

# SOME EXAMPLES OF RECENT PROGRESS OF BEAM-DYNAMICS STUDIES FOR CYCLOTRONS

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## Abstract

Two subjects are highlighted. The first is the problem of high space charge effects in cyclotrons. The second is the progress in development of tools and simulations for industrial and medical cyclotrons at IBA.

## INTRODUCTION

This paper reviews examples of recent progress in beam dynamics studies for cyclotrons. The paper is not extensive, but rather follows developments of two subjects during the past years and tries to add an educational accent. The first is the problem of high space charge. This subject is at this moment very important, in view of possible applications of (compact) cyclotrons for ADS, or for generating high fluxes of neutrinos for experiments such as IsoDAR or Daeδalus (see this conference [1]). Examples are given of the increased understanding based on (semi-)analytical models and of the latest developments and achievements based on numerical simulations. The second subject is the progress in simulations for industrial and medical accelerators as has been achieved at our company during the last years.

## SPACE CHARGE IN CYCLOTRONS

One can distinguish roughly three types of intensity limiting space charge effects in cyclotrons: i) the problem of beam blowup due to the weak vertical focusing in the cyclotron center, ii) the problem of loss of turn separation due to space charge induced energy spread and iii) the problem of space-charge induced halo for poorly matched beams.

### *Space Charge Effects in the Cyclotron Center*

This paragraph refers to papers of Rick Baartman, Thomas Planche and Yi-Nong Rao [2–5]. In the central region of a (compact) cyclotron, the azimuthal magnetic field variation falls quickly to zero and therefore the magnetic vertical focusing becomes very weak. The beam current is limited by the space charge vertical defocusing and the resulting vertical losses on the central region.

The incoherent vertical tune shift inversely depends on the kinetic energy of the particle. Thus increasing the injection energy helps to increase the current limit. This is done for example at PSI (870 keV) and at Triumf (300 keV). A drawback is that losses at these higher injection energies can already be harmful. Furthermore such high injection energies can not be done in smaller machine. For compact cyclotrons, one has to rely on the weak electric focusing of the RF gaps.

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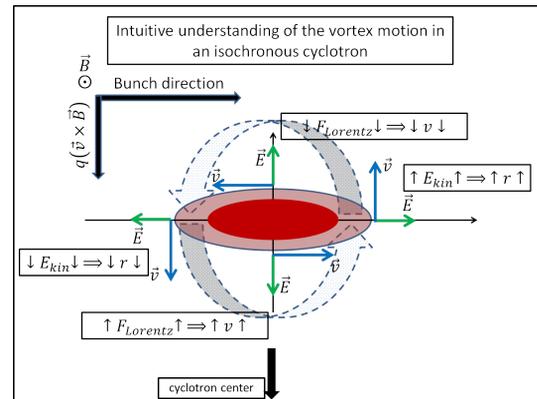


Figure 1: Intuitive understanding of the vortex motion

This focusing is RF-phase dependent. The bunch-center must cross the gap on the falling RF slope. This means that the head of the bunch experiences weaker focusing than the tail. By this, space charge causes progressive loss of the bunch head [2]. When using a (spiral) inflector, the beam needs to be strongly compressed in three dimensions (by bunching and transverse focusing). The beam at the inflector exit is therefore strongly mismatched with respect to the weak vertical focusing. The complicated spiral inflector optics strongly correlates the 6D phase space. Both effects result in strong emittance growth [5].

### *Loss of Turn-Separation Due to Space Charge*

This paragraph refers for some part to a nice overview paper of Rick Baartman [6] in the 2013 Vancouver cyclotron conference. In an isochronous cyclotron, the space charge effect induces a vortex motion and an increase of energy spread in the bunch. An intuitive understanding of this is obtained from Fig. 1. Here the bunch is moving clockwise from left to right. Due to the outward directed space charge force, the leading particles gain energy. In an isochronous cyclotron they can only move to higher radius. Trailing particles lose energy and move to lower radius. The radially exterior particles experience a reduction of centripetal force and fall behind in phase. The opposite happens for the interior particles.

The bunch density shows an effect of macroscopic rotation (vortex). If the space charge equipotential curves would be similar to that of the ellipse and the space charge force would remain perfectly linear, then the elliptical bunch-shape would be conserved. Generally this is not the case and the bunch starts to deform (spiralize) such that the outer part of the bunch will rotate slower than the core.

