COOL'23, Oct. 8-13, 2023 Montreux-CH

Theoretical and Simulation Study of Dispersive Electron Cooling

He Zhao

hezhao@impcas.ac.cn

Institute of Modern Physics, Chinese Academy of Sciences

中国科学院 CHINESE ACADEMY OF SCIENCES

中国科学院近代物理研究所 Institute of Modern Physics, Chinese Academy of Sciences

Outline

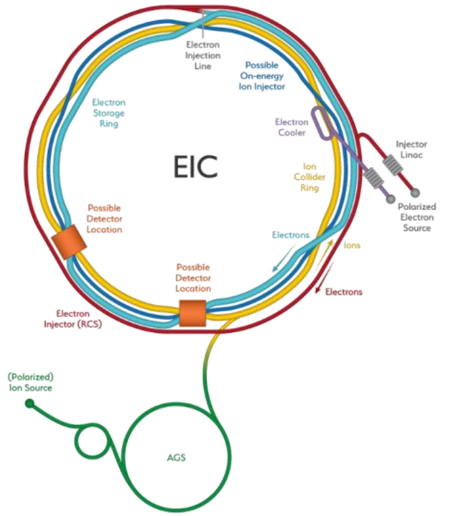
- Introduction
- Dispersive electron cooling
  - Analytical model based on linear friction force
  - Methods and Monte Carlo results
  - Simulation results
- Electron dispersion
  - E-beam velocity distribution
  - Ring-cooler simulation for EIC
- Summary

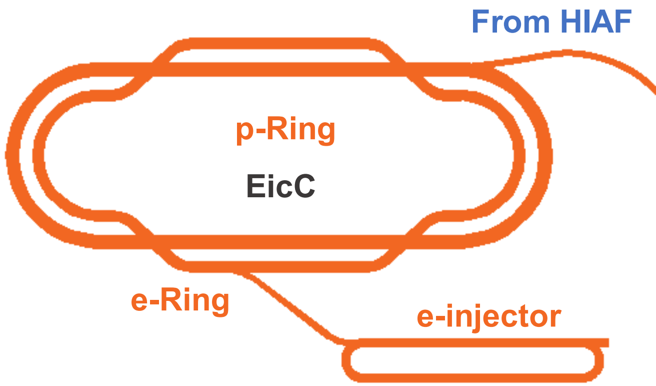
High energy electron cooling

■ Hadron Cooling is a must to achieve the EIC high-luminosity goal

Suppress the IBS to preserve emittance and luminosity

- **Electron Ion Collider (BNL EIC)**
  - Variable CM energies from ~20 to ~100 GeV
  - High collision luminosity ~$10^{33\text{-}34}$ $cm^{-2}s^{-1}$
  - **Proton energy is 100 GeV to 275 GeV**
- **Polarized Electron Ion Collider in China (EicC)**
  - CM energy is between 15 GeV and 20 GeV
  - Luminosity with light to heavy ions is ~$10^{33\text{-}34}$ $cm^{-2}s^{-1}$
  - **Proton energy is up to 20 GeV**





■ Electron cooling is one of the most important methods for EIC

Proposals for EIC cooling

- Induction-Linac based e-cooler (FNAL)
- ERL circulator Ring based e-cooler (Jlab)
- Single energy storage ring e-cooler (BNL)
- Duel energy ring e-cooler (JLab&ODU)

E-beam energy is 10-150 MeV

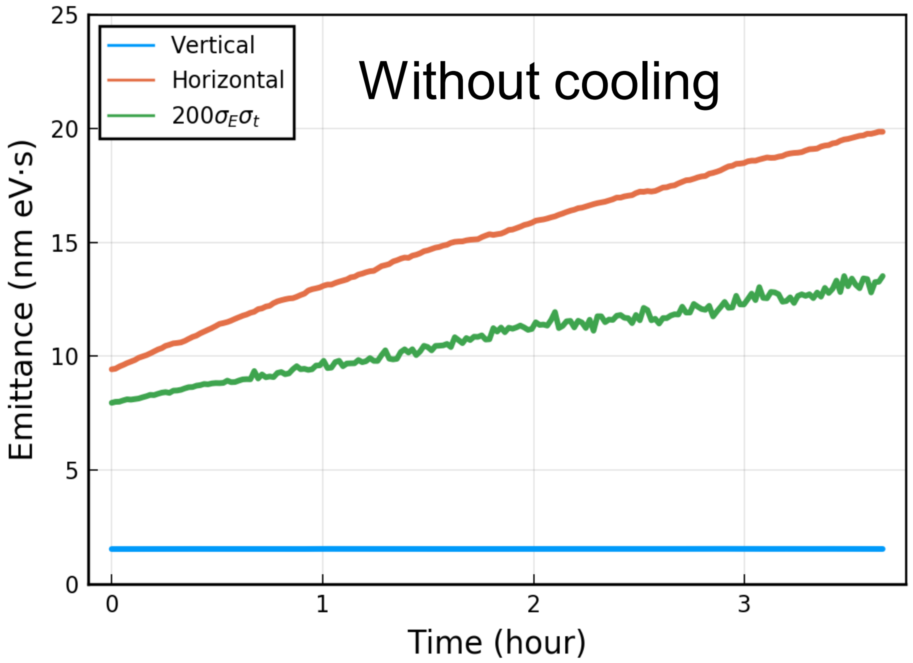
Cooling rates asymmetry

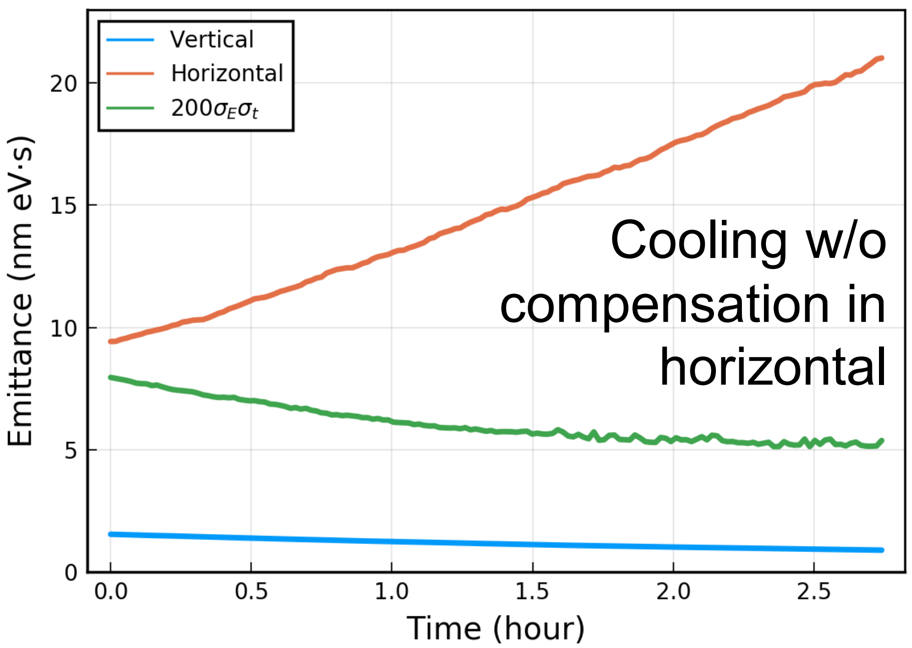
The transverse cooling is much slower compared with the longitudinal one.

$$p_\perp/mc = \gamma\theta_\perp \gg \sigma_\gamma/\gamma = p_\parallel/mc \qquad kT_\perp/kT_\parallel \propto \gamma^2$$

- Cooling simulation on the EIC 275 GeV proton beam using a single energy Ring Cooler (preliminary design).

