

# STATUS OF FFAS (MODELLING AND EXISTING/PLANNED MACHINES)

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## Abstract

Since their rebirth two decades ago, great progress has been made in Fixed Field alternating gradient Accelerator (FFA) design, with different optical concepts and technological developments. Several machines have been built, and others are planned. The talk will review the recent progress around the world.

## INTRODUCTION

A Fixed Field alternating gradient Accelerator (FFA) is defined as a circular particle accelerator, with a static guide field, like cyclotrons, and focusing and defocusing elements alternated to provide a strong focusing similar to modern synchrotrons. This principle is not new and came shortly after the discovery of the alternating gradient focusing [1] in the 1950s, in Japan [2], in USSR [3] and USA [4, 5], independently. Electron models were built shortly after its discovery by the Midwestern Universities Research Association (MURA) group [6–8] in the USA, but after it closed in 1967, FFA development was paused for about 40 years. If cyclotrons benefitted from the spiral geometry developed with FFAs [4, 5], pulsed synchrotrons, more suitable to reach the energy frontier, were favoured over FFAs for high energy physics. However, the importance of the beam power over the final energy has been growing recently, since intense sources of secondary particles from high-power proton beams are now a priority in several fields. An FFA would be a good candidate for such proton drivers since it can indeed reach relativistic energies, contrary to cyclotrons, and fixed field gives the possibility for higher repetition rates and at a more energy efficient operation than in Rapid Cycling Synchrotrons (RCS).

Since the rebirth of the FFAs twenty years ago, several machines have been designed and built in Japan [9–13], and in the UK [14], with several of them still in activity, like the ADS complex [15] and the MERIT experiment [16] at Kyoto University and the 150-MeV ring at Kyushu University [17]. Several machines are planned for the near future or being commissioned at the moment. At CERN, the nuSTORM project aims to study neutrino interactions with a muon-decay racetrack ring composed of FFA magnets [18]. In the USA, to demonstrate the feasibility of the use of FFA arcs in the eRHIC project [19], the Cornell-BNL Energy recovery linac Test Accelerator (CBETA) is under commissioning [20, 21]. In the UK, a major upgrade of the ISIS synchrotron is currently under study [22] and the FFA option is under consideration. A test ring is planned at RAL to demonstrate the capability of the FFA to deliver a high-power and short pulse proton beam for spallation neutrons [23].

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Several simulation codes are now available to model FFAs. Since the beam orbit in an FFA moves spatially with momentum, synchrotron simulation codes, which assume a central orbit independent of momentum, are unsuitable for studying FFAs. However, cyclotron codes and more generally static field codes including OPAL [24], Zgoubi [25], SCODE, MUON1 and FIXFIELD among others are now used to design FFAs. The first three integrate space charge to study high intensity effects.

FFAs are usually designed with an increasing radius, but excursion can be also done in the vertical direction. It has been first proposed in 1955 [27] as an “electron cyclotron”. It has been rediscovered recently [28]. Vertical FFA (vFFA) could be an asset when it comes to accelerate ultra-relativistic particles, because of its quasi-isochronicity. This arrangement has several other advantages. First, it results in an orbit radius independent of momentum, like synchrotrons. Second, the horizontal dispersion function and the momentum compaction factor are zero, with infinite transition energy. Third, the scaling property is separated from the geometrical arrangement of the lattice footprint. In principle, the ring could have any shape and it would still be possible to maintain a scaling property as long as the vertical magnetic field satisfies the design shape of scaling magnets. Finally, a rectangular shape for the main magnets and the coil geometry is simpler compared to the spiral magnet of horizontal FFA.

This paper will present the new concepts in terms of lattice first for the horizontal excursion FFA, and then for the vertical excursion FFA.

## HORIZONTAL EXCURSION FFA

### DF Spiral

To keep the linearised transverse motion equations independent of momentum, the vertical component in the horizontal mid-plane of the magnetic field  $B_z$  varies with radius  $r$  according to the so-called scaling law, following

$$B_z = B_0 \left( \frac{r}{r_0} \right)^k \mathcal{F} \left( \theta - \ln \frac{r}{r_0} \tan \zeta \right), \quad (1)$$

with  $k$  the constant geometrical field index,  $r_0$  the reference radius,  $B_0$  the field at that radius,  $\zeta$  the constant logarithmic spiral angle and  $\mathcal{F}$  an arbitrary fringe field fall-off function.