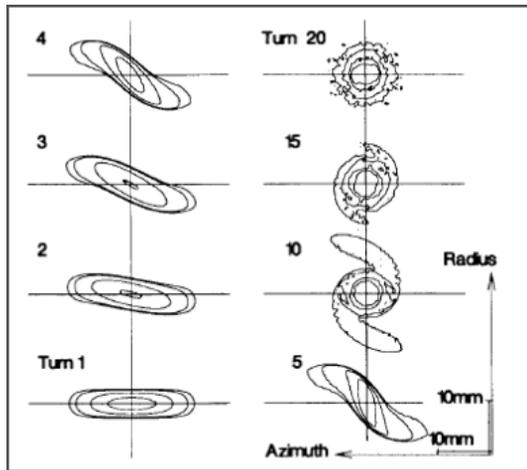


Figure 2: Simulation by Stefan Adam [7] of a 1 mA bunch accelerated in the PSI injector II

Stefan Adam from PSI first started with efforts to simulate the longitudinal space charge effects in their injector and ring cyclotron [8]. Figure 2 shows a later (1995) simulation [7] of a 1 mA bunch accelerated in the PSI injector II, with an initial phase width of  $15^\circ$ , during 20 turns. It is seen that the core of the bunch rotates faster than the envelope, resulting in an initial deformation of the bunch. After 10 to 15 turns a round beam emerges with an intense core, surrounded by halo. Halo reduces with better initial matching

The vortex effect was first recognized in 1969 by Mort Gordon [9] who explained that, due to the Coriolis forces in a rotating frame, the particles in the bunch execute a steady state velocity pattern which is directed along the equipotential curves of the space charge potential. However, at the same time he found that this effect could be neglected in the then known practical case where the bunches were much longer than wide. In 1981 Werner Joho [10] elaborated further on Gordon's idea, using a model of multiple turns with constant azimuthal length (sector- or pie-model) to calculate the space charge induced energy-spread and its effect on the turn-separation. This resulted in a formula showing an intensity limit proportional to the cube of the energy gain per turn (or RF-voltage; so a  $1/n^3$  dependence;  $n$  is turn number). This formula is still found to be correct although the sector model has been invalidated later with the numerical confirmation of the vortex motion at PSI.

In 1988 the author of this paper [11] applied the 3D beam-envelope approach of Sacherer [12] to derive differential equations for the full set of second moments of a space charge phase space in an Azimuthally Varying Field (AVF) cyclotron. One of the outcomes was a proof of existence of stationary round bunches. Another was that such bunches follow envelope equations that are similar to the Kapchinsky-Vladimirsky (KV) equations.

In 2001 Bertrand and Ricaud [13] used an elegant and simple model of a spherical non-relativistic bunch in a homogeneous magnetic field. The solution of the particle linear equations of motion show the existence of two

modes of oscillation. The motion is found to be stable if the total charge  $Q$  in the bunch is smaller than a threshold  $Q_{max} = \pi\epsilon_0(m/q)\omega_c^2 r^3$  (where  $\omega_c$  is the cyclotron frequency and  $r$  the bunch radius). It is interesting to see that this can be also formulated as follows:

$$\omega_p < \frac{1}{2}\sqrt{3}\omega_c, \quad (1)$$

where  $\omega_p$  is the plasma-frequency ( $\omega_p = \sqrt{nq^2/m\epsilon_0}$  and  $n$  is particle density). This suggests that the plasma-frequency  $\omega_c$  is a critical parameter.

In his 2013 paper [6] Baartman explores further the models of Kleeven and Bertrand/Ricaud and provides a deeper insight into the vortex physics. The two modes are interpreted as coupled betatron ( $r, P_r$ ) and dispersion ( $E, \Phi$ ) motion. For  $Q \ll Q_{max}$  the betatron oscillations are fast and the energy oscillations slow. For  $Q = Q_{max}$  the acceptance approaches zero and both frequencies are equal to  $1/2$ . This is a beam with zero emittance and laminar flow. Figure 3 shows the tunes of both modes.

