \mathbf{F}\,\psi$ with $\mathbf{F} = \mathrm{const}$ substitutes "real" position-dependent $\mathbf{F}(s)$.
- Beam-matrix $\Sigma = \langle \psi\psi^T \rangle$ changed by lattice:
  $\Sigma_{n+1} = \mathbf{M}\,\Sigma_n\,\mathbf{M}^T$.
- Use symplectic unit matrix **J** and define $\mathbf{S}_n \equiv \Sigma_n\,\mathbf{J}$:
  $\mathbf{S}_{n+1} = \mathbf{M}\,\mathbf{S}_n\,\mathbf{M}^{-1}$: Symplectic transport is a similiarity transformation.
- Matched beam $\mathbf{S}_{n+1} = \mathbf{S}_n = \tilde{\mathbf{S}} \Rightarrow \tilde{\mathbf{S}}$ and **M** commute.
- $\Rightarrow$ $\tilde{\mathbf{S}}$ and **M** share a system of **Eigenvectors**.
- $\Rightarrow$ $\tilde{\mathbf{S}} = \tilde{\mathbf{S}}(\mathbf{M}, \varepsilon_i)$ (Emittances ε_i are Eigenvalues of **S**).

Matching with Space Charge

- **F** depends on lattice **and** beam size (self-interaction by space-charge).
- $\Rightarrow$ Transfer Matrix $\mathbf{M} = \mathbf{M}(\mathbf{F}, \mathbf{S})$.
- Matching becomes a "circular problem":
  We need to know $\tilde{\mathbf{S}}$ to determine $\mathbf{M}$.
  We need to know $\mathbf{M}$ to determine $\tilde{\mathbf{S}}$.
- Solution can (only?) be obtained iteratively:
  1. Knowing the emittances ε_i, guess $\mathbf{S}_0 = \Sigma_0\, \mathbf{J}$.
  2. Compute $\mathbf{M}_0(\mathbf{F}, \mathbf{S}_0)$.
  3. Compute $\mathbf{S}_1(\mathbf{M}_0, \varepsilon_i)$.
  4. Compute $\mathbf{M}_1(\mathbf{F}, \mathbf{S}_1)$
  5. Compute $\mathbf{S}_2(\mathbf{M}_1, \varepsilon_i)$ etc.
  6. If space charge is small enough, the sequence converges:
  7. Then $\mathbf{S}_{n+1} \approx \mathbf{S}_n$.

(see Ref. [10, 11, 12, 13]).

Computation I

Single particle dynamics (axial motion decoupled, treated separately):

- Pathlength along orbit s
- Radial coordinate $x = r(\theta) - r_0$ and $x' = \frac{dx}{ds}$.
- Longitudinal position $z = r_0\,(\theta - \theta_0)$.
- Momentum deviation $\delta = \frac{\Delta p}{p_0}$.
- Put in state vector $\psi = (x, x', z, \delta)^T$ in local co-moving curvilinear coordinates.
- Define $h = 1/r_0$ as curvature of orbit.
- Use CFC (const foc. channel) approximation: $\psi' = \mathbf{F}\,\psi$
- 1-Turn Transfer Matrix $\mathbf{M}$ defined by $\psi_{n+1} = \mathbf{M}\,\psi_n$

Computation II

The Hamiltonian matrix **F** is

$$\mathbf{F} = \begin{pmatrix} \cdot & 1 & \cdot & \cdot \\ -k_x + K_x & \cdot & K_{grad} & h \\ -h & \cdot & \cdot & \frac{1}{\gamma^2} \\ K_{grad} & \cdot & K_z\,\gamma^2 + K_{rf} & \cdot \end{pmatrix}$$

Focusing terms, space charge terms (defoc.), dispersive coupling $h = 1/r_0$, rf voltage grad/rf B-field, drift terms in black.

- $K_{rf} > 0$: "Debunching" phase.
- $K_{rf} < 0$: "Bunching" phase.

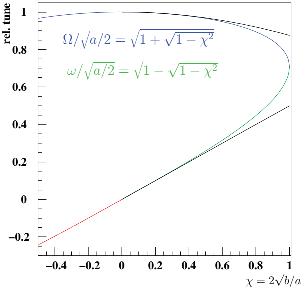
The motion is stable, if the Eigenvalues of **F** are purely imaginary. (and the *Eigenfrequencies* are real).