He Zhao, PRAB 24, 043501





Cooling rates asymmetry

The transverse cooling is much slower compared with the longitudinal one.

$$p_\perp/mc = \gamma\theta_\perp \gg \sigma_\gamma/\gamma = p_\parallel/mc \qquad kT_\perp/kT_\parallel \propto \gamma^2$$

Solutions:

1. Magnetized cooling (adiabatic collision with Larmor circle)
2. **Dispersive electron cooling** (transfer cooling from long. to trans.)
   - Dispersion function in the cooling section
   - <u>Transverse gradient of the longitudinal cooling force</u>

-- J. Bosser, NIM A, 441 (2000) 60
-- M. Beutelspacher, NIM A, 512 (2003) 459
-- H. Zhang, IPAC2018, TUPAL072, 2018
-- Y. Derbenev, EIC hadron cooling workshop, Fermilab, 2019

A simple example

Assumptions

- An off-momentum particle passing the cooling section with a dispersion **D**, and only consider the longitudinal cooling with a linear cooling force **$\Delta\delta_p = -\lambda\delta_p$**
- The particle coordinate **x** remain unchanged during passing the cooling section.

$$\boldsymbol{x}_{\beta 2} = \boldsymbol{x} - D\delta_{p2} = \boldsymbol{x}_{\beta 1} + D\lambda\delta_{p1}$$

λ: longitudinal cooling coefficient

Q: How does the betatron oscillation x_β change with only longitudinal cooling?

➢ **λ is a constant**

$$x_{\beta 2} = x_{\beta 1}$$

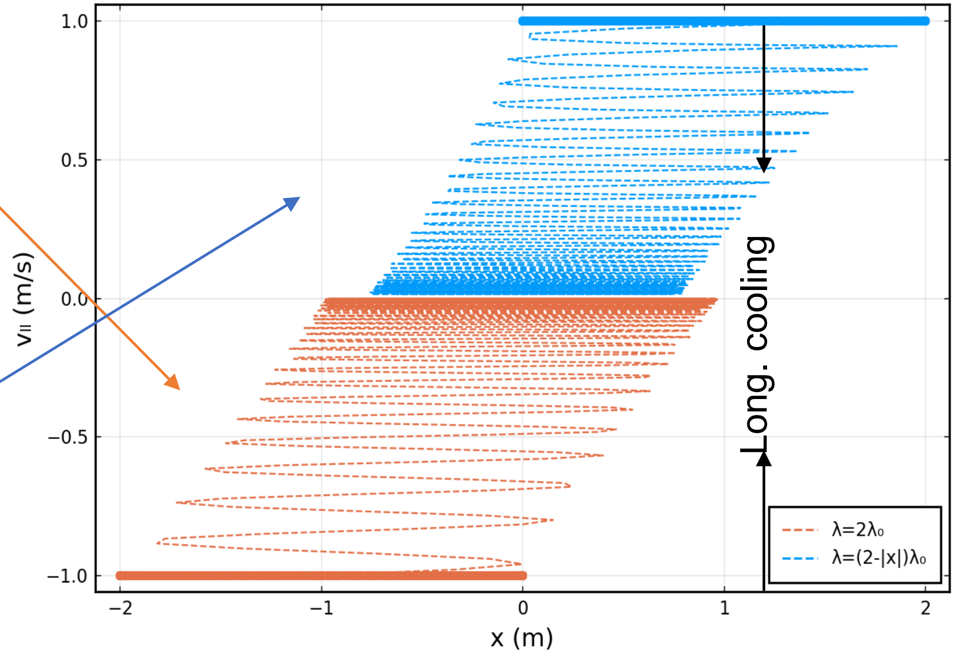
No transverse cooling, since x_β is independent with δ_p

➢ **λ with x-gradient** (e.g. $\lambda(x) = (M - |\boldsymbol{x}|)\lambda_0$)

$$x_{\beta 2} \simeq (1 - \lambda_0 |D\delta_{p1}|) x_{\beta 1}$$

Transverse amplitude damping from longitudinal cooling, i.e. dispersive cooling

$$(x_\beta = 1\ m,\ D = 1\ m,\ \delta_p = \pm 1,\ \lambda_0 = 0.01)$$



Dispersive electron cooling

■ Factors related to the transverse gradient

- Energy offset $\boldsymbol{\delta_o}$
- Trans. displacement $\boldsymbol{x_o}$
- Space charge $\boldsymbol{K_{sc}}$
- Density distribution $\boldsymbol{n_e}(x, y, z, D\delta)$

■ Linear model

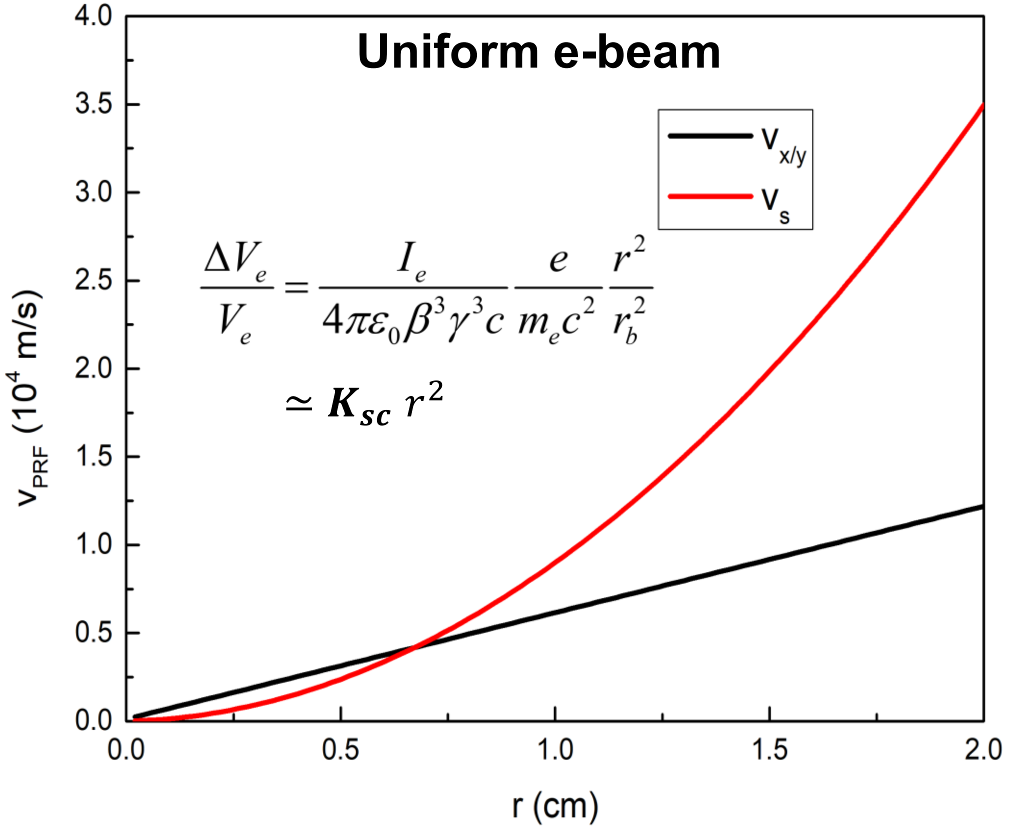
- Single particle dynamics
  - Linear friction force: $\Delta \boldsymbol{u} = -C n_e \boldsymbol{u}$
  - Longitudinal cooling:

$$\Delta\delta \simeq -C_p n_e(\delta - \delta_e - \delta_o) \qquad \delta_e = K_{sc}(x^2 + y_\beta^2)$$