Baartman also derives a formula for the intensity limit of separated turn cyclotrons which applies if the injected bunch is sufficiently short such that the vortex motion causes the bunch to curl up into a single droplet.

$$I_{max} = \frac{h}{2g_r\xi^3\beta^3\gamma v_x^4} \frac{V_{rf}^3}{V_m^2 Z_0}. \quad (2)$$

This formula shows a dependence with a third power of the RF-voltage  $V_{rf}$ , but also the scaling with respect to the particle type (mass  $V_m = mc^2/q$ ), the RF harmonic mode  $h$ , the tune  $v_x$ , and the relativistic parameters  $\beta$  and  $\gamma$ .

Acceleration effects are considered and a qualitative threshold is found for the vortex motion to take place: below the threshold the bunches maintain their phase length (thus bunch length increasing like  $R\delta\theta$ ), but above it the bunch length remains constant and thus decreases in phase length. This threshold is  $2\pi\Delta v_r \ll \delta\beta/\beta$ , stating that the space charge induced tune shift must be (considerably) larger than the relative velocity increase due to acceleration.

The vortex effect makes that bunches with a high length to width ratio break up into small approximately circular

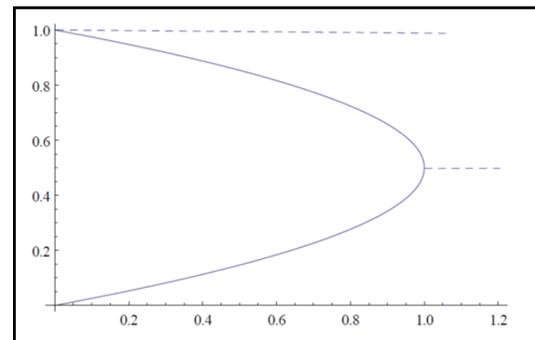


Figure 3: Tunes  $\nu_{r\pm} = \frac{1}{2}(1 \pm \sqrt{1 - Q/Q_{max}})$  of the two modes as a function of  $Q/Q_{max}$

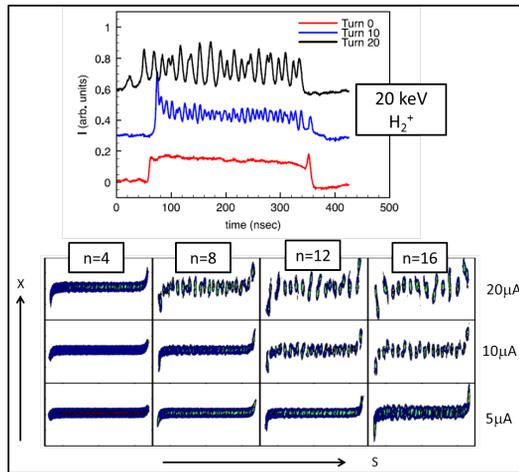


Figure 4: upper: longitudinal bunch profiles measured on a fast Faraday cup in the SIR at MSU for a bunch peak current of  $9.3 \mu\text{A}$ . lower: beam simulations in the SIR with the PIC tracking code CYCO. Each frame contains a median plane projection of the bunch and has a size of  $5 \text{ cm} \times 45 \text{ cm}$  ( $x \times s$ ). Droplets start to appear quickly, depending on the peak current (from Pozdeyev [14]).

droplets. This was shown both experimentally and by simulations in the 2003 thesis study of Pozdeyev [15] on the Small Isochronous Ring (SIR) at MSU (see Fig. 4).

Recently Antoine Cerfon *et al.* have introduced a new fluid dynamics approach in which they directly obtain an approximate solution of the collisionless Vlasov equation [16–18]. The main simplifications and assumptions made are the following: i) the space charge is not too strong such that  $\omega_p/\omega_c < 1$  (equivalent to saying that the incoherent tune shift is small). ii) the beam size is mainly determined by dispersion and little by emittance (close to laminar phase space), iii) a non-relativistic, 2-dimensional coasting beam in homogeneous B-field is considered. The first assumption makes that the time-scale associated with betatron oscillations is much smaller (faster) than the time scale associated with space charge. This allows to apply an averaging procedure to the Vlasov equation that results in a simple fluid-like equation for the particle density  $n$ . Together with the Poisson equation this gives two simple coupled 2D partial differential equations that describe the radial-longitudinal space-charge induced vortex motion in the isochronous cyclotron:

$$\frac{\partial n}{\partial t} + \delta^2 (\nabla\phi \times \mathbf{e}_z) \cdot \nabla n = 0, \quad (3)$$

$$\Delta\phi = -n, \quad (4)$$

where  $\delta = \omega_p/\omega_c$ . Since the vector  $\nabla\phi \times \mathbf{e}_z$  in Eq. (3) is proportional to  $\vec{E} \times \vec{B}$ , the vortex motion can be intuitively understood as the nonlinear advection of the bunch by the  $\vec{E} \times \vec{B}$  velocity field (similar to Gordon's idea). Note that the  $\delta^2$ -term in the above equation can just be eliminated by a proper time scaling. This implies that (within the assumed

approximations) the beam intensity does not affect the nature but only the time scale of the vortex motion.