There are two types of zero-chromatic horizontal FFA, the radial type (case where  $\zeta = 0$ ) and the spiral type. In the radial sector FFA, the alternating gradient is achieved with reverse bend magnets, while designing a lattice with logarithmic spiral FFA magnets gives a constant edge focussing independent of the beam momentum. Spiral FFAs thus have a smaller circumference than radial sector FFAs,

but the possibility of adjusting focussing in the transverse plane is very limited. The field gradient of the main magnets could be changed if pole-face winding coils are used. However, this would only allow adjustment of the transverse tune in a very confined range, which could be a problem for the initial commissioning, especially for high current proton accelerators where the tune depends on the beam current.

A DF spiral lattice, which features normal and reverse bending magnets with a spiral edge angle, is a compromise between machine circumference size and tune flexibility [26]. In the same way, this method could be used in cyclotrons to offer better control over dynamics and would be useful in high-intensity machines. The number of cells is chosen as a multiple of 5 to give the largest resonance-free space between an integer and a quarter integer. A systematic 5th order resonance indeed coincides with an integer when the periodicity of the lattice is a multiple of 5, while space charge driven resonances at a quarter integer prohibit an operating tune just above a quarter integer. Almost equal horizontal and vertical tunes are chosen because it is the empirical best operating point of the most recently built high current accelerators, e.g. SNS and J-PARC. Parameters of an example of such a lattice is given Table 1, designed for ISIS-II.

Table 1: Parameters of DF Spiral 1.2 GeV FFA

Parameter	Value
Kinetic energy	0.4 - 1.2 GeV
Reference radius	24 m
Number of cells	25
Packing factor	0.35
Straight section	3.58 m
Spiral angle	62 deg
k-index	20.6
Ratio Bd/Bf strength	-0.443
Orbit excursion	0.8 m
Cell tune (H, V)	(0.2073, 0.2098)
Ring tune (H, V)	(5.18, 5.24)
Transition gamma	4.6

Figure 1 shows the top view of the DF spiral lattice. Figure 2 shows the vertical magnetic field strength along the closed orbit and the beta function at the extraction momentum.

### Tilted Sector FFA

The main problem of the spiral geometry is the difficulty to manufacture the main magnets, especially for a superconducting design. It is also a challenge to incorporate square shape elements in drift sections, like cavities. A tilted sector type of magnet would be an advantage to solve these issues, while keeping edge focusing and thus vertical stability for high energy machines. A scheme of such a machine is presented in Fig. 3. However, a numerical solution of the field is necessary to control the tune with such a technique, since analytical solution is not available in this case.

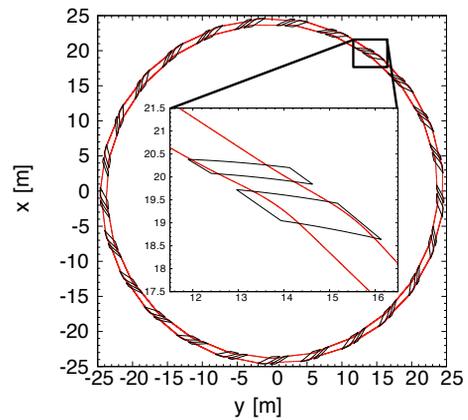


Figure 1: Spiral FFA ISIS upgrade lattice with 25 cells with closed orbits of injection and extraction momenta.

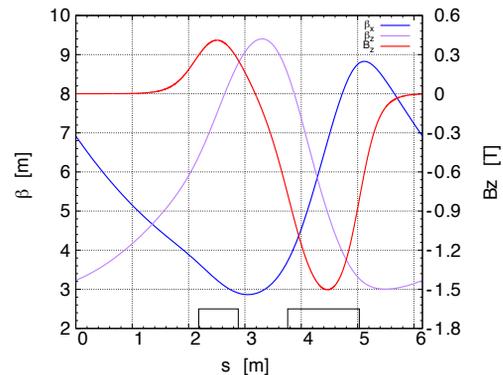


Figure 2: Vertical magnetic field on the median plane and beta functions of the DF spiral cell at extraction energy (1.2 GeV).