  - Transverse cooling:

$$\Delta\epsilon_x \simeq -D x_\beta \Delta\delta/\beta_x + \beta_x x' \Delta x'$$

$$\epsilon_x = (x - x_o - D\delta)^2/2\beta_x + \beta_x x'^2/2 \qquad \Delta x' = -C_x n_e x'$$



Dispersive electron cooling

■ Factors related to the transverse gradient

- Energy offset $\boldsymbol{\delta_o}$
- Trans. displacement $\boldsymbol{x_o}$
- Space charge $\boldsymbol{K_{sc}}$
- Density distribution $\boldsymbol{n_e}(x, y, z, D\delta)$

■ Linear model

$$\langle \Delta\delta^2 \rangle = -2C_p\langle n_e\delta^2\rangle + 2C_p\delta_o\langle n_e\delta\rangle + 2C_pK_{sc}x_o\left(x_o\langle n_e\delta\rangle + 2\langle n_e x_\beta\delta\rangle + 2D\langle n_e\delta^2\rangle\right)$$

$$\langle \Delta\epsilon \rangle = -C_x\epsilon_0\langle n_e\rangle + \underbrace{\frac{C_pD}{\beta_x}\langle n_e x_\beta\delta\rangle}_{\text{E-beam density } n_e} - \underbrace{\frac{C_pD\delta_o}{\beta_x}\langle n_e x_\beta\rangle}_{\text{Energy offset } \delta_o} - \frac{C_pDK_{sc}x_o}{\beta_x}\underbrace{\left(x_o\langle n_e x_\beta\rangle + 2\langle n_e x_\beta^2\rangle + 2D\langle n_e x_\beta\delta\rangle\right)}_{\text{Space charge } K_{sc} \text{ and displacement } x_o}$$

- Redistribution factor $\quad \lambda_p = \frac{\langle\delta\rangle}{\delta_{ip}} \quad \lambda_x = \frac{\langle\Delta\epsilon_x\rangle}{\epsilon_{ix,\mathrm{rms}}} \quad k_p = \frac{\lambda_p}{\lambda_{p,D_i=0}} \quad k_x = \frac{\lambda_x}{\lambda_{x,D_i=0}}$

- For a given electron and ion beam distribution, the dispersive cooling can be modeled analytically. (Ion: Gaussian; Electron: Gaussian/Uniform)

Case-1: Gaussian e-beam with energy offset $\boldsymbol{\delta_o}$ and beam displacement $\boldsymbol{x_o}$

- Gaussian e-beam itself can realize the dispersive cooling (density gradient)
- The product of energy offset and beam displacement keeps negative value

$$n_e = n_{e0} exp[-\frac{(x_\beta + x_o + D\delta)^2}{2\sigma_{ex}^2} - \frac{y_\beta^2}{2\sigma_{ey}^2} - \frac{s^2}{2\sigma_{es}^2}]$$

$$a = \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2}$$

$$b = \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2 + D^2\delta_p^2}$$

$$k_p = e^{-\frac{x_o^2}{2b^2}} [\frac{a^3}{b^3} + \frac{a}{b^5} D^2 \delta_p^2 x_o^2 + \frac{a}{b^3} D\delta_o x_o]$$

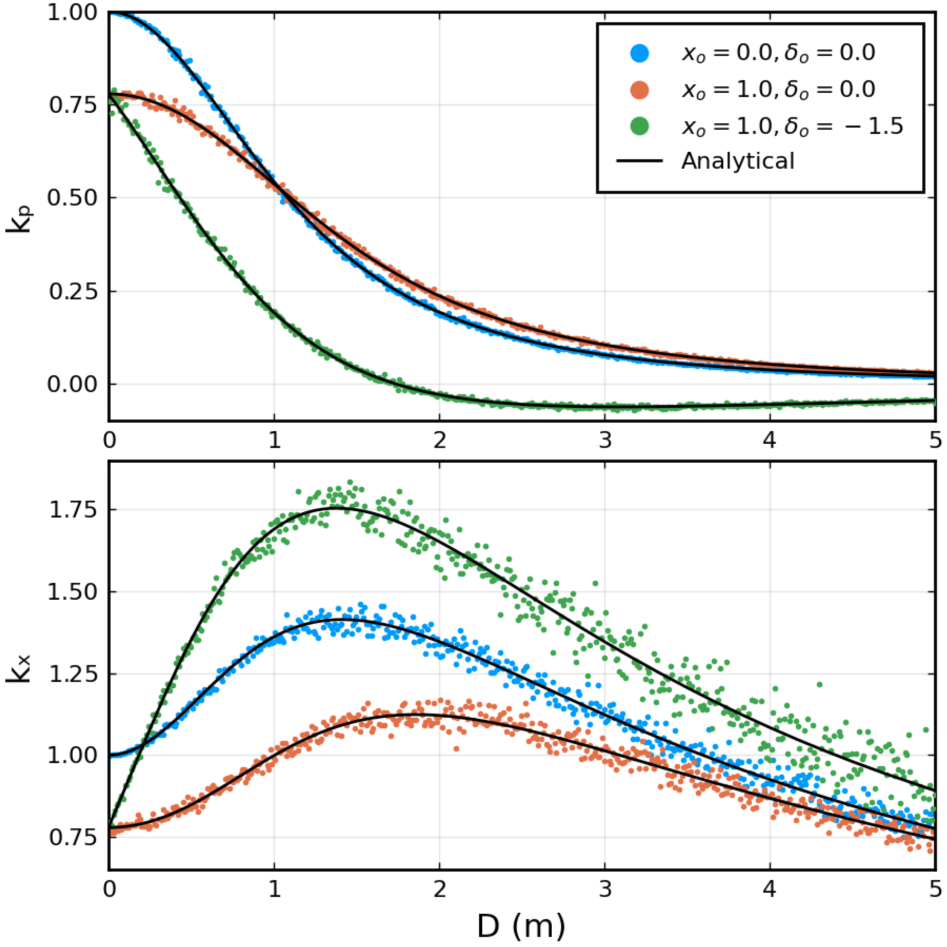
$$k_x = e^{-\frac{x_o^2}{2b^2}} [\frac{a}{b} + \frac{C_p a}{C_x b^5} D^2 \delta_p^2 (b^2 - x_o^2) - \frac{C_p a}{C_x b^3} D\delta_o x_o]$$