The strength of the model is that it provides an interpretation of complicated PIC simulation results and identifies the basic contributing mechanisms without the need of large supercomputers and long computing times. Figure 5 shows two examples of simulations. Of course the approach does not give the quantitative precision as PIC codes such as for example OPAL. Such precision is needed for actual designs. Both approaches are complementary. Another observation made by Cerfon is that the fluid equations are isomorphic to the two dimensional Euler equations for a fluid of uniform density. This means that results known from fluid dynamics theory on the behaviour of isolated vortices can be directly interpreted in the language of beam dynamics in cyclotrons. Based on this, predictive statements can be made such as for example: i) *round beams with monotonically decreasing density profiles are stable to finite perturbations* ii) *elliptic beams with smooth, monotonically decreasing density profiles are subject to spiraling and axisymmetrization* iii) *elliptic bunches with too high aspect ratios break up into smaller bunches due to Rayleigh's inviscid shear instability*.

For high-intensity cyclotrons with an ESD, turn-separation at extraction is crucial (avoid septum losses) and a good matching is needed at injection such that the vortex-effect occurs quickly, resulting in circular bunches. For too long bunches their sizes increase and a large halo devel-

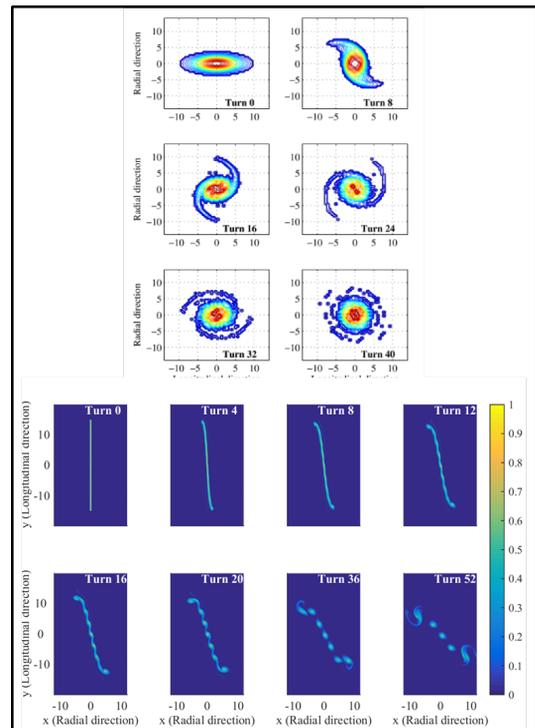


Figure 5: Upper: formation of a round beam core surrounded by a low density halo as simulated with the fluid-dynamic approach by Cerfon [17] for  $\delta^2 = 0.2$ . Lower: simulation of the beam break-up [18].

ops, resulting in high extraction losses. For the PSI injector II cyclotron the vortex motion is so strong that very short bunches are obtained such that the flattop system is no longer needed [19] (and is actually used for acceleration). In the PSI ring cyclotron the space charge effect is not strong enough to produce the circular bunches. Here the relative phase of the flattop cavities is detuned such that the energy gain in the tail of the bunch is larger than in the head, thereby counteracting the linear part of the longitudinal space charge force and the related energy spread [20].