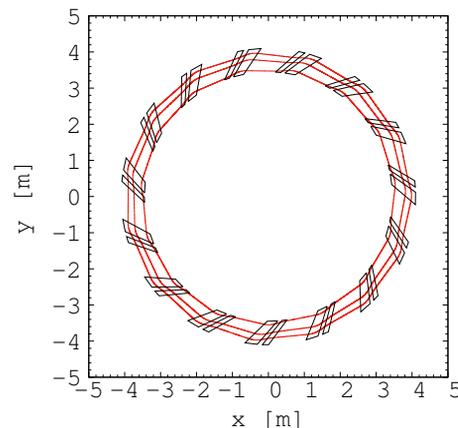


Figure 3: Scheme of a tilted sector FFA solution.

## VERTICAL EXCURSION FFA

### Simulation

In a vFFA, the particle motions in horizontal and vertical planes are no longer independent, making numerical simulation necessary to study beam dynamics in such machines. There is no established code designed to model vFFA, so extensive effort is under way to develop such codes. The

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field used in the simulation code is expanded in Cartesian coordinates  $(h, v, l)$  in terms of polynomial in the horizontal direction  $h$  from the ideal mid-plane field (the mid-plane for a vFFA is a zero-displaced plane in the horizontal direction) so that the fields satisfy Maxwell's equations. In this case, the zero-chromaticity condition can be obtained with an exponential increase of the magnetic field in the vertical direction  $v$ . In the mid-plane  $h = h_0$ , the field is then defined as

$$\begin{cases} B_{h0}(h_0, v, l) = 0 \\ B_{v0}(h_0, v, l) = B_0 e^{m(v-v_0)} \mathcal{F}(l) \\ B_{l0}(h_0, v, l) = \frac{B_0}{m} e^{m(v-v_0)} \mathcal{F}'(l) \end{cases}, \quad (2)$$

with  $m$  the constant normalised field gradient,  $\mathcal{F}$  an arbitrary fringe field fall-off function. It is worth noticing that the parameters of the magnet for its design include both vertical and longitudinal components in the mid-plane.

Parameters of the test ring lattice planned to study the feasibility for ISIS-II presented above are presented in Table 2. Figure 4 shows the top and side view of the vFFA test ring

Table 2: Parameters of Test Ring vFFA

Parameter	Value
Kinetic energy	3 - 12 MeV
Reference radius	3.9789 m
Number of cells	10
Packing factor	0.32
Straight section	1.0 m (long), 0.5 m (short)
m-index	$1.6 \text{ m}^{-1}$
Ratio Bd/Bf strength	-0.47
Orbit excursion	0.4 m
Cell tune (H, V)	(0.19, 0.16)
Transition gamma	infinite

cell. Figure 5 shows the magnetic field components along the orbit at the extraction momentum.

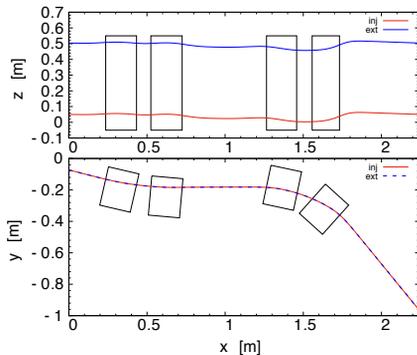


Figure 4: vFFA test ring cell from the side (top part) and from the top (bottom part).

## SUMMARY

The contribution of FFAs to cyclotrons has been tremendous over the years, with the spiral geometry to increase

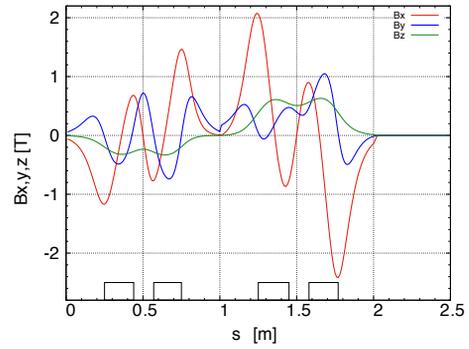


Figure 5: Magnetic field components on the closed orbit in the vFFA test ring cell at extraction momentum.

vertical stability, the addition of negative bend to increase control over dynamics, and the vertical excursion to have an isochronous machine for relativistic particles.

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