H. Zhao and M. Blaskiewicz, PRAB 24, 083502

Y. Derbenev, EIC hadron cooling workshop, Fermilab, 2019

Comparison between Monte-Carlo and analytical results

(σ_{ex} = 1 m, σ_{ix} = 1 m, δ_p = 1, C_p/C_x = 2)



Case-2: Gaussian e-beam with Space Charge K_{sc} and beam displacement x_o

- The quadratically momentum deviation due to the e-beam space charge, and an inward displacement create the transverse gradient.

$$a = \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2} \qquad b = \sqrt{\sigma_{ex}^2 + \sigma_{ix}^2 + D^2\delta_p^2} \qquad c = \sqrt{\sigma_{ex}^2 - \sigma_{ix}^2 - D^2\delta_p^2}$$

$$k_p = e^{-\frac{x_o^2}{2b^2}}\left[\frac{a^3}{b^3} + \frac{a}{b^5}D^2\delta_p^2 x_o^2 - \frac{a}{b^5}DK_{sc}x_o(2\sigma_{ex}^2 b^2 - x_o^2c^2)\right]$$

$$k_x = e^{-\frac{x_o^2}{2b^2}}\left[\frac{a}{b} + \frac{C_p a}{C_x b^5}D^2\delta_p^2(b^2 - x_o^2) + \frac{C_p a}{C_x b^5}DK_{sc}x_o(2\sigma_{ex}^2 b^2 - x_o^2c^2)\right]$$

$\delta_e(r) = K_{sc}r^2$ is not exactly correct for Gaussian e-beam

J. Bosser, NIM A, 441 (2000) 60
M. Beutelspacher, NIM A, 512 (2003) 459

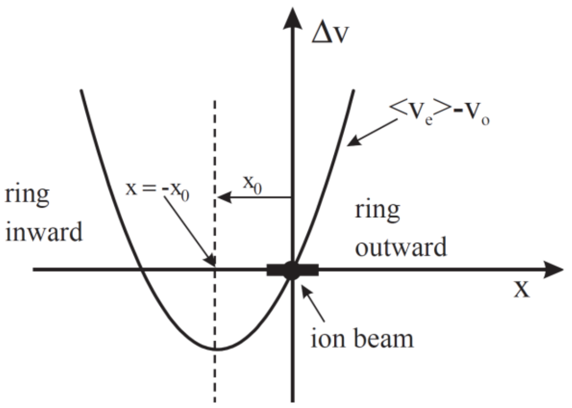
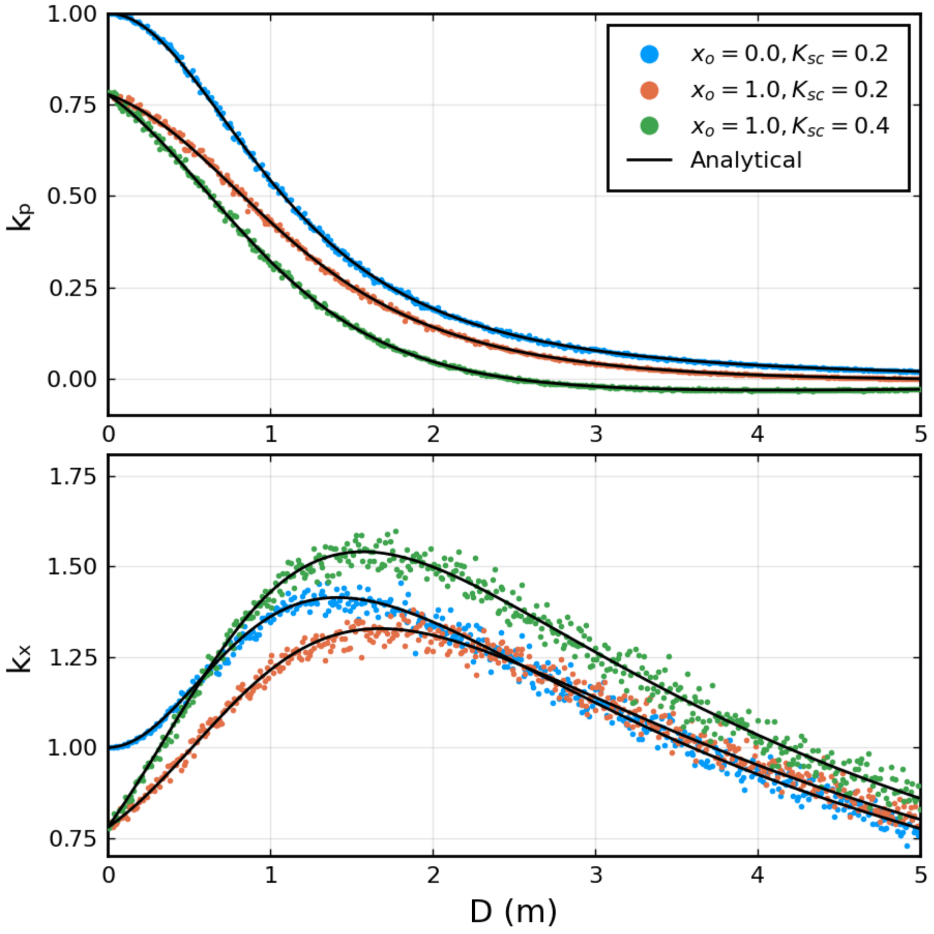


Fig. 2. Creating a horizontal gradient of the longitudinal electron cooling force. Due to the space charge of the electron

(σ_{ex} = 1 m, σ_{ix} = 1 m, δ_p = 1, C_p/C_x = 2)



Case-3: Uniform e-beam with Space Charge K_{sc} and beam displacement x_o

- **E-beam with a radius R_e**

Particles oscillate between the inside and outside of the e-beam can generate the transverse gradient.

$$m = Erf[\frac{R_e}{\sqrt{2\sigma_{ix}^2}}]$$

$$n = \sqrt{\sigma_{ix}^2 + D^2\delta_p^2}$$

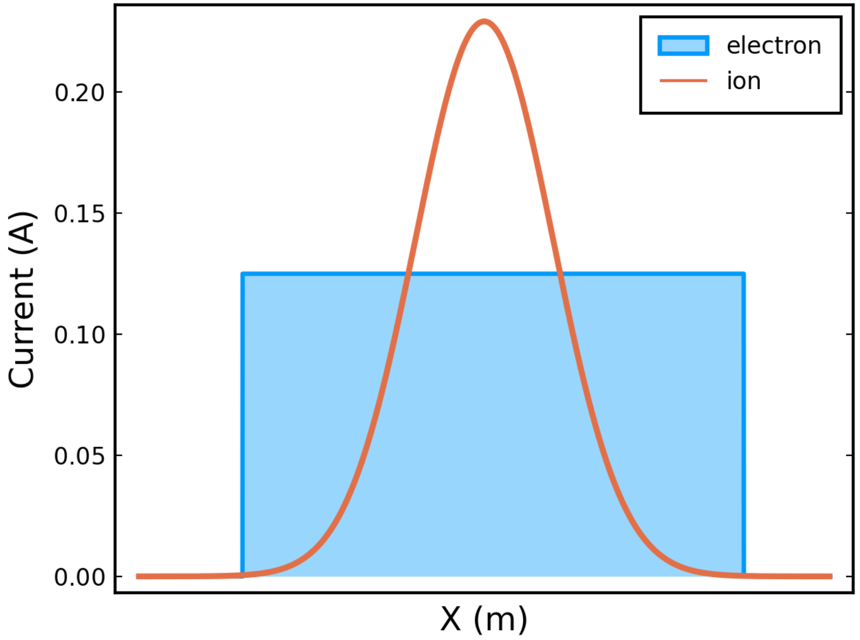
$$a = Erf[\frac{R_e + x_o}{\sqrt{2}n}] + Erf[\frac{R_e - x_o}{\sqrt{2}n}]$$