A reference in the numerical simulation of space charge effect is the code OPAL (Object oriented Parallel Accelerator Library). It is a PIC space charge tracking code, for large accelerator structures and is developed mainly at PSI [21]. It extensively relies on parallel processing and is able to simulate large numbers of accelerated particles (order  $10^6$ ) in cyclotrons resulting in very precise beam density and profile predictions. It has been (and still is) used extensively in simulations of the PSI injector II cyclotron as reported in this conference by Anna Kolano [22]. It also has been applied for other cyclotrons such as for example the PSI ring cyclotron [20,23], the CIEA 100 MeV H- cyclotron [24], the proposed IsoDAR injector cyclotron and the Daedalus superconducting ring cyclotron [25], but also for other machines such as FFAG's [26]. The code remains under further develop-

ment. It can simulate the space charge effect of neighboring bunches [23]. Besides space charge it has various other simulation applications such as wake-fields, multipacting and particle-matter interaction. An update on OPAL is given also in this conference by Andreas Adelman [27]. Figure 6 shows some results of a OPAL simulation of the IsoDAR cyclotron.

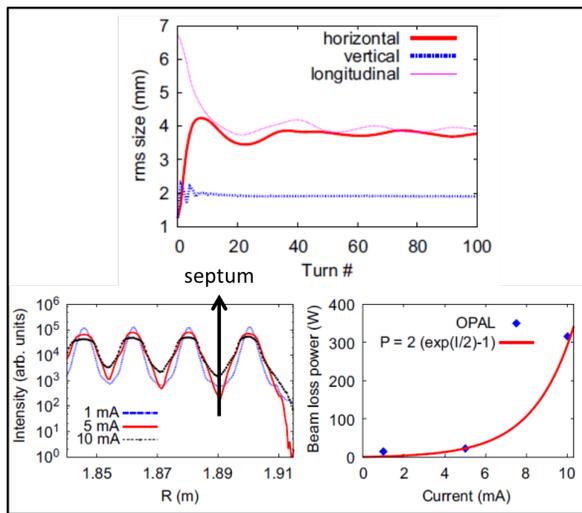


Figure 6: OPAL simulation of IsoDAR. Upper: a long Gaussian bunch (aspect ratio 5/1,  $\Delta\Phi=40^\circ$ ) of  $I_{ave} = 5$  mA  $H_2^+$  is coasting at 3 MeV. After 10 to 15 turns, the bunch obtains the circular match due to the vortex effect. This suggests that, as in the PSI injector II, flattop cavities are not needed and 4 accelerating cavities can be placed in the valleys. Lower: the use of a large number of macroparticles ( $10^6$ ) allows to predict with high dynamic range ( $10^4$ ) the density profile in between the last two turns and to estimate the losses on the septum (0.5 mm) of the ESD as function of the beam current and also to optimize collimators placed closer to the center to reduce beam-halo. Based on such simulations it is claimed that IsoDAR can extract the 5 mA of  $H_2^+$

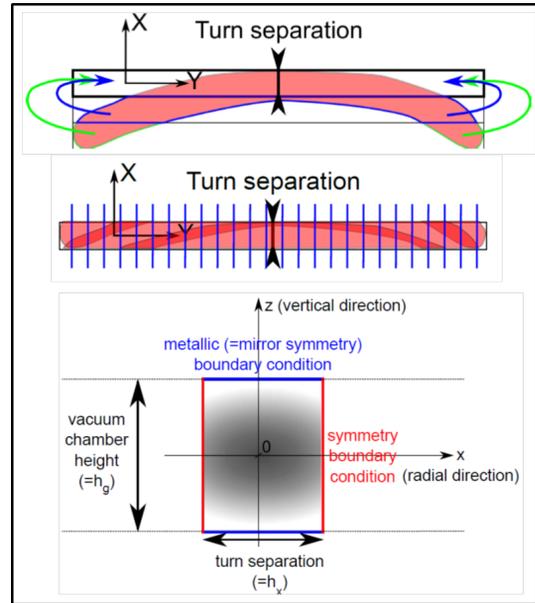


Figure 7: Illustration of the use of radial periodicity in the space charge code TRICYCLE [4, 28].

For cyclotrons using stripping extraction (such as  $H^-$  at Triumf or  $H_2^+$  in the proposed Daedalus ring-cyclotron [25]) a large turn-separation  $\Delta R$  at extraction is not needed and the space charge induced energy spread is not harmful for extraction. The relation between energy and radius remains unique (except for the small contribution of emittance) and the quality of extracted beam is not really affected. These machines can accept a large phase width at injection (up to  $60^\circ$ ). Bunches are long and also may have a large radial extend due to energy spread. At large radii, turns overlap and the effects of neighboring turns become essential. The neighboring bunch feature is available in OPAL and has been used successfully in simulations of the PSI ring cyclotron [23] and the CIAE 100 MeV  $H^-$  cyclotron [24]. For the Triumf case, with extreme long bunches, OPAL would require a very large number of particles and a large grid. Thomas Planche *et al.* [4, 28] made a code (TRICYCLE) which uses periodic boundary conditions in the radial direction to solve the Poisson equation (see Fig. 7). This is allowed assuming that bunch shape evolves slowly turn by turn. This considerably reduces computation time, because only one bunch needs to be followed during the simulation.