Eigenfrequencies (Tunes)

Compute the *Eigenfrequencies* Ω and ω of **F** [4]:

Technical Details:

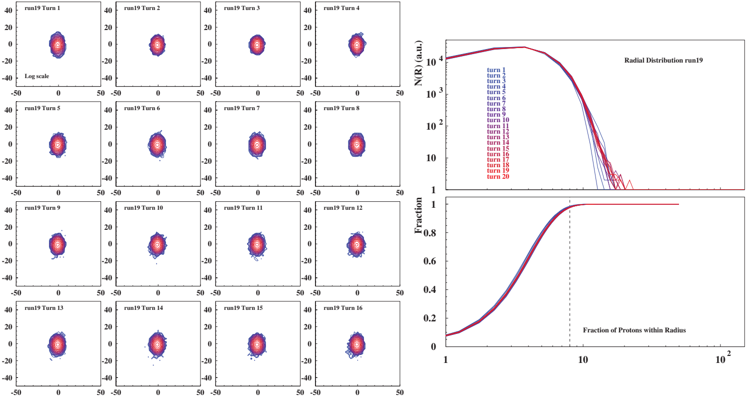
$$
\begin{aligned}
a &\equiv -\mathrm{Tr}(\mathbf{F}^2)/2 = \Omega^2 + \omega^2 \\
b &\equiv \mathrm{Tr}(\mathbf{F}^2)^2/8 - \mathrm{Tr}(\mathbf{F}^4)/4 = \Omega^2\,\omega^2 \\
\chi^2 &= 4\,b/a^2 \\
\Omega &= \sqrt{\frac{a}{2} + \sqrt{\frac{a^2}{4} - b}} = \sqrt{a/2}\,\sqrt{1 + \sqrt{1 - \chi^2}} \\
\omega &= \sqrt{\frac{a}{2} - \sqrt{\frac{a^2}{4} - b}} = \sqrt{a/2}\,\sqrt{1 - \sqrt{1 - \chi^2}} \\
\omega &\in \mathbb{R} \Rightarrow b \geq 0 \\
b\,\gamma^2 &= K_x\,(K_{rf} + K_z\,\gamma^2) - K_{grad}^2 > 0
\end{aligned}
$$



Ω is real, iff $b \leq a^2/4$.
ω is real, iff $0 \leq b \leq a^2/4$.

(Field error ε has been discussed before [10, 11], here neglected $\varepsilon \approx 0$).

Example: Coasting Beam in PSI Injector 2



Results for a matched coasting beam of 1 MeV in Injector 2.

OPAL [5, 6]: Matched gaussian *coasting* beam is stable.

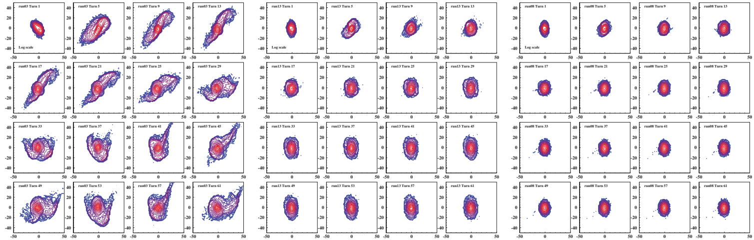
Questionable Assumptions

- "Adiabatic approximation is valid" ($\Delta E/E \ll 1$). Is it really?
  Joho's N^3-law [7] suggests to use highest possible voltage!
  In Injector 2 we have $\Delta E \approx E$ at injection!
- "Vortex motion is stable". Wrong: It is metastable.
  If the bunch has expanded spatially, the space charge force and the space charge tune decrease and the bunch does not "shrink" back to original size. Distortions of a matched beam induce, due to the non-linearity of the space charge force, bunch deformation followed by filamentation and emittance increase.
- "RF can be neglected". But what about injection and the first turns? What, if the phase is not zero? Then there should be (de-) bunching effects. Also a strong voltage gradient will likely disturb the vortex effect.

Adiabatic Approximation I

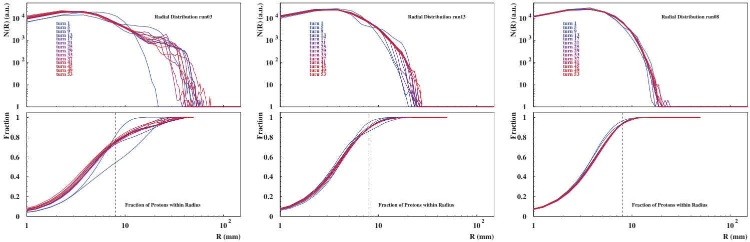
Using OPAL simulations to test the simplified linear model:
Fast acceleration versus adiabatic approximation.