$$b = \frac{e^{-\frac{(R_e - x_o)^2}{2n^2}}(R_e - x_o) + e^{-\frac{(R_e + x_o)^2}{2n^2}}(R_e + x_o)}{n^3}$$

$$c = \frac{e^{-\frac{(R_e + x_o)^2}{2n^2}} - e^{-\frac{(R_e - x_o)^2}{2n^2}}}{n}$$

$$k_p = \frac{a}{2m} - \frac{D^2\delta_p^2 b}{\sqrt{2\pi}m} + \frac{DK_{sc}x_o}{\sqrt{2\pi}m}(2n^2 b - \sqrt{2\pi}a - x_o c)$$

$$k_x = \frac{a}{2m} + \frac{C_p}{C_x}[\frac{D^2\delta_p^2 b}{\sqrt{2\pi}m} - \frac{DK_{sc}x_o}{\sqrt{2\pi}m}(2n^2 b - \sqrt{2\pi}a - x_o c)]$$



- **Infinite R_e**

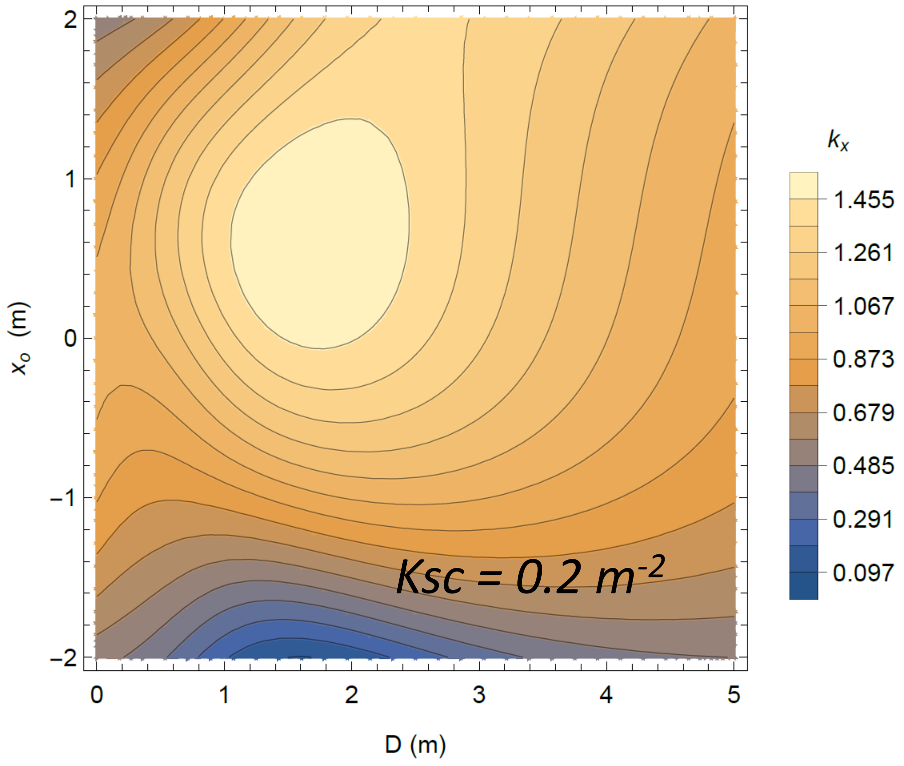
$$k_p = 1 - 2DK_{sc}x_o$$

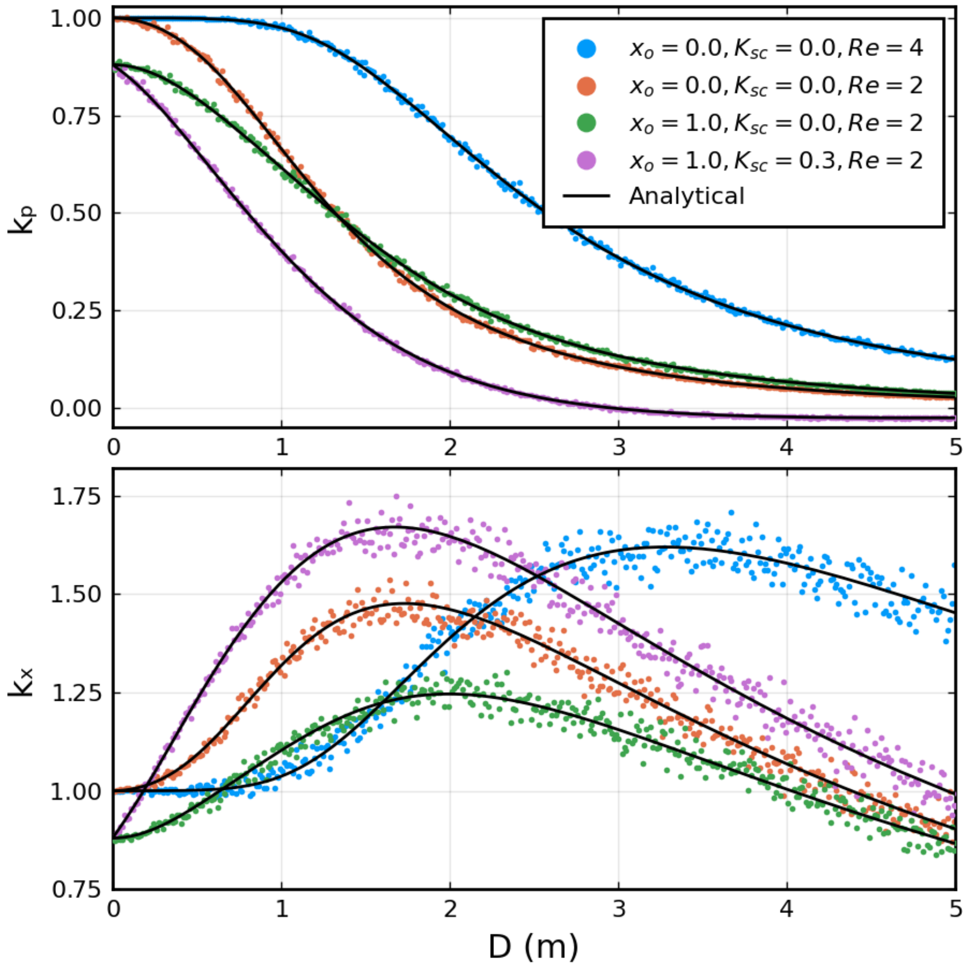
$$k_x = 1 + 2DK_{sc}x_o C_p/C_x$$

Case-3: Uniform e-beamwith Space Charge $\boldsymbol{K_{sc}}$ and beam displacement $\boldsymbol{x_o}$