Bunches are sliced radially by a box with a width of the turn separation  $\Delta R$ . Parts of the bunch that are outside of the box, are returned into it by a radial shift of  $\Delta R$ . The box is sliced longitudinally to create a number of 2D surface-charge density distribution (in X-Z plane) For each slice the

3D Poisson equation is solved by FFT with proper boundary condition ( $x \Rightarrow$  periodic,  $y \Rightarrow$  open,  $z \Rightarrow$  metallic). A simulation result is shown in Fig. 8.

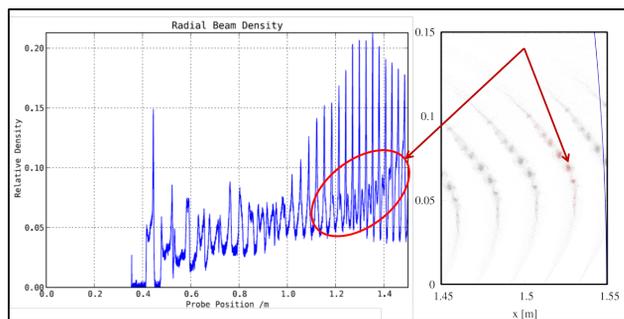


Figure 8: Observation of the beam break-up in a measured radial probe track (left) and TRICYCLE simulation (right) of a high space charge bunched beam (410  $\mu$ A) during the first 30 turns in the Triumf cyclotron central region [28].

For the DAE $\delta$ ALUS/IsoDAR studies, extensive injection line space charge simulations have been done. This paragraph refers to a paper made by Daniel Winklehner *et al.* [29]. The project calls for 50 mA of  $H_2^+$  injected from the ion source into the LEBT. Simulations need to also take into account space charge compensation: the effect that slow electrons from the beam-residual gas interaction are trapped in the beam potential well, thereby reducing space charge. The Particle-In-Cell (PIC) code WARP [30,31] as developed at LBNL/LLNL was used. The space charge compensation depends on the beam line vacuum. The pressure distribution along the line was simulated with the Monte Carlo program MOLFLOW [32]. The space charge compensation is estimated at each time-step, based on an analytical/empirical modeling of the physical processes. Initial particle distributions were obtained from self-consistent 3D ion source extraction simulations, done by the Catania group with the code KOBRA-INP [33]. The dual-particle ( $p$  and  $H_2^+$ ) DC ion beam in the LEBT is transported through a series of transversal slices along the  $z$ -axis. From the different possible field-solvers in WARP, the XY-slice solver was used (longitudinal fields are small and ignored) The simulations were bench-marked against measurements done on a LEBT test-stand provided by Best Cyclotron Systems (BCS) in Vancouver (see Fig. 9).

As is shown in Fig. 9, even with the long chain of simulations needed (i) ion source extraction, ii) beam line vacuum, iii) space charge neutralization and iv) dual particle beam transport with space charge), a good fit with measurements was obtained. For IsoDAR, this is not the end of the journey because the beam transport, transmission, optics and matching through the inflector and cyclotron central region (CR) is a crucial and very complex 3D space charge problem. This question needs to be answered, in order to be able to estimate the highest intensity achievable with such a cyclotron. Efforts are ongoing to include the spiral inflector and the CR into OPAL. Alternatively, RFQ injection has been explored [34]. The strong mismatch between the RFQ