$V = 100\,\%$ $V = 50\,\%$ $V = 25\,\%$



Adiabatic Approximation II

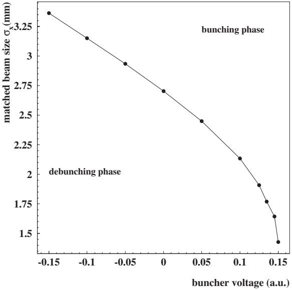
$V = 100\,\%$ $V = 50\,\%$ $V = 25\,\%$



If adiabatic condition is not fulfilled due to high accelerating voltage, the beam halo increases. The beam must be cleaned up by beam collimation [9].

(De-)Bunching by RF Voltage I

- RF-phase $\phi > 0$: Bunch lags behind. $V_{head} > V_{tail}$ ("debunching")
- RF-phase $\phi < 0$: RF lags behind. $V_{head} < V_{tail}$ ("bunching")

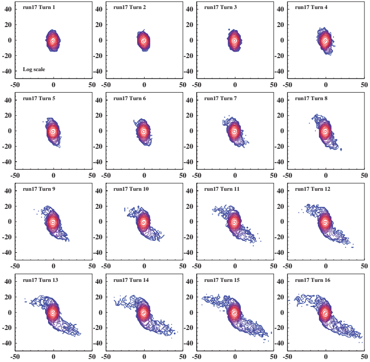


- Matched beam size increases with debunching voltage.
- Matched beam size decrease with bunching voltage.
- Compromize: Accelerate at phase $\phi = 0$.

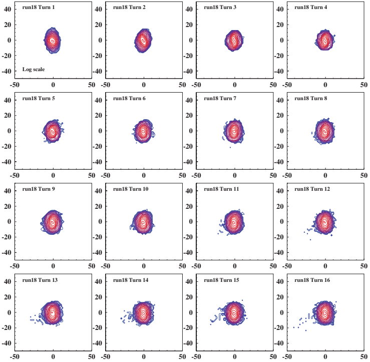
(De-)Bunching by RF Voltage II

- RF-phase $\phi = -90^\circ$: No acceleration, “bunching” phase.
- RF-phase $\phi = 90^\circ$: No acceleration, “debunching” phase.

$V = 10\,\%$, $\phi = -90^\circ$, bunching

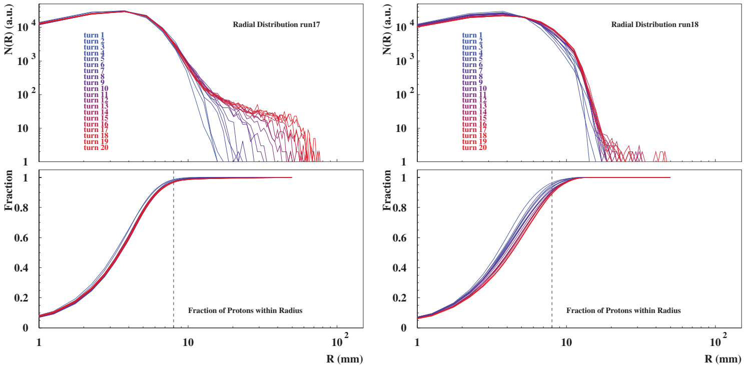


$V = 10\,\%$, $\phi = 90^\circ$, debunching



(De-)Bunching by RF Voltage III

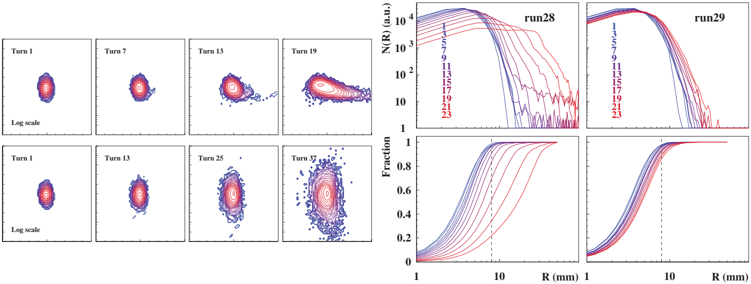
$V = 10\,\%$, $\phi = -90^\circ$, "bunching". $V = 10\,\%$, $\phi = 90^\circ$, "debunching".