- **E-beam with a radius R_e**

σ_{ex} = 1 m, σ_{ix} = 1 m, δ_p = 1, C_p/C_x = 2



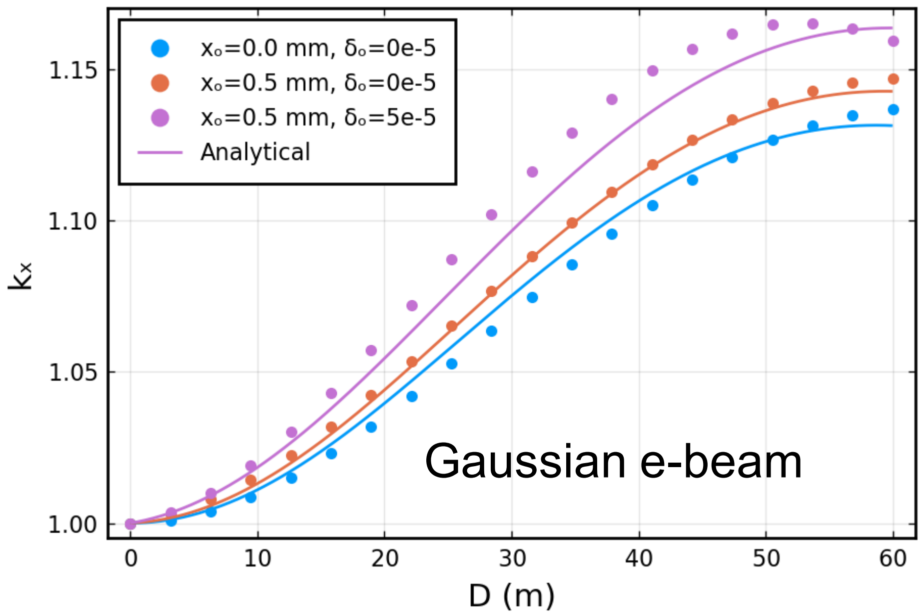


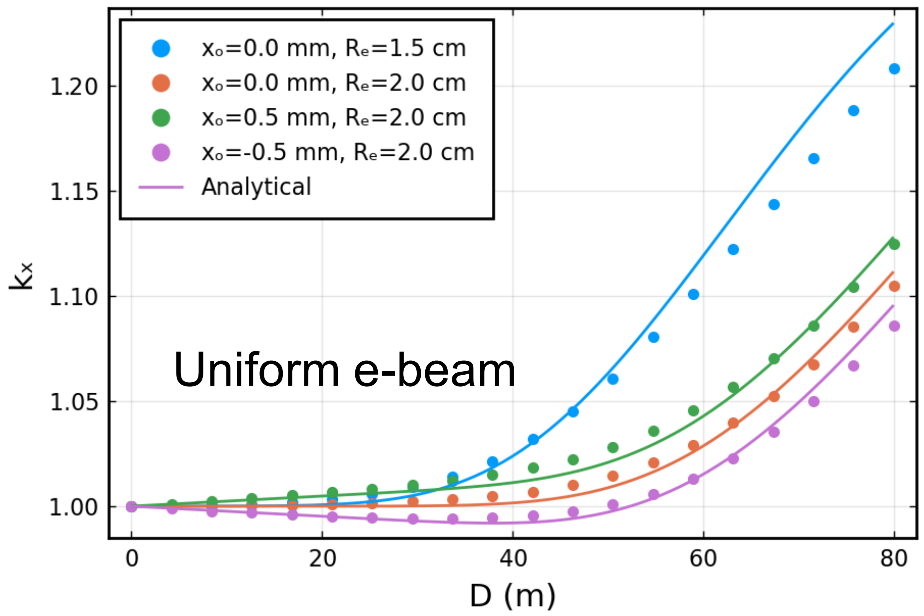
Simulation results

- TRACKIT code
  https://github.com/hezhao1670/ECool-TRACKIT

- Using low energy ion beam cooling to effectively include space charge K_{sc} = $0.2\sim0.3\ m^{-2}$

	$^{56}Fe^{26+}$	Electron	
Circumference (m)		128.8	
Length of cooler (m)		3.4	
Transverse dis.	Gaussian	Gaussian	Uniform
Longitudinal dis.	Coasting	DC	DC
Energy (MeV/u)	35.0	0.0192	0.0192
Beam current (mA)	0.5	50.0	30.0
Beam radius (cm)	1.0/0.5	2.0	2.0
RMS ϵ_x/ϵ_y (μm)	0.5/0.1	–	–
RMS δ_p	1.0×10^{-4}	–	–
β_x/β_y @cooler (m)	25/25	–	–
Long. temp. (eV)	–	5.0×10^{-5}	5.0×10^{-5}
Tran. temp. (eV)	–	0.5	0.5
B field in cooler (Gs)	–	500	500
B field error	–	2.0×10^{-4}	2.0×10^{-4}





High energy dispersive cooling

- Space Charge effect of e-beam can not by applied
- Energy offset and displacement (Mismatch) may not be acceptable
- E-beam density distribution is much important for the dispersive cooling at high energy, a Gaussian e-beam is preferable.

$$\langle \Delta\delta^2 \rangle = -2C_p\langle n_e\delta^2\rangle + 2C_p\delta_o\langle n_e\delta\rangle + 2C_pK_{sc}x_o\left(x_o\langle n_e\delta\rangle + 2\langle n_e x_\beta\delta\rangle + 2D\langle n_e\delta^2\rangle\right)$$

$$\langle \Delta\epsilon \rangle = -C_x\epsilon_0\langle n_e\rangle + \frac{C_pD}{\beta_x}\langle n_e x_\beta\delta\rangle - \frac{C_pD\delta_o}{\beta_x}\langle n_e x_\beta\rangle - \frac{C_pDK_{sc}x_o}{\beta_x}\left(x_o\langle n_e x_\beta\rangle + 2\langle n_e x_\beta^2\rangle + 2D\langle n_e x_\beta\delta\rangle\right)$$

E-beam density n_e — Energy offset δ_o — Space charge K_{sc} and displacement x_o

Electron dispersion

- E-beam distribution for friction force calculation

$$f_e \propto f_{ex} f_{ey} f_{ez} \quad \Rightarrow \quad f_e(\boldsymbol{r}, \boldsymbol{u_e}) = n_e(\boldsymbol{r}) f_{v_e}(\boldsymbol{u_e})$$

$$f_{ex} = exp\left[-\frac{1}{2\epsilon_{ex}}\left(\frac{1+\alpha_{ex}^2}{\beta_{ex}}\hat{x}^2 + 2\alpha_{ex}\hat{x}x' + \beta_{ex}x'^2\right)\right]$$

$$f_{ey} = exp\left[-\frac{1}{2\epsilon_{ey}}\left(\frac{1+\alpha_{ey}^2}{\beta_{ey}}y^2 + 2\alpha_{ey}yy' + \beta_{ey}y'^2\right)\right]$$

$$f_{ez} = exp\left[-\frac{(\delta-\delta_{off})^2}{2\sigma_{ep}^2} - \frac{s^2}{2\sigma_{es}^2}\right],$$