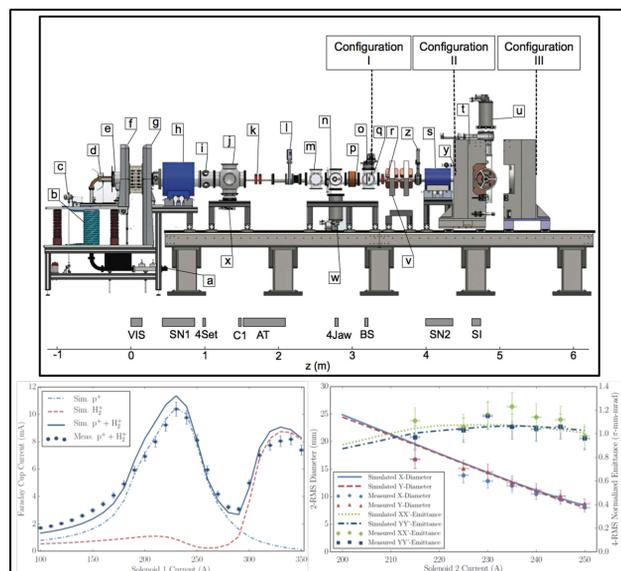


Figure 9: Upper: layout of the test stand of BCS as used for benchmarking the IsoDAR LEBT simulations. Lower left: comparison of measured and simulated beam current at the Faraday cup (placed at  $z=3.2$  m from the Catania Versatile Ion Source (VIS) extraction aperture.) versus solenoid SN1 excitation. Lower right: a similar comparison of beam emittance and diameter (at  $z=4.8$  m) as function of the second solenoid SN2.

extracted beam and the inflector acceptance seems problematic however.

## SOME RECENT PROGRESS AT IBA

During the last few years IBA made quite some efforts in order to precisely simulate orbits and beams in all their different types of accelerators. The program phase-motion integrates the coupled motion of orbit-center and longitudinal phase space over multiple pulses (several  $10^5$  RF periods) in the proton-therapy superconducting synchro-cyclotron S2C2. This is reported in this conference by Jarno Van de Walle *et al.* [35]. The Advanced Orbit Code (AOC) [36] facilitates design studies of critical systems and processes in medical and industrial accelerators. Examples are : i) injection into and extraction from cyclotrons, ii) central region, beam capture and longitudinal dynamics studies in synchro-cyclotrons, iii) studies of resonance crossings, iv) stripping extraction, v) beam simulation from the ion source to the extraction, vi) beam transmission studies in gantries, vii) calculation of Twiss- functions, viii) space charge effects.

For the space charge option, a particle-to-particle solution was chosen. Here the self-field acting on one particle is obtained as the sum of contributions of all other particles. Of course this slows down the calculation for large number of particles  $N$  (computing time scales as  $N^2$ ). But the cyclotron injection and central region problem (and also the Rhodotron [37]) does not require very high precision (a few percent is already good) and therefore it is not required to use a very

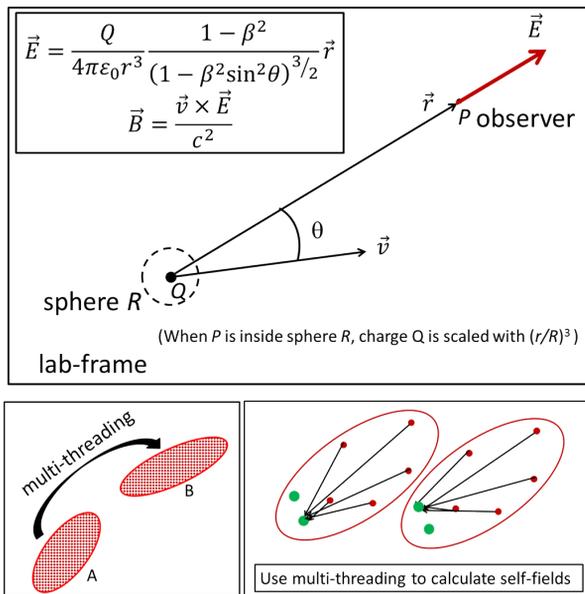


Figure 10: Illustration of calculation of self-fields in AOC (upper) and the use of multi-threading in both the particle integration (lower left) and SF-calculation (lower-right)

large number of particles (10000 can be enough). A big advantage of the chosen method is that one can immediately include the space charge option together with the existing fully 3D features of the  $(\vec{E}, \vec{B})$ -fields and with a complex 3D shape of a reference orbit (such as for the spiral inflector) already available in AOC.

