

SIMULATION AND DETECTION OF THE HELICAL ION-PATHS IN A SMALL CYCLOTRON

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Abstract

The small cyclotron COLUMBUS, which was developed in high school Ernestinum Coburg in cooperation with the Institute of Research of Jülich and the University of Applied Sciences of Coburg, is a particle accelerator for education and teaching purposes.

Since its installation, the cyclotron has been continuously upgraded and is now part of the newly created Student Research Centre of the University of Coburg.

In the cyclotron hydrogen ions are accelerated and their positions recorded after a few revolutions by a Faraday cup, which is moved by a linear translator in radial direction across the trajectories of the ions.

This thesis presents a MATLAB simulation of the orbits of the accelerated hydrogen-ions. In contrast to the simpler common school model that approximates the tracks in the acceleration gap by straight tracks, the presented simulation considers the deflection of the ions by the magnetic field in the acceleration gap. So a more realistic picture of the paths can be drawn, which will help to adjust the cyclotron and explore the initial orbits of the ions in detail.

INTRODUCTION

The COLUMBUS project began in 2012 and was first presented at the Cyclotrons 2013 in Vancouver [1]. At this time, however, no jet operation was possible. The first beam was detected in April 2014.

As part of a master's thesis a linear translator was developed in order to move the detector, a Faraday-cup in a radial direction behind the dummy dee. In addition to the registered ions the corresponding x-position of the cup is measured, too [2].

To give students a clear picture of the acceleration processes of the ions in the cyclotron, the ion trajectories are calculated and then visualized in a diagram. This allows qualitative and later quantitative predictions of experiments.

THE STRUCTURE OF THE SIMULATION

As shown in Fig. 1 the simulation consists of three sections, the input-layer, the simulation-layer and the presentation-layer.

The Input-Layer

The input-layer incorporates three groups of parameters which define the ions, the experiment and the geometry of the cyclotron. So the simulation can be easily adapted to different situations.

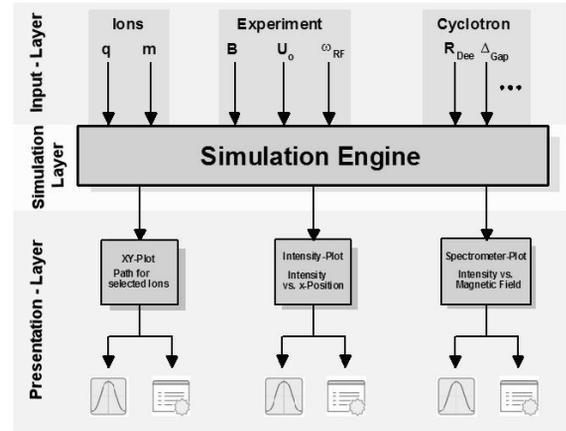


Figure 1: Structure of the Simulation.

The Simulation-Layer

In this layer there is the Simulation Engine, a MATLAB program which calculates all variables such as the positions, the velocity, the radius and the cyclotron-frequency ω_{ZF} of the ions during the acceleration process.

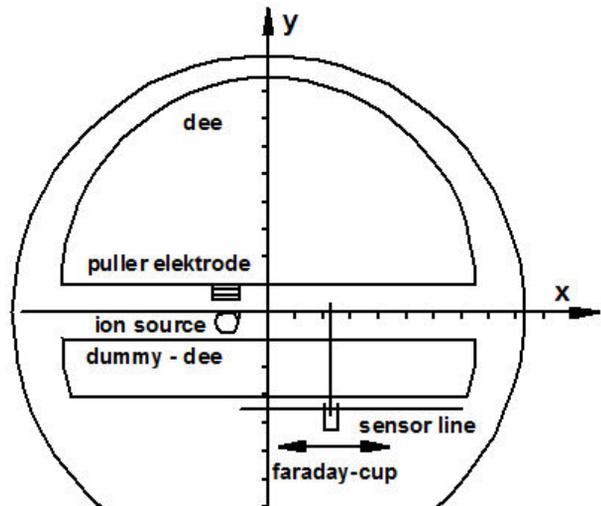


Figure 2: Cartesian coordinate system of the Cyclotron.

For this purpose a coordinate system has been introduced, which has its origin (0, 0, 0) in the center of the vacuum chamber, i.e. Fig. 2. The magnetic field is directed parallel to the z-axis of the coordinate system. The plain $z = 0$, parallel to the chamber bottom, represents the plain of the path of the accelerated ion beam. The ion source is located at position (-15, 5, 0) (all figures in mm), a Faraday-cup as an ion detector moves along the line $y = -31$ mm, the so-called sensor-line. This line ex-

tends parallel to the lower edge of the dummy dee with a distance of about 1 mm.

