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NUMERICAL STUDY OF THE WIGGLER-BASED MICROBUNCHING AMPLIFIER FOR EIC*

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Abstract

An amplifier of microbunching instability in the electron beam employing wiggler magnets is considered. A lattice design is described. The impact of the SASE FEL resonance is analysed. A setup for macro particle tracking and a method for microbunching gain determination is presented. Calculations demonstrate feasibility of a broad band amplifier with a large gain.

INTRODUCTION

Coherent electron cooling [1] using a plasma-cascade amplifier (PCA) [2] can provide significantly faster cooling of hadrons than the conventional method of microwave stochastic cooling due to the wide bandwidth of a pickup, a kicker, and an amplifier. The PCA creates unstable plasma oscillations by varying the transverse beam size along the beam line, thus modulating the plasma frequency. An alternative approach to the amplifier is to modulate the plasma frequency by a sequence of wiggler magnets separated by weak chicanes [3]. Numerical simulations of the gain function are done using OPAL-FEL code [4] and the electron beam parameters projected for the Electron Ion Collider.

LATTICE OF THE AMPLIFIER UNIT

The basic cell of the Wiggler Enhanced Plasma Amplifier (WEPA) is composed of a wiggler and quadrupole triplet, with the triplet providing transverse matching into subsequent cells. Parameters of the wigglers used in all simulations presented here are given in Table 1.

Table 1: Wiggler Parameters

Parameter	Value	Unit
Wiggler length ℓ_W	1.188	m
Wiggler period λ_W	3.3	cm
Wiggler parameter K_w	1.5	
Wiggler peak magnetic field B_{peak}	0.487	T
Bending radius ρ	1.076	m
Wiggler magnetic gap	16.0	mm

Chicanes are placed to convert the accumulated energy modulation into additional density modulation after a set of cells. Two identical chicanes are used for convenience. We refer to a set of several wiggler and triplet cells, followed by chicanes as a single amplifier *unit*. The total length occupied by the chicanes is equal to the wiggler length such as to minimize a perturbation to the lattice. The basic chicane parameters are given in Table 2.

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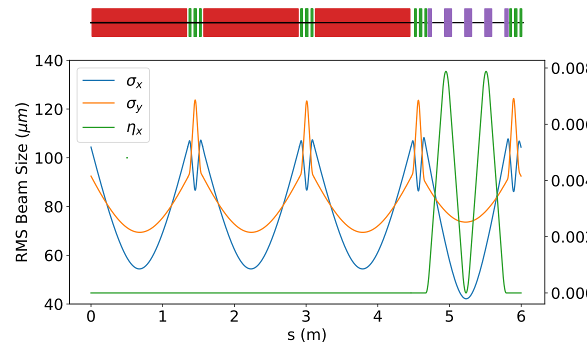

Figure 1: Lattice of a single amplifier unit with three cells followed by two chicanes. Transverse RMS beam sizes and dispersion function are shown. Locations of wigglers, triplets, and chicanes are shown at the top.

Table 2: Chicane Parameters

Parameter	Value	Unit
R_{56}	0.5×10^{-3}	m
Dipole length l_b	0.05	m
Dipole bend angle θ_b	1.96	degrees
Dipole field	0.04	T
Chicane leg L	0.18	m

Accounting for the wiggler focusing and transverse space charge effects necessitated slight modifications to the matched Twiss functions and focusing strengths of the quadrupoles comparing to a lattice in which drifts are used instead of the wigglers. To ensure that a wiggler and triplet cell is matched a numerical optimization of the initial transverse conditions of the bunch and the quadrupole strengths in the triplet was carried out with particle tracking through one cell in OPAL-FEL.

The main concerns related to use of the chicanes in the lattice are the nonlinear compression terms and coherent synchrotron radiation (CSR). Because of a small R_{56}, the weak dipoles are used and the overall impact of non-linearity and CSR is small. Indeed, the growth in energy spread in the chicane after the second amplifier unit for a bunch with peak current of 165 A and with an initial modulation wavelength of 2.88 µm, where the gain in the second unit is close to 80, is around 7 %. The simulations for the chicane were carried out using the particle tracking code `elegant` [5]. The resulting matched beam with lattice overlaid is shown for a three-cell unit in Fig. 1. For simulations involving more

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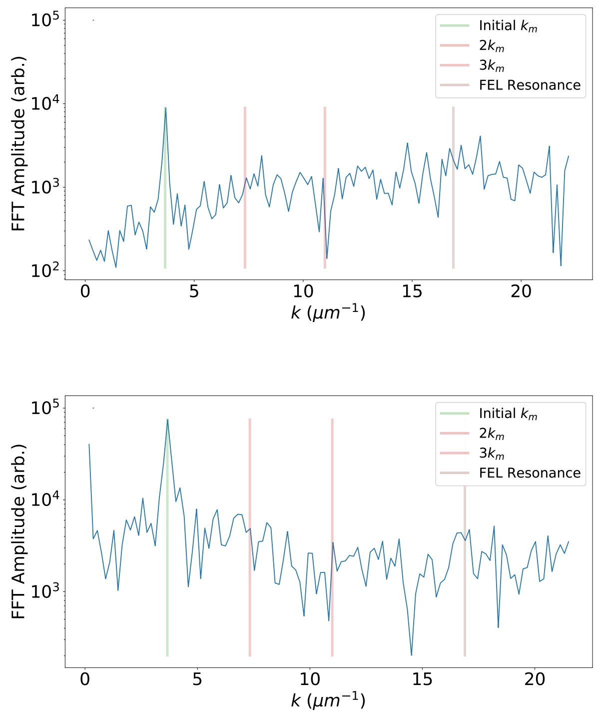

Figure 2: Peak current spectrum of a bunch initially modulated at a frequency with the wave vector $k_m = 3.67\,\mu\mathrm{m}^{-1}$ show after one wiggler (top), after three wigglers and the chicane (bottom).

than one unit the matching between units was performed with a zero-length matrix transformation.

SASE FEL RESONANCE

It was found that it is important to select the wiggler parameter $K_w = 1.5$ such that the range of the microbunching spectrum utilized in the process of hadron cooling is below the SASE FEL resonance wave vector k_{sase}=16.89 $\mu\mathrm{m}^{-1}$. For initial density modulation with the wavelengths nearby the FEL resonance the gain was observed to drop considerably. An additional consideration, even when the resonance lies outside the region of interest on the gain spectrum, is to ensure that the microbunching at this wave vector does not grow much beyond the noise. This is one of the reasons to divide the WEPA lattice into a set of cells with several wigglers followed by a chicane. Passage through the chicanes partially destroys the accumulated microbunching growth due to incoherent energy spread in the electron beam.

Shown in Fig. 2 are the current spectra for the bunch after the first and third wigglers. The gain between wiggler one and three of the FEL resonance is approximately 1.5. After the chicane, the amplitude of the resonance is reduced by 20 %.

SIMULATION SETUP

To study gain in a systematic fashion the longitudinal distribution of the electron bunch is prepared with a core of uniform density n_0. Added on top of this is a small sinusoidal density modulation with amplitude of Δn. So that the density in the core is $n_C(z) = \Delta n \sin kz + n_0$ for a modulation wave vector of $k = 2\pi/\lambda$. To ameliorate the impact of space charge from the finite length of the bunch the tails of the longitudinal distribution are sampled from the error function $\mathrm{erf}(z/\hat{\hat{z}})$ where $\hat{z}$ is typically several tens of micron long. This results in a piece-wise density function for the two tails and core. To sample from this distribution the cumulative distribution function (CDF) for this piece-wise density function is numerically calculated on a uniform grid of 10 000 points. The inverse CDF is then obtained by interpolating the inverse of the CDF grid values using a third-order spline interpolation. Samples are then drawn from a Halton sequence [6] on the unit interval and evaluated with the inverse CDF to complete the longitudinal distribution creation.

We also employ a "tail refresh" procedure whereby the tails of the bunch are cut off after each wiggler and replaced with fresh tails draw from the same erf distribution as described above. The amplitude of the distribution is matched to the edges of the core on either end to ensure that no significant jumps in the current are introduced. This procedure allows even badly distorted tails to be replaced in an automatic fashion. This allows the impact of the core from spurious space charge forces at the bunch edges to be reduced and improve the accuracy of the gain measurement.

GAIN DETERMINATION

To measure gain particles in the tails of the longitudinal distribution are cut, leaving the bunch core. The longitudinal distribution is binned, with bin size typically chosen such that the resonant frequency is outside the Nyquist limit and cannot be resolved. Determination of the gain relies on the numerical analysis of fundamental frequencies (NAFF) algorithm [7] as implemented in the sddsnaff [1] and NAFFlib [8] libraries. The NAFF algorithm, in general, begins by taking the fast Fourier transform (FFT) of the signal to find the approximate frequencies of interest. The frequencies f and their corresponding amplitudes may then be resolved to high accuracy by maximizing the Fourier integral:

$$\phi(f) = \left\langle \psi(t), e^{i2\pi f} \right\rangle = \frac{1}{T}\int_0^T \psi(t) e^{-i2\pi f t} x(t) dt, \qquad (1)$$

where $\psi(t)$ is the time domain signal of interest and $x(t)$ is a windowing function. For both sddsnaff and NAFFlib the Hanning window was used. Finally, to get the fractional modulation depth the frequency component amplitude from FFT or NAFF must be divided by the average density.

[1] https://ops.aps.anl.gov/manuals/SDDStoolkit/SDDStoolkitsu63.html

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WEPA GAIN SIMULATION RESULTS

Base electron bunch parameters used in simulation are given in Table 3. Grid and macroparticles parameters are given in Table 4.

Table 3: Electron Beam Parameters

Parameter	Value	Unit
Beam energy, $\mathscr{E}_0$	157	MeV
Peak current	50, 100, 165	A
Relative energy spread, $\sigma_{\mathscr{E}}$	$2{\times}10^{-4}$	
Normalized emittance, ϵ_x/ϵ_y	2.2, 2.2	µm
Average beta-function, β_x/β_y	0.75, 0.84	m
rms beam size, σ_x/σ_y	72, 77	µm

Table 4: Simulation Parameters

Parameter	Value	Unit
Domain size, L_x, L_y, L_z	1700, 1700, 90	µm
Grid dimensions, Δ_x, Δ_y, Δ_z	10, 10, 0.0286	µm
Macroparticles, N_p	8×10^6	

Here we present the results from simulations of the microbunching amplifier with two amplification units. Each unit comprises a series of three identical wigglers joined by matching triplets. After each set of wigglers a series of two chicanes is used to convert the developed energy modulation into density modulation. Simulations use tail-refresh method and tracking through the chicane with CSR included. The chicane section was simulated in `elegant` and includes nonlinear terms in the transport and CSR.

In simulations the gain was observed to saturate after three wiggler sections. This result differs from theory, which predicts that gain should steadily increase with longer (more) wigglers. Since this saturation was consistent across simulation parameters the choice was made to use units of three wigglers for multi-unit simulations, to reduce compute time.

Figure 3 shows the microbunching gain in simulations using the electron bunch with the flat-top peak current of 165, 100, and 50 A.

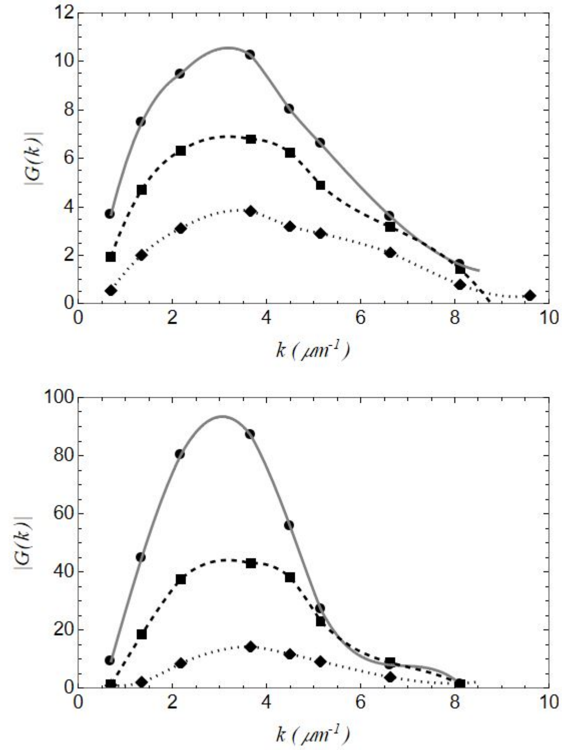

Figure 3: Gain after amplifier Unit 1 (top panel) and after amplifier Unit 2 (bottom panel) calculated using 165, 100, and 50 A peak currents. Data points are connected with solid, dashed and dotted lines for convenience.

CONCLUSION

Gain in density modulation has been studied through simulations using the OPAL-FEL code for the wiggler-enhanced plasma amplifier. Care was taken to control for nonphysical numerical effects and limitations in the simulation to achieve accurate determination of the gain as a function of the wavelength of initial density modulation. These results will be compared with the analytical calculations in a forthcoming publication.

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