At IBA one wants to also simulate the industrial CW electron accelerator Rhodotron. This machine (in the range from 5 to 10 MeV) has high average beam power in the range of 20 kW to 700 kW. There is some similarity to cyclotrons in the sense that i) the beam is re-circulating several (order 10) times into the same coaxial accelerating-cavity (see Fig. 11), ii) the machine runs with an RF similar to cyclotrons (100-200 MHz) and iii) one orbit period is equal to an RF period. In this machine i) electrons are accelerated from low energy (30 keV) to fully relativistic (10 MeV); ii) one bunch may be very far from mono-energetic, and iii) the direction of particle velocities in the bunch may differ over  $180^\circ$  in the re-circulating dipole magnets. Therefore, a fully relativistic approach was chosen (a Lorentz frame moving with the bunch as commonly used in PIC codes, can not be well defined due to ii) and iii) above).

The relativistic  $\vec{E}$ -field from a moving point charge is radially directed but not isotropic and the magnetic field is perpendicular to  $\vec{E}$  and  $\vec{v}$  (see Fig. 10). In order to avoid singularities in the calculations of self-fields, a virtual sphere  $R$  is placed around the charge  $Q$ . For an observer inside this sphere,  $Q$  is scaled with  $(r/R)^3$ . Here  $R$  is estimated from the rms volume of the bunch and the number of particles in the bunch. The evolution of the self-fields (SF) is iteratively determined as follows: i) calculation of self-fields at A: SF(A), ii) integration of particles from A to B assum-

ing constant SF along the path, iii) calculation of SF(B), iv) re-integration from A to B applying linear interpolation between SF(A) and SF(B), v) repeating of iii) and iv) until the error is considered small enough. This method is not necessarily slower (may even be faster) than a single evaluation approach, because the step from A to B may be taken larger. In most practical cases 2 iterations are sufficient. Both particle integration and SF calculation is done using multi-threading. Two examples of a AOC space charge calculation are shown in Fig. 11 and Fig. 12.

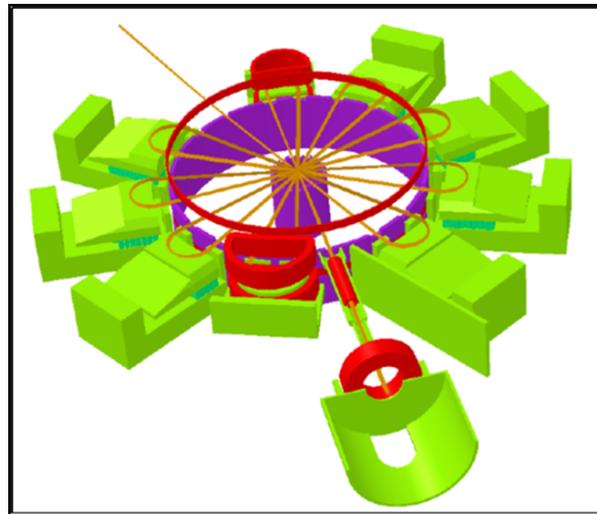


Figure 11: AOC space charge simulation of a  $I_{ave}=10$  mA electron beam in the TT50 Rhodotron, starting at 35 keV from the egun and accelerated upto 10 MeV during 10 passes through the cavity and the re-circulating magnets.

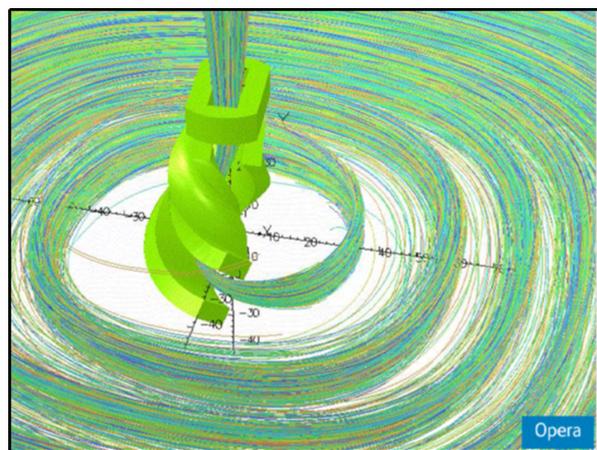


Figure 12: AOC space charge simulation of a bunched  $I_{ave}=2$  mA  $H^-$  beam (5000 particles), injected in the axial bore of the CYCLONE®70, going through the cyclotron spiral inflector and accelerated during 5 turns in the central region. For clarity, the dee-structure and magnet structure are not shown. The structure of the program is such that an RF buncher and injection line focusing elements could be included in a rather straightforn manner.

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