- "Bunching" RF-phase: More halo, but more compact core.
- "Debunching" RF-phase: Few halo, but larger core.
- Best compromize: RF-phase close to zero.

Strong RF Voltage Gradients

- $\langle R \rangle \approx R_0$.
- $V(R) \propto (R - R_0)$, RF-phase $\phi = 0°$: Positive $V' > 0$.
- $V(R) \propto (R - R_0)$, RF-phase $\phi = 180°$: Negative $V' < 0$.



- Positive $V' > 0$: Bunch deforms quickly.
- Negative $V' < 0$: Bunch size increases continuously.
- Best: $V' \approx 0$.

Summary

- Linear model allows to compute matching conditions.
- Adiabatic approximation not valid in center (of Injector 2).
    - $\Rightarrow$ bad conditions for smooth acc. of matched beam.
    - $\Rightarrow$ halo formation difficult to avoid.
    - $\Rightarrow$ beam collimation required.
    - $\Rightarrow$ high energy gain required (space for collimators).
    - $\Rightarrow$ self-matching of beam by filamentation unavoidable.
    - $\Rightarrow$ some emittance increase is unavoidable.
- Strong voltage gradients at low energy are potentially harmful.
- (De-) bunching by $\phi <> 0$ does not (always) help to stabilize beam.
- It is difficult to predict beam/halo formation without simulations (OPAL).

Thank You.

[1] M.M. Gordon "The longitudinal space charge effect and energy resolution"; Proc. of the 5th Int. Cycl. Conf., Oxford, 1969; Proceedings ed. R. W. McIllroy, Butterworth, London, UK (1971), pp. 305-317.

[2] R. Dölling; Proc. of HB2010, Morschach, Switzerland (2010).

[3] K.R. Meyer, G.R. Hall and D. Offin; "Introduction to Hamiltonian Dynamical Systems and the N-body Problem", Springer (2000).

[4] C. Baumgarten, arXiv:1703.02893.

[5] J. J. Yang, A. Adelmann, M. Humbel, M. Seidel, and T. J. Zhang, Phys. Rev. ST Accel. Beams 13, 064201 (2010).

[6] Y. J. Bi, A. Adelmann, R. Dölling, M. Humbel, W. Joho, M. Seidel, and T. J. Zhang, Phys. Rev. ST Accel. Beams 14, 054402 (2011).

[7] W. Joho, High Intensity Problems in Cyclotrons, Proc. 5th intl. Conf. on Cycl. Appl., Caen 1981.

[8] Frank Hinterberger, Physik der Teilchenbeschleuniger, 2. Auflage, Springer, Heidelberg 2008.

[9] A. Kolano *et al* NIM A 885 (2018).

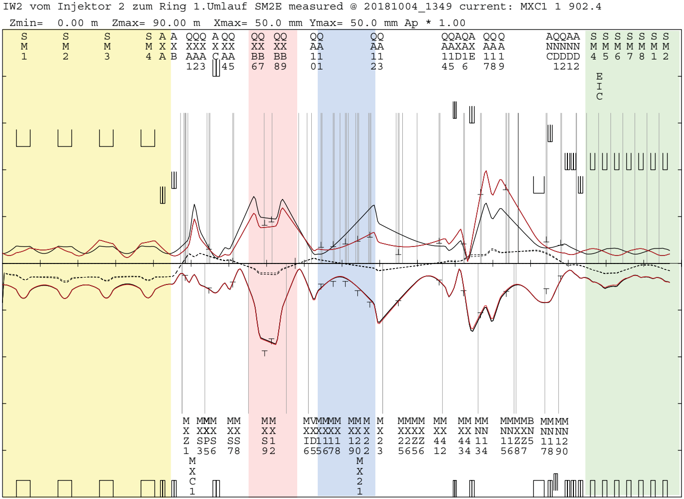
[10] C. Baumgarten; Phys. Rev. ST Accel. Beams 14, 114201 (2011).

[11] C. Baumgarten; Contr. to Cyclotron Conf. 2013.

[12] C. Baumgarten; Phys. Rev. ST Accel. Beams 15, 124001 (2012).

[13] C. Baumgarten; arXiv.org:1205.3601 (2012).

The Usefulness of Matching



Horizontal beam parameters: From matched beam. Vertical parameters σ_y,σ_{py} and $\mathrm{cov}(3,4)$ fitted to measurement.