$$\hat{x} = x - x_{\text{off}} - D_e\delta$$

$$f_{v_e} = \frac{(2\pi)^{-3/2}}{\sigma_1\sigma_2\sigma_3\sqrt{1-\rho^2}}\exp\left\{-\frac{(u_y-\bar{u}_y)^2}{2\sigma_2^2} - \frac{1}{2(1-\rho^2)}\cdot \times\left[\frac{(u_x-\bar{u}_x)^2}{\sigma_1^2} + \frac{(u_p-\bar{u}_p)^2}{\sigma_3^2} - 2\rho\frac{(u_x-\bar{u}_x)(u_p-\bar{u}_p)}{\sigma_1\sigma_3}\right]\right\}$$

$$\bar{u}_x = -\frac{\gamma\alpha_{ex}\epsilon_{ex}(x - x_{off} - D_e\delta_{off})}{\epsilon_{ex}\beta_{ex} + D_e^2\delta_{ep}^2}$$

$$\bar{u}_y = -\frac{\gamma\alpha_{ey}y}{\beta_{ey}}$$

$$\bar{u}_p = \frac{D_e\delta_{ep}^2(x - x_{off}) + \epsilon_{ex}\beta_{ex}\delta_{off}}{\epsilon_{ex}\beta_{ex} + D_e^2\delta_{ep}^2}$$

$$\sigma_1^2 = \frac{\epsilon_{ex}\gamma^2}{\beta_{ex}}\left(1 + \frac{\alpha_{ex}^2 D_e^2\delta_{ep}^2}{\epsilon_{ex}\beta_{ex} + D_e^2\delta_{ep}^2}\right)$$

$$\sigma_2^2 = \frac{\epsilon_{ey}\gamma^2}{\beta_{ey}}$$

$$\sigma_3^2 = \frac{\delta_{ep}^2\epsilon_{ex}\beta_{ex}}{\epsilon_{ex}\beta_{ex} + D_e^2\delta_{ep}^2}$$

$$\rho = \frac{\alpha_{ex}D_e\delta_{ep}}{\sqrt{\epsilon_{ex}\beta_{ex} + D_e^2\delta_{ep}^2(1+\alpha_{ex}^2)}}$$

Friction force

Electron dispersion

- Electron dispersion also contributes to the redistribution effect, but only when there is ion dispersion

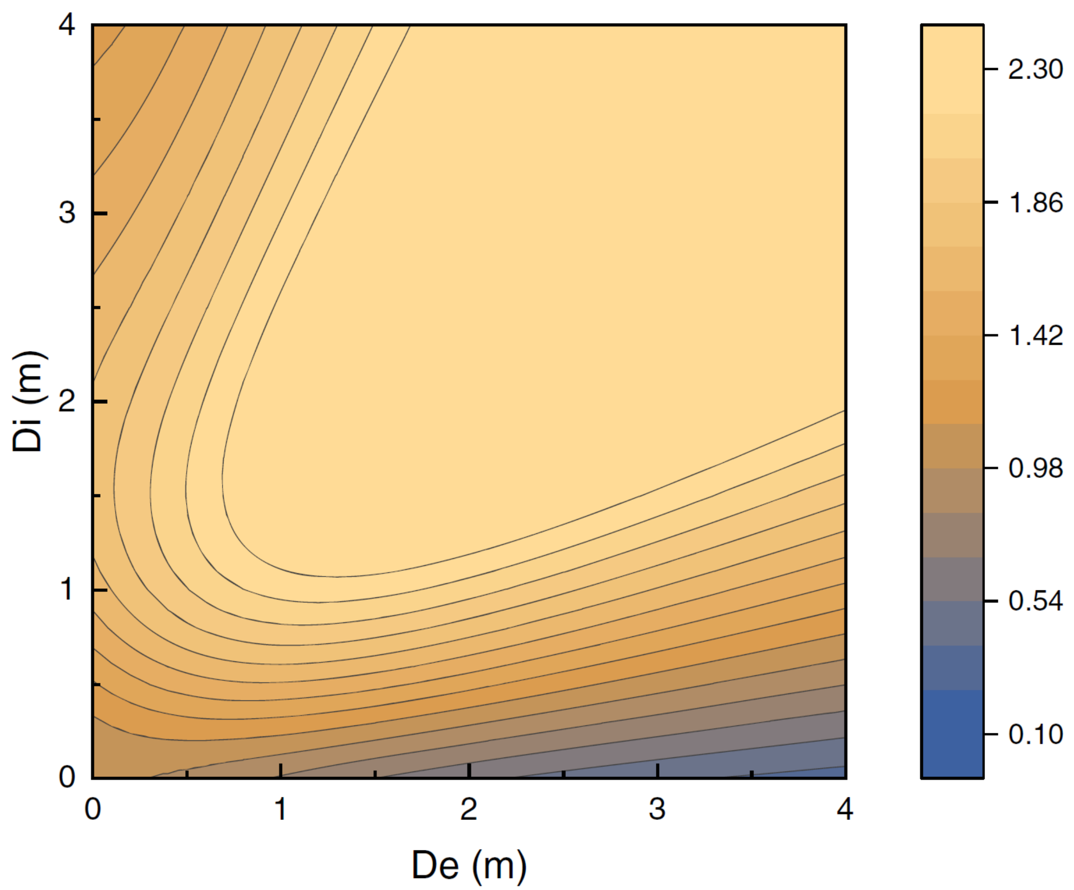
$$k = \frac{D_e \delta_{ep}^2}{\sigma_{ex}^2 + D_e^2 \delta_{ep}^2} \qquad g = \frac{\sqrt{\sigma_{ex}^2 + D_e^2 \delta_{ep}^2}}{\sigma_{ex}}$$

$$L = \sigma_{ex}^2 + \sigma_{ix}^2 + D_i^2 \delta_{ip}^2 + D_e^2 \delta_{ep}^2$$

$$k_x = \left(L + \frac{gC_p}{C_x} D_i^2 \delta_{ip}^2 + \boxed{\frac{gC_p}{C_x} D_i D_e \delta_{ep}^2}\right) \sqrt{\frac{\sigma_{ex}^2 + \sigma_{ix}^2}{L^3}}$$

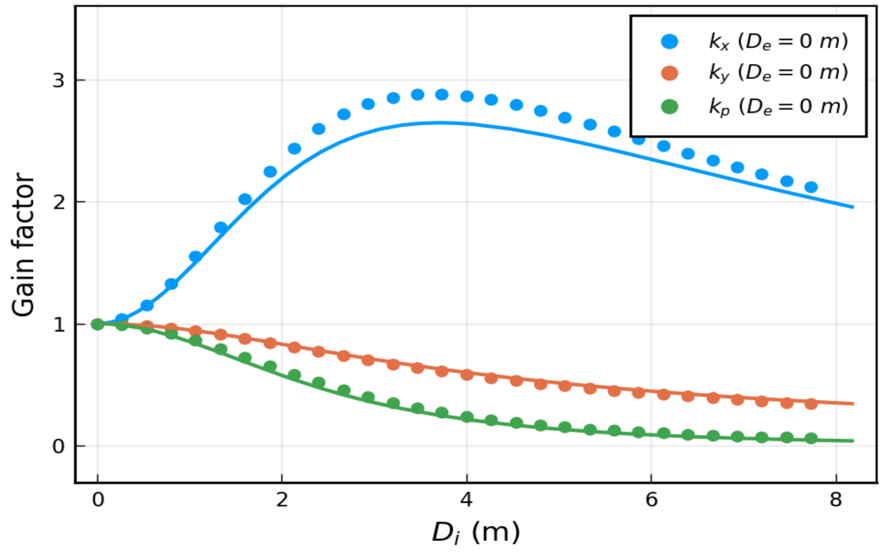
$$k_p = g(L - D_i^2 \delta_{ip}^2 - D_i D_e \delta_{ep}^2) \sqrt{\frac{\sigma_{ex}^2 + \sigma_{ix}^2}{L^3}},$$

$\sigma_{ex} = \sigma_{ix} = \delta_{ip} = \delta_{ep} = 1$, $C_p/C_x = 3$

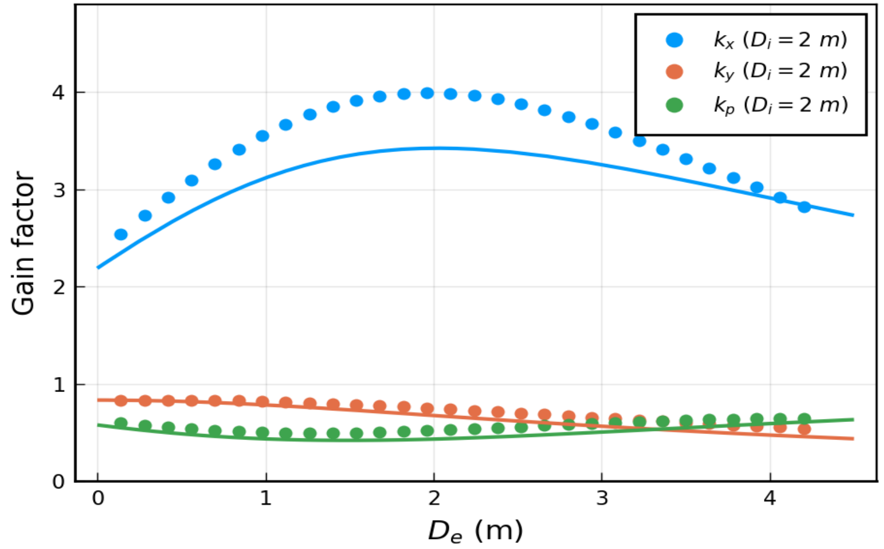


Simulation on the ring cooler for EIC

- 275 GeV proton beam cooling using 149.8 MeV e-beam in a ring-based cooler.



Name	Electron	Proton
N	3e11	6.9e10
Emittance (x/y) [nm]	21/18	9.6/1.5
β* @ cooling section [m]	**153/275**	**100**
Rms size [mm]	**1.8/1.6**	**1.0/0.4**
Rms ang. spread (rest frame)	**3.4e-3/2.4e-3**	**2.9e-3/1.1e-3**
dp/p	8.9e-4	6.6e-4
Rms length [cm]	12	6
L_cool [m]	170	170

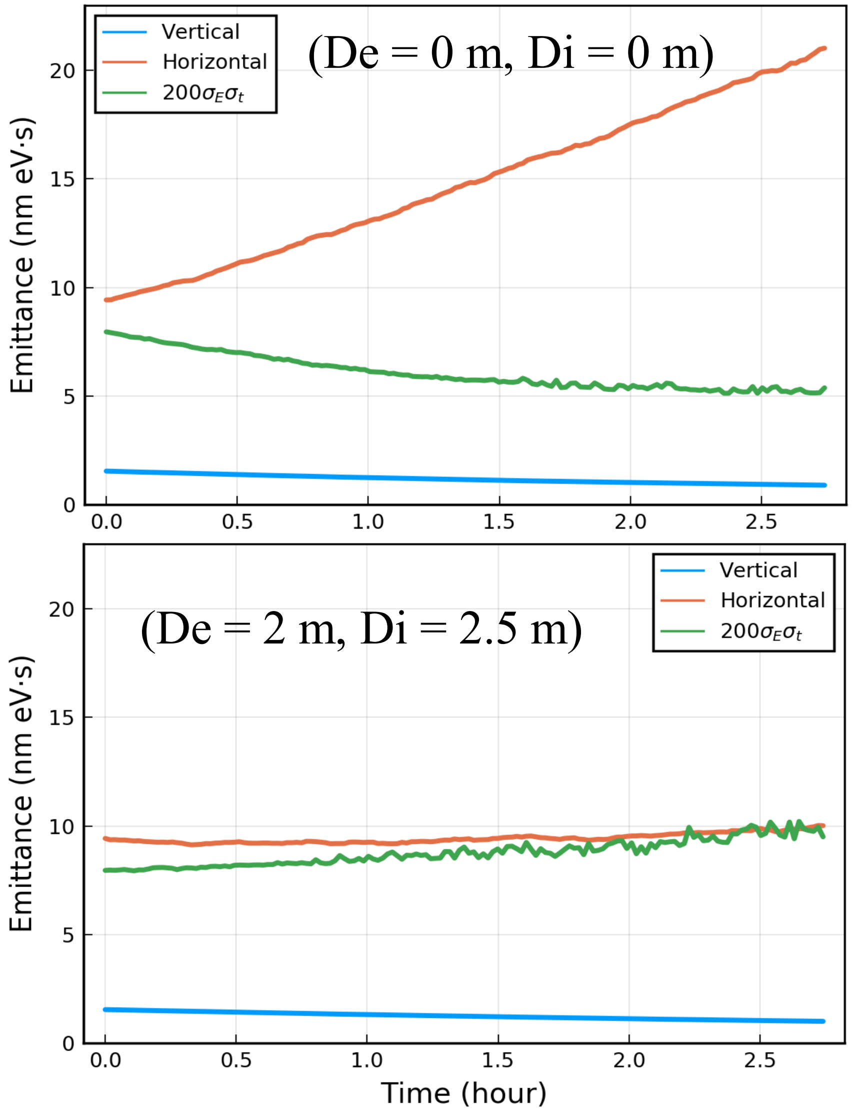


H. Zhao, PRAB 24, 043501

Simulation on the ring cooler for EIC

- 275 GeV proton beam cooling using 149.8 MeV e-beam in a ring-based cooler.
- Dispersive cooling is essential to realize the high energy beam cooling.

Name	Electron	Proton
N	3e11	6.9e10
Emittance (x/y) [nm]	21/18	9.6/1.5
β* @ cooling section [m]	**153/275**	**100**
Rms size [mm]	**1.8/1.6**	**1.0/0.4**
Rms ang. spread (rest frame)	**3.4e-3/2.4e-3**	**2.9e-3/1.1e-3**
dp/p	8.9e-4	6.6e-4
Rms length [cm]	12	6
L_cool [m]	170	170



H. Zhao, PRAB 24, 043501

Summary

- Dispersive electron cooling is an effective scheme to redistribute the cooling rate between transverse and longitudinal direction.
- We demonstrated that beam energy offset, displacement, density distribution and space charge effect of the e-beam all contribute to the rate redistribution in dispersive cooling.
- Also, the electron dispersion contributes to the redistribution effect.
- An linear model of the redistribution effect is introduced, which agree with the Monte-Carlo calculation and numerical simulation for both Gaussian and Uniform e-beam.
- In the EIC and EicC hadron beam cooling, the dispersive cooling should be an essential method to realize the cooling requirements.