Each acceleration process consists of two phases:

- Acceleration phase in the gap
- Deflection phase in the dee/dummy dee

During the acceleration phase the energy and therefore the velocity of the ions increases due to the electric field $\vec{E} = (0; E_y; 0)$. Considering the influence of the magnetic field $\vec{B} = (0; 0; B_z)$, the following approach in the acceleration gap is made:

$$m \cdot \vec{a} = \sum \vec{F}_i = \vec{F}_C + \vec{F}_L = q \cdot \vec{E} + q \cdot (\vec{v} \times \vec{B}) = q \cdot (\vec{E} + \vec{v} \times \vec{B}) \quad (1)$$

This results in the following system of coupled differential equations:

$$\ddot{x} = \frac{q}{m} \cdot \dot{y} \cdot B_z = \dot{y} \cdot \omega_{ZF} \quad (2.1)$$

$$\ddot{y} = \frac{q}{m} \cdot E_y - \frac{q}{m} \cdot \dot{x} \cdot B_z = \frac{q}{m} \cdot E_y - \omega_{ZF} \cdot \dot{x} \quad (2.2)$$

Here is $\omega_{ZF} = \frac{q}{m} B_z$ the rotational frequency of the ions, which is known to be independent of the radius r of the orbit.

With the approach $E_y = \hat{E} \cdot \cos(\omega_{RF} \cdot t - \varphi)$ it is possible to solve the DGL in general, but here it is only done numerically [3]. From the solutions of $y(t)$, the solutions for

$$x(t) = \int [y(t) \cdot \omega_{ZF} + \dot{x}(t_0) - y(t_0) \cdot \omega_{ZF}] dt$$

are also determined numerically. ω_{RF} is the frequency of the acceleration voltage, which is equal to ω_{ZF} for resonant acceleration and t_0 is the time, when an ion enters or leaves the dee/dummy dee.

For a unique solution one needs four other constants, which are calculated from the location $\vec{r}_o = \vec{r}(t_0)$ and the velocity $\vec{v}_o = \vec{v}(t_0)$ at the entrance or exit of the dee.

In the dee itself, or, in and behind the dummy dee there is no accelerating electric field so that the ion trajectories can be described by equations of a circle with radius ρ :

$$x(t) = \rho \cdot \cos(\varphi_{in} - \omega_{ZF} \cdot [t - t_0]) + x_M \quad (3.1)$$

$$y(t) = \rho \cdot \sin(\varphi_{in} - \omega_{ZF} \cdot [t - t_0]) + y_M \quad (3.2)$$

$$\text{with } \varphi_{in} = \frac{\pi}{2} + \arctan\left(\frac{v_{yo}}{v_{xo}}\right) \quad (3.3)$$

$$\text{and } \rho = \sqrt{\frac{v_{xo}^2 + v_{yo}^2}{\omega_{ZF}^2}} \quad (3.4)$$

From the known entry position of ions $\vec{r} = (x_o; y_o)$ into the dee, the position of the center $\vec{r}_M = (x_M; y_M)$ can be calculated using Eq. (3.1) and Eq. (3.2) for the time t calculated with Eq. (3.3) and finally the complete circular path until the ions leave the dee is determined.

The change of differential- and circuit-equations is completed as long as the calculated ions meet a virtual obstacle. Such an obstacle can be the ion source, a shield or the vacuum chamber itself. In this case the calculation of the ion trajectory is complete.

The Presentation-Layer

The simulation engine described above has three modules for the evaluation:

- XY-Plot
- Intensity-Plot and
- Spectrometer-Plot

Every module - apart from the graphical output - allows also a data export to a file so that the measured values can be processed by external programs.

The XY-plot allows a selection of different traces at eligible speeds (Fig. 3).

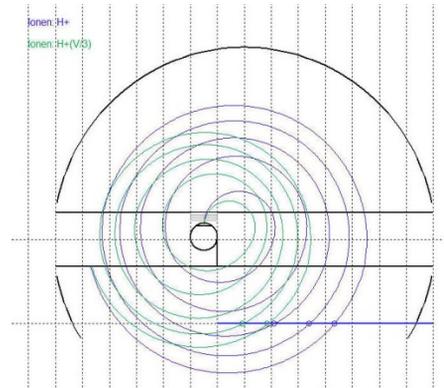


Figure 3: Helical paths of Ions.

So here only the ion paths of protons with $v = v_{max}$ and $v_{max}/3$ are drawn, where v_{max} is the maximal velocity at the given acceleration voltage. It would also be possible to display paths with $v_{max}/5$ and the orbits of H_2^+ - ions at these speeds (Fig. 4).

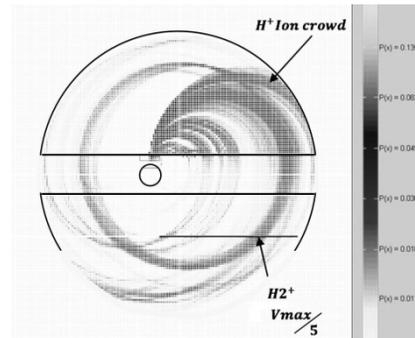


Figure 4: Initial ion-paths.

Another form of a plot i.e. Fig. 4 examining an initial trajectory of accelerated ions. If $B = 66 \text{ mT}$, the simulation predicts a set of closely spaced trajectories of H_2^+ - ions that have $1/5$ of the maximum possible speed.

This meets the corresponding measurement (Fig. 5), which shows a remarkable signal in that region:

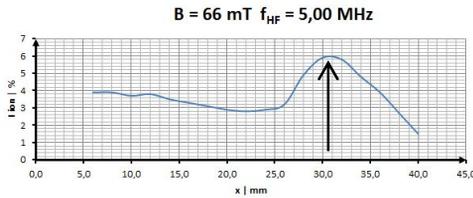


Figure 5: Experimental Diagram.

In the corresponding plot one can see how close the orbits are to each other. So it is obvious that at an Intensity-Plot $I = I(x)$ in reality will be an almost continuous chart as shown in Fig. 6. This is qualitatively confirmed by the model, too.

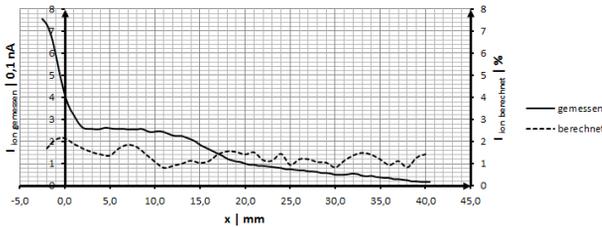


Figure 6: Intensity Plot.

The last module is used for the cyclotron working as a spectrometer. In this experiment the beam is measured in dependence of the magnetic field at a fixed position of the detector. Here one can identify the ions in the beam (Fig. 7).

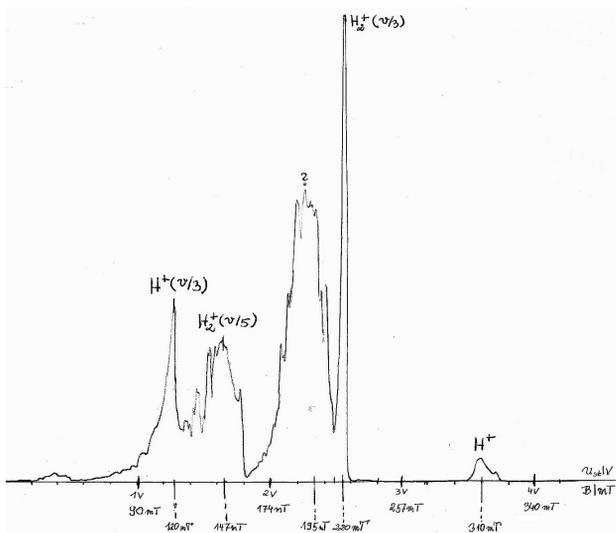


Figure 7: Spectrometer Experiment.

The Spectrometer-Plot (Fig. 8) shows the calculated probability as a measure for the beam-current vs. the magnetic field and allows to simulate this experiment.

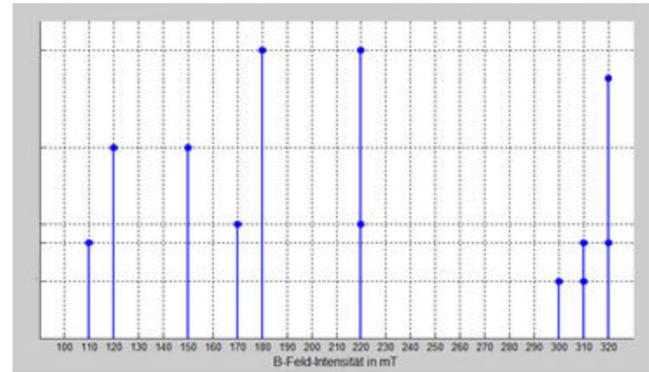


Figure 8: Spectrometer Plot.

CONCLUSION

Of course, the quantitative analysis of the results must be considered very carefully, because there are some assumptions in the simulation that are not met in reality, such as a constant ion beam or the conservation of particles, which is not satisfied, due to the lack of focusing.

Finally, the measurement accuracy in the determination of some parameters was not considered in the simulation.

Nevertheless, the present simulation offers qualitatively a good idea of the acceleration process in the cyclotron and the rest will be a challenge for the future.

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REFERENCES

- [1] Ch. Wolf, M. Frank, E.Held, "COLUMBUS- a Small Cyclotron for School and Teaching Purposes", in Proc. Cyclotrons2013, Vancouver, BC, Canada.
- [2] D. Görtler, Ch. Wolf, M. Precht, R. Bauer, "Detektion der Spiralbahnen in einem Klein-Zyklotron" in *Vakuum in Forschung und Praxis*, Vol. 28 Nr. 3, Juni 2016, pp.26 - 29
- [3] Ernst-Abbe-Hochschule Jena, http://www.eah-jena.de/~dathe/MATLAB/WS09_10/Blatt